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## Systemic Cycles of Accumulation and Chaos in the World Capitalist System: A Missing Link

Giorgos Galanis , Christian Koutny and Isabella Weber

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Giorgos Galanis[1], Christian Koutny[2], Isabella Weber[3]

## Abstract

We re-examine the Systemic Cycles of Accumulation (SCA) of Arrighi (2010) and Arrighi and Silver (1999) which provide a framework for the analysis of the cyclical patterns of geographical expansion of trade and production and the related shifts of hegemonic power within the world capitalist system. Within the SCA framework, the last stage of a hegemonic cycle is characterized by what is called 'systemic chaos' the drivers of these chaotic socioeconomic dynamics have not been explicitly analyzed. This article fills this gap by providing a link between the accumulation process, and systemic chaos. Our approach provides the missing detailed understanding of how systemic chaos is an outcome of the contradictory socioeconomic dynamics of capital accumulation itself while being based on the key insights of the SCA framework of hegemonic cycles.

**Keywords:** chaos, socioeconomic dynamics, hegemony, spatial fix, systemic cycles of accumulation

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[1] School of Business and Management, Queen Mary University of London
[2] Kingston University
[3] Department of Economics, University of Massachusetts, Amherst

# 1.    Introduction

A rich literature within comparative and historical sociology has focused on hegemonic transitions as a theoretical lens to interpret geographical and socioeconomic reconfigurations (e.g. Cox, 1987; Frank et al 1996; Gilpin, 1981; Gunitsky, 2014; Peet, 2002). This has obtained new relevance in the context of the ongoing shifts in the global economic order. The influential work of Arrighi (2010) and Arrighi and Silver (1999) on the theory of Systemic Cycles of Accumulation (SCA) provides an analytical framework where historical capitalist development is explained by hegemonic cycles that emerge from internal contradictions of capital accumulation which continuously evolve as capital overcomes socioeconomic limits to accumulation. The accelerating disintegration of the global political and economic order raises the question of the stability of the current world system with new urgency. In Arrighi and Silver's framework, the immanent instability of the present juncture can be characterized by what is known as chaos, a state of a complete lack of order. However, the concrete link between the drivers of this state of disorder and the general processes which define the SCA remains loose. In this paper we show that by expressing the SCA accumulation process through a logistic map, we are able to consistently replicate the different SCA phases and establish the link between chaos and the rest of the SCA dynamics.

The notion of 'chaos' has been introduced to the study of world systems by Abu-Lughod (1989) who compared world systems with weather systems; while Rosenau (1990) applied the concept of chaos to changes in world politics. Arrighi (2010) and Arrighi and Silver (1999) build on these previous contributions and introduce the concept of 'systemic chaos' to capture systemic behavior during hegemonic transitions and the related spatial re-organization of trade and production within the world system. Both from a mathematical, and a political economy viewpoint, what is most interesting is not the observation that there are chaotic dynamics as such, but to develop an understanding of the specific forces and their interaction which drive the chaotic behavior of a system (which can be physical, social, political, economic etc). Identifying the drivers of chaos has been a key focus in the research of Rosenau (1990), Arrighi (2010), as well as Arrighi and Silver (1999). They do so by studying hegemonic shifts in the past and highlight that their 'investigation has sought clues as to what [the] underlying patterns [of the chaotic world] might be in the present turbulence by uncovering the underlying patterns in comparable past instances of systemic change' (Arrighi and Silver, 1999, 22).

Arrighi and Silver's analytical framework known as Systemic Cycles of Accumulation (SCA) produces a repeated pattern in the dynamics of the accumulation of capital where an initial phase is characterized by stable accumulation concentrated around a specific geographic location which is followed by a transition characterized by a spatial reorganisation where the system exhibits turbulent dynamics. The increasing geographical expansion and intensification is on one hand able to overcome existing barriers to capital accumulation but

at the same time further destabilises the capitalist system as a whole. The end of a SCA overlaps with the end of a given hegemonic center and the birth of a new one starting with stable expansion. This cyclical behavior is shown in the graph from Arrighi (2010) reproduced below.
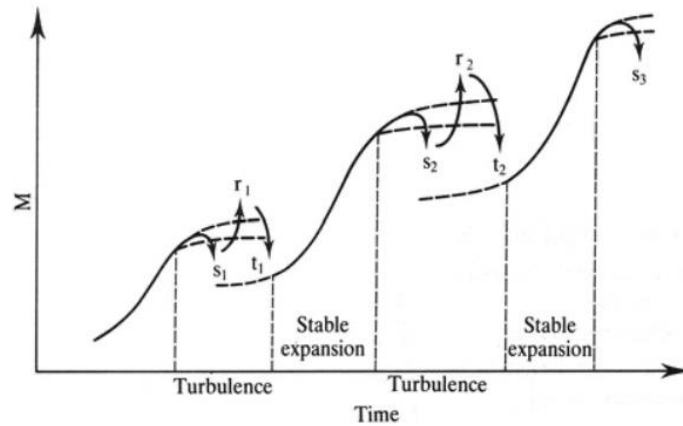
Figure 1



Figure 1: This is figure 3.10: "Metamorphosis Model of Systemic Cycles of Accumulation" from Arrighi (1994)

As Arrighi (2004, 2007) also argues, the SCA framework shares some of the key insights of Harvey's (1981, 2003), theory of 'spatio-temporal fixes' and 'switching crises' according to which spatial expansion and geographic reorganization can *temporarily* overcome crises which are the outcome of so-called barriers to accumulation. But this spatial expansion at the same time creates new crisis tendencies. The fact that the spatio-temporal fix does not provide a permanent solution provides a useful intuition regarding why according to the SCA approach, the process of accumulation is turbulent. However, even when we add Harvey's insights, it remains open what the precise dynamic is, that forges the path from turbulence to chaos. This leads to the more general question of whether there is a theoretically consistent way to add this missing link between turbulence and chaos to the SCA framework.

By linking the SCA framework to the logistic map, we show that the foundational socioeconomic dynamics described by Silver and Arrighi in relation to the initial phases of global capital accumulation can indeed give rise to chaos even in a strict mathematical sense of this term. More precisely, in the theory of SCA each long cycle is composed of two phases: The first phase (A-phase) exhibits stability while the second one (B-phase) is characterized by turbulence and geographic shifts. The B-phase also corresponds to a period of expansion of financial activities across sectors and space where financial markets become the dominant source for profits.[4] We show that the B-phase can transition into chaos if the limits to capital accumulation are pushed beyond a certain threshold, in other words, if financialized capitalism overexpands.

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[4] Arrighi (2010) also refers to this financial expansion as 'financialization'.

The logistic map is not only an elegant mathematical choice for our purpose of establishing a link from turbulence to chaos due to its simplicity. The logistic map also resonates directly with Arrighi's (2010) own graphic depiction of the accumulation process as illustrated in the figure above. The logistic map represents the evolution of a variable over time, where the specific dynamics of the process depend on the value of a what is called a bifurcation parameter. Arrighi and Silver (1999), too, refer to a bifurcation point at which the world system changes its behavior qualitatively. In our interpretation of the logistic map, the bifurcation parameter corresponds to barriers to capital accumulation. More specifically, the dynamics of the bifurcation parameter are such that, as the capitalist system overcomes barriers to accumulation through a variety of expansions which can take social, political, financial technological and geographic forms, the value of the parameter increases. As the value of the parameter grows, the dynamic behavior of the whole socioeconomic system changes. These changes occur in distinct phases in Arrighi and Silver's theory as well as in the logistic map. As a result, we can show that the barriers to accumulation and the related push to overcome these limits, can be systematically interpreted using the logistic map and can be linked to the phases in the SCA. The last phase characterized by chaotic dynamics – as discussed in Arrighi (2010) and Arrighi and Silver (1999) – now arises directly from the preceding turbulent phase.

In addition to working out the concrete link between chaos and the rest of the dynamics characterising the SCA, our work makes a number of additional contributions within the broader field of socioeconomics. First and foremost, our reinterpretation of SCA provides a framework to integrate different layers of social change (e.g. territories, place, space, time, scale, social, economic (Jessop et al, 2008) and provides a language to interpret the non-linear behavior in socioeconomic change in the context of hegemonic shifts. Within this approach, we strengthen the systemic link between SCA, spatio-temporal fixes and switching crises. Arrighi (2007) points to the synergies between the SCA framework and Harvey's spatial analysis. This paper aims to provide a step towards a systematic integration of these two foundational contributions through a careful analysis of their relationship in the different phases of cycles of accumulation.

Second, while our approach provides an analytical link between the historical patterns analysed within the SCA framework, we do not claim that the outcome of the process that we analyse is predictable or predetermined. We show how the logistic map can be used as an interpretative tool, that can help us clarify the phases and transitions in the SCA theory on its own terms. The way in which the bifurcation parameter moves is exogenous to the logistic map and depends on political and socioeconomic processes that are historically contingent and shaped by human agency. If the barriers of accumulation are overcome as a result of these socioeconomic processes determined outside our formulation, the logistic map can provide us with a heuristic to interpret the dynamics that arise. As a result, our observation is not only confirming the consistency between the rise of systemic chaos and the accumulation process in SCA but also reformulates the B-phase as being composed of two distinct sub-phases – turbulence and chaos – that prepare the hegemonic shift.

The structure of the rest of the article is as follows. The next section presents the key points which define the SCA framework and its connection to the spatio-temporal fix, and introduces the logistic map and its bifurcation parameter, relating the latter to the spatial, social and economic limits of accumulation. We argue that the logistic map, is an appropriate representation of the general intuition regarding the (over)accumulation dynamics in the SCA. In Section 3, we show how the increase of the value of the bifurcation parameter to overcome accumulation limits corresponding to spatial shifts in production and trade activities, leads to different phases which correspond to the different phases in SCA, including the chaotic one. The last section provides a concluding discussion.

## 2.      Systemic Cycles of Accumulation

The first two phases that we observe in the logistic map perfectly fit the 'A' and 'B' phases of the SCA framework. We argue that this chaotic phase, 'endogenously' evolves from the dynamics of turbulence: capital continues to further extend the various forms of expansion in the turbulent phase to overcome new barriers to accumulation. In this process, turbulence eventually gives way to chaos. Thus, turbulence and chaos should be seen as two *sub-phases* within the B-phase of capitalist expansion of various socioeconomic forms. In the phase of turbulence, the role of the spatio-temporal fix as one of the ways in which limits to accumulation are overcome becomes more prominent and a switching crisis is set in motion. Nonetheless, the hegemon remains initially dominant as the financial center of the capitalist system. In the chaos phase these two processes – the financial dominance of the outgoing hegemon and the dominance of the emerging hegemon in production – come into conflict. This conflict breeds chaos and the accumulation process becomes erratic as the global socioeconomic order starts to disintegrate. This prepares the geographic shift in hegemony. As a result, our observation is not only confirming the consistency between the rise of systemic chaos and the accumulation process in SCA but also reformulates the B-phase as being composed of two distinct sub-phases – turbulence and chaos – that prepare the hegemonic shift.

## 2.1 General Framework

In the *Long Twentieth Century* Arrighi (2010) presents an analysis of hegemonic cycles in the capitalist world system. For Arrighi, '[t]he concept of "world hegemony" … refers specifically to the power of a state to exercise functions of leadership and governance over a system of sovereign states' (ibid., 28). This power is derived at least in parts by the capacity of the hegemonic state to lead the world system in a way that is perceived by other states as being not only in the interest of the hegemon but also conducive to a more general interest (Arrighi, 2007, 149). SCA drive hegemonic cycles and rest on Braudel's (1984) notion of

reoccurring financial expansions.[5] In such reoccurring financial expansions, profits which are initially ploughed back into the further expansion of trade and production but eventually cannot be invested without jeopardising profit margins, lead capitalist agencies to hold larger portions of their incoming cash flows in liquid form.

Arrighi argues that over the last 700 years the capitalist world system has experienced four different SCA each of which corresponds to a different world hegemon and a different geographic location of the core of the capitalist world system – the Italian city states, the Netherlands, Great Britain and most recently the United States. SCA start with an initial material expansion and endogenously lead to financial expansion where large sums of capital are transferred to the rising global center of accumulation, providing the economic foundation for the hegemonic transition. Importantly, each of the Dutch, British, and US hegemonies have been the outcome of a competitive struggle for world leadership. This struggle creates the conditions for chaos and hegemonic crises, gives rise to new SCA and new hegemons, and also creates an ever-greater global concentration and consolidation of economic and political power. Each cycle exceeds the previous one in terms of the mass of accumulated capital and geographical size as well as complexity of organization and integration of previously externalized processes.

According to Harvey (1981) geographical expansions of financial and commercial capital provide a "spatial-fix" allowing to temporarily overcome economic crises related to low profitability. Importantly, Harvey (1981, 2003) does suggest that the spatial fix is linked to the permanent process of capital accumulation and hence is not just a spontaneous solution to acute crises related to the accumulation process. The link between the spatial-fix and finance is spelled out in *The New Imperialism*, where Harvey (2003) includes the temporal aspect of the spatial-fix and coins the term "spatio-temporal fix". A switching crisis constitutes the acute form of the spatio-temporal fix where capital is devalued in one place and future income thus starts to flow from other geographic regions (Harvey, 2003, 121-122). There are clear overlaps between the SCA, spatio-temporal fixes and switching crises. In fact, Arrighi (2007) points to the link between Harvey's concept and SCA. However, he refrains from a systematic incorporation of his and Harvey's conceptualizations. We aim to do so here, and show that by consolidating them, the transition from tranquillity to chaos, can also be supported coherently within this expanded framework.

Arrighi (2007, 25) takes inspiration from Adam Smith's concept of the extent of the market as limiting the degree of the social division of labor[6] and as he points out, rather than seeing the economic independent of the political, Smith had built his analysis on Hobbes' insight that wealth and power are inseparable. Arrighi (2005a, 2005b) also stresses similarities to Arendt (1958) who links the accumulation over and above what can be invested profitably to a spatial-expansion of capitalism. There, "superfluous men" and "superfluous capital" search

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[5] Also see Arrighi (2001).

[6] See Weber (2019) for a detailed analysis of the dynamic co-evolution of the extent of the market and the division of labor in Smith.

for employment abroad as a solution to the internal contradictions that created that surplus. The idea of such an 'overaccumulation crisis' has roots both in Smith and Marx[7] and is fundamental in SCA. The fact that "[o]nly the unlimited accumulation of power could bring about the unlimited accumulation of capital" (Arendt, 1958, 137) highlights the significance of political organization for the expansion of capital.

Due to the key role of accumulation, each of the different phases of an SCA should be understood in relation to the dynamics of the circuit of capital; the process from a monetary investment via production and trade to a monetary revenue that exceeds the initial investment yielding a profit for the capitalist. In SCA this general formula is reinterpreted as a general theory of the organization of society and the evolution of the world capitalist system as a whole. Thereby, the circuit of capital serves as a lens to analyze the historical development of the capitalist system while linking to Marx's theory of crisis and falling rate of profit. Importantly, each systemic cycle of accumulation corresponds to a specific political, geographic and economic configuration of the world capitalist system. In each cycle, the initial phase of material expansion eventually culminates in a conflict between a falling rate and a growing mass of profit as the limits of accumulation under a given political and economic configuration are approached (Arrighi, 2010, 232-233). This generates a drive towards an intensified spatio-temporal fix as well as a switching crisis signalling a hegemonic transition. But rather than leading immediately to a collapse of global hegemony, a financial expansion creates a *belle époque* where the declining global economic center switches to financing an economic expansion in a different geographical location. Now, profitability is temporarily restored and the limits to accumulation of capital are pushed back further.

With the framework we establish in this work we are able to explicitly show how the SCA framework effectively demonstrates that the spatial and institutional barriers of the world market are constantly overcome through geographical expansion and reorganization of the system as such, *as a result* of dynamics originating from the accumulation of capital *itself*.

## 2.2 The role of chaos

The phase of SCA in which a spatial relocation and intense financial expansion pushes the socioeconomic accumulation barriers, signals the beginning of the transition from one hegemon to the next. This transition is characterized by what Arrighi calls *systemic chaos*. More specifically "'[c]haos" and "systemic chaos," […] refer to a situation of total and apparently irremediable lack of organization' (Arrighi, 2010, 31). Furthermore, he provides a summary of possible channels through which *systemic chaos* can emerge: "It is a situation

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[7] According to Smith there is a general tendency for profits to fall when too much capital is competing as a result of general overaccumulation. This drives up wages and squeezes profits. Another reason for falling profits is overproduction: Marx suggests that when wages are too low, aggregate demand is falling short of supply pushing down profits. For a discussion on the tendency of the profit rate to fall see Foley (1986) and Fine and Saad-Filho (2016) among others.

that arises because conflict escalates beyond the threshold within which it calls forth powerful countervailing tendencies, or because a new set of rules and norms of behavior is imposed on, or grows from within, an older set of rules and norms without displacing it, or because of a combination of these two circumstances." Simply put, it is a situation of escalating conflicts and a collapse of predominant rules and norms. Arrighi highlights the role of systemic chaos to hegemonic transition: "As systemic chaos increases, the demand for 'order' – the old order, a new order, any order! – tends to become more and more general among rulers, or among subjects, or both" (Arrighi, 2010, 31).

Note that the quote above provides both a definition of (systemic) chaos through its characteristic of "total and irremediable lack of organization" and also describes the possible channels (conflict and lack of rules and norms) that can lead to chaos and through this to a hegemonic transition and a geographic reorganization of the world system. Even though the relationship between (systemic) chaos and the notion of chaos in mathematics and physics is not explicitly discussed by Arrighi (2010) or Arrighi and Silver (1999), the property of chaotic systems to appear as random or unpredictable can be related to what they refer to as a "lack of organization". More specifically the characteristic of apparent unpredictability is sufficient to prevent a system from being organized. Given the role of accumulation within the SCA framework, the lack of organization is necessarily linked to the accumulation process: a lack of order and organization undermines stable capital accumulation and leads to chaotic accumulation dynamics.

## 2.3   Logistic map

Arrighi (2010, 1996) and Arrighi and Silver (1999) see the SCA as a theoretical project that aims to show how the evolution of the world capitalist system through the succession of different hegemons is not random but can be explained through an analysis of world capitalism as a *historical social system.* Thereby, the ambition is for "This reconstruction [to] proceed gradually through a comparative analysis of successive systemic cycles of accumulation and through heavy borrowing from whichever theoretical construct can provide the most plausible and parsimonious explanation of the observed patterns." (Arrighi, 1997, 159). Following this same approach, we assume the *most plausible parsimonious* functional form that can explain the observed patterns.

We represent the accumulation process through what is known as a *logistic map* which provides a simple expression able to replicate different types of behaviors relevant for the SCA analysis. Let $x_t$ be the level of accumulation at time $t$, then accumulation dynamics are represented through a logistic map as

$$x_{t+1} = r\, x_t\, (1 - x_t),$$

where $r$ is a parameter which defines both how fast capital accumulation grows at a given point in time but also the limit that the value $x_t$ can take (in the long run). This rate of growth

is determined outside the above equation as a result of socioeconomic processes – as we already pointed out in the Introduction. As we discuss in more detail below, for relatively low values of $r$, this mapping captures an S curve (logistic) evolution of $x_t$, which is characterized initially by fast growth and then by a slowdown, while for higher values of $r$, the evolution becomes very different. More specifically, there are threshold values for $r$. When the threshold values are crossed the behavior of the process changes *qualitatively*, or put differently a *bifurcation occurs*. This means that $r$ is a bifurcation parameter the value of which defines the qualitative behavior of the process – here that of accumulation. The value of the parameter itself is, however, determined outside the system of equations. Hence, we can link socioeconomic processes that correspond to changes in the value of the bifurcation parameter to qualitative changes in the behavior of the system as a whole.

While the logistic map can lead to chaos, it is far from clear whether this is a good representation of the SCA accumulation process. In order to show this, we need to derive two things in parallel. On the one hand we have to demonstrate that the different stages within the SCA phases correspond to the different *bifurcations* (or changes in qualitative behavior of the process) of the logistic map as the bifurcation parameter grows. On the other hand, we need to show that the reason for which the bifurcation parameter grows is also consistent with the SCA framework. In the next subsection we present the main intuition regarding the latter. Building on this in the following section we will show the former.

## 2.4 Profitability and the bifurcation parameter

The SCA analysis rests on the notion of a falling rate of profit which is in contradiction to the simultaneous growth of the mass of profits. Considering that there is initially an exponential increase[8] of cash-flows to be absorbed into the circuit of capital, each SCA eventually reaches the socioeconomic and geographical limits of what can be reinvested profitably. Approaching those limits decreases the average rate of profit and intensifies competition for the more profitable outlets. A nevertheless increasing mass of profits exacerbates this problem, accelerating the speed at which the limits are approached.

As it is shown in the reproduction of figure 3.7 from Arrighi (2010) in Figure 2, the accumulation process takes an S-shape logistic form in the SCA framework, where the returns to scale are initially increasing (A-phase) and then decreasing (B-phase). In the S-curve the world system reaches the second near-horizontal stage that marks the crisis of accumulation which signals the nearing end of a hegemonic cycle.

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[8] Harvey (2021) demonstrates why the law of compound growth can consistently be applied to the general formula of capital and that therefore capital grows exponentially. However, independent of that argument, economic output and indeed the stock of capital has grown exponentially over the last 700 years which is the period of SCA.
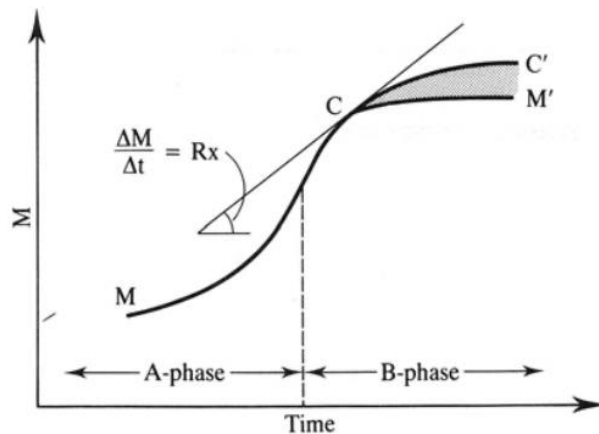
Figure 2



Figure 2: this is Figure 3.7 from Arrighi (1994)


The slope $\frac{\Delta M}{\Delta t}$ captures how fast accumulation grows, which Arrighi calls the rate of return on the stock of capital invested in trade. In other words, the slope $\frac{\Delta M}{\Delta t}$ captures the rate of profit at any point in time and at any given level of accumulation. Hence, in the A-phase the profit rate is rising, but in the B-phase it is falling. In other words, for higher values of M (or $x_t$ in terms of the logistic map), the slope of $\frac{\Delta M}{\Delta t}$ will become increasingly less steep, capturing the tendency of the rate of profit to fall. The mass of profits will continue to grow up to a point where the slope of the curve is horizontal. According to the logistic map representation $x_t$ corresponds to M, and $r$ captures the slope Rx (or $\frac{\Delta M}{\Delta t}$) and through this the maximum level of accumulation.[9]

The value of the bifurcation parameter depends on the acuteness of the barriers related to accumulation. Thus $r$ can be thought as a parameter capturing the force capital can employ to overcome the accumulation barriers at any point in time which is the result of complex social and political processes. $r$ extends the expansion of capital and increases as the geographical and organizational efforts are increased. Implicitly, it also captures the proximity and severity of the socioeconomic barriers to accumulation.

This parameter, does not only define the speed and maximum level of accumulation, but as we discuss in the next section in detail it also characterizes the dynamics of the process itself. An increase of $r$ at some point $x_t$, will be able to change the slope corresponding to the rate of return of the stock of capital at that point and hence increase the rate of profit; but as long

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[9] Even though there is a correspondence between M in Figure 2 and $x_t$, we use $x_t$ instead of M in order to highlight that we do not offer a general theory of the accumulation process in SCA (or in Marx) but rather that the simple logistic map can capture the key SCA dynamics.

as this increase is relatively small, it will not change the general shape of the curve. This means that while the increase of $r$ may allow for a relaxation of the severity of the crisis of accumulation, the overall tendency does not disappear. Given that $r$ also captures the spatial extent of the accumulation process, based on the above we can see why the fix towards overcoming accumulation barriers is spatio-temporal as in Harvey (1981, 2003).

In sum, the crisis of accumulation that occurs towards the end of the A-phase triggers a systemic push to overcome the barriers to capital accumulation and leads to a bifurcation in the start of the B-phase, as shown in Figure 2. However, overcoming the fall of profitability does not last for long as the slope will change again becoming less vertical leading to a continuous need for $r$ to increase. In the next section we show how this increase in the bifurcation parameter leads to qualitative changes capturing the different stages of the SCA. Furthermore, we show how chaos endogenously follows the turbulent stage within SCA as the value of the bifurcation parameter increases.

# 3.    Stages within SCA

## 3.1 Stable accumulation

The stable accumulation phase begins with initial investments that establish a "particular bloc" of agencies and rules that form a new hegemony with a new geopolitical center. In this phase capital shapes the new geographical space in its image and subordinates it to its laws and needs. In this initial part of the stable phase the profit rate is still low (this corresponds to the initial horizontal part of the S-curve shown in Figure 2). But once hegemony is established a virtuous cycle of an ever-increasing division of labor is kicked off (this is the first part of the S-curve where the slope is higher than one[10]). However, this virtuous cycle is not without limits and a continuous expansion will sooner or later lead to a situation where profit margins will be under threat.

Profitability, the very force of the initial expansion, eventually creates the conditions that set an end to the process: "Decreasing returns set in; competitive pressures on the system's governmental and business agencies intensify; and the stage is set for the change of phase from material to financial expansion." (Arrighi 1996, 155). This corresponds to the B-phase of the curve where the gradient is less than one and the geographical expansion is intensified. In order to be able to show the correspondence with the logistic map, we present two cases with relatively low values of $r$. In Figure 3 For $r = 1.2$ while in Figure 4, $r = 2$.
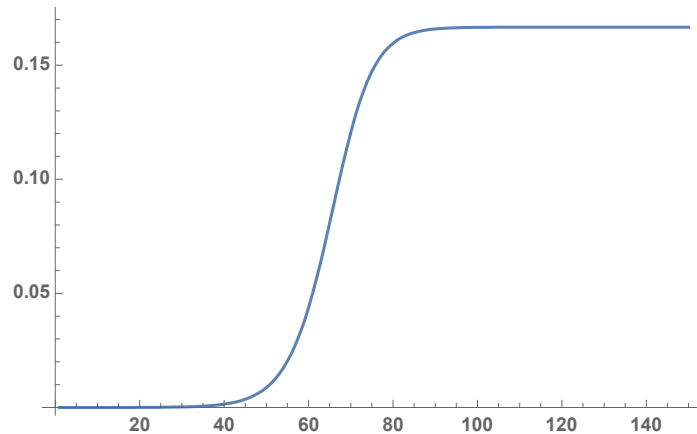
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[10] Or more than 45 degrees.

Figure 3



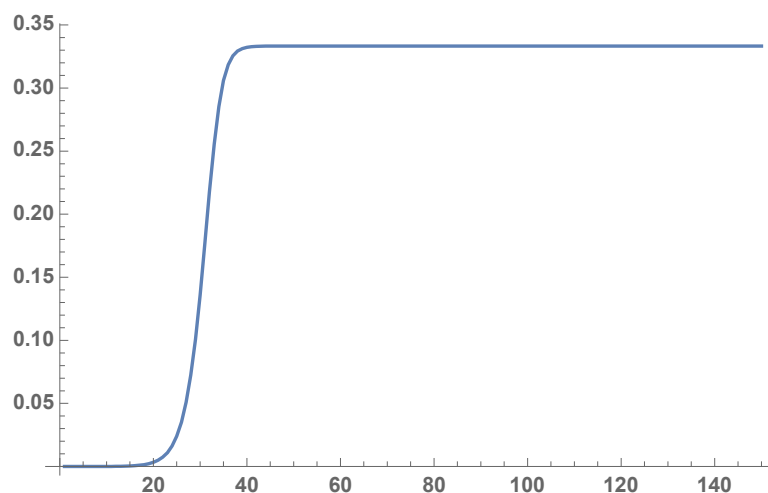Figure 3: The material expansion phase of stable accumulation, $r = 1.2$

Figure 4



Figure 4: The initial financial expansion phase, r=2

Both graphs show a logistic (S-shape) accumulation process as the one shown in Figure 2. While in Figure 3 of the initial material expansion phase the maximum value of for $x_t$ is around 0.17 and reaches the maximum value in around 40 time steps, in Figure 4 the maximum value is around double, while this is reached in around 20 time steps. This quick burst of accumulation is when the first barriers to accumulation are overcome but "because higher profits mean an increase in the mass of capital looking for profitable employment and the tendency towards overaccumulation is exacerbated, but now on an expanding geographical scale" (Harvey, 1981, 7), the solution is only temporary and will be more difficult to solve next time.

The material expansion and the financial expansion figures in Arrighi (2010) can be thought of as the two different parts of Figure 2, where the first corresponds to the M-C-M' part and the second to the M-C-C' part of the graph.[11] Thus, the two lines in the B-phase in Figure 2 correspond to two different values of $r$ in the logistic map. We should also mention here that there is no qualitative change in the behavior of the process, hence a bifurcation has not occurred.

Note that the logistic shape of Figure 4 also demonstrates that the tendency for the rate of profit does not disappear as $x_t$ increases over time, but that there is a continuous need for $r$ to increase over time, hence highlighting the temporal aspect of this spatio-temporal fix. Put differently, the logistic shape of the accumulation process implies that the rate of return will be (increasingly) diminishing once $x_t$ crosses a threshold value (corresponding here to the start of the B-phase) but this can be overcome through an increase of $r$ until a new threshold level of $x_t$ and so on.

This highlights a continuous need for spatial expansion in order to overcome accumulation barriers. As Marx puts it: "The tendency to create the *world* market is directly given in the concept of capital itself. Every limit appears as a barrier to be overcome. … [but] … it does not follow by any means that it has really overcome it, and, since every such barrier contradicts its character, its production moves in contradictions which are constantly overcome and just as constantly posited" (Marx, 1973, 408-410)

## 4.2 Turbulent accumulation

The extended expansion is in fact "a sign of 'autumn' (Braudel, 1984, 246)" as Arrighi and Silver (1999, 31) argue drawing on Braudel, and is largely driven by what Harvey (1981, 2003) calls a "switching crisis". As more and more capital from the declining hegemon is channeled into liquid assets the world enters the B-phase of the cycle of accumulation. Finance is no longer grounded in production and the continued expansion gives way to what has come to be known as financialization: As money is exchanged for more money, profits are derived from speculation and accumulation becomes turbulent. At the same time, a new material expansion phase sets off in the ascending parts of the world. This material expansion is initially driven by the spatio-temporal fix and the resulting relocation of production from the declining hegemon but increasingly takes on a life of its own. The interstate and interenterprise competition for more and more mobile capital intensifies. Financialization and intensified international competition give rise to further turbulences. In the beginning, these turbulences are local but as the dynamic unfolds, they eventually become global.

This type of destabilizing dynamics is depicted by Arrighi (2010) in two figures (reproduced below) to capture two types of turbulence: *local* turbulences, which initially increase but

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[11] Arrighi (1996, 157) notes that "every phase of financial expansion is indeed characterized by the emergence of a newly successful MCM' circuit". We can therefore think of MCC' in his graph as a new circuit of capital in relation to the initial MCM'.

eventually fade out when the accumulation curve returns to a stable increasing trajectory; and *systemic* where the turbulence is increasing over time without returning to a stable path. The nature of finance contributes to this intensification and spread of turbulence; finance being inherently international, it distributes capital to wherever profits are highest and thereby undermines the stability of global hegemony. Hence while the type of instability portrayed in the first graph of Figure 5 is temporary the second one, appears to be more long term or even permanent. In the case of systemic turbulence there seem to be no endogenous forces that lead the world system back to a stable path.
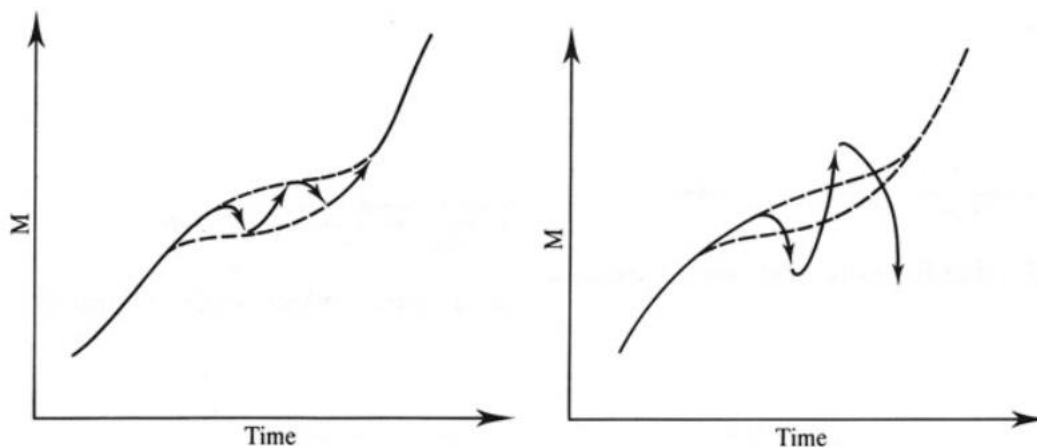
Figure 5



Figure 5: Figures 3.8 "Local Turbulence" and 3.9 "Systemic Turbulence" from Arrighi (1994)

In order to show the correspondence between the behavior captured in Figure 5 and the dynamics generated by the logistic map as $r$ increases, we need to check whether an increase of the value of the bifurcation parameter in the logistic map will first lead to local turbulence and then to systemic turbulence corresponding to the two graphs of Figure 5. The graph in Figure 6 portrays the case of $r = 2.95$ (which is higher than the previous value).
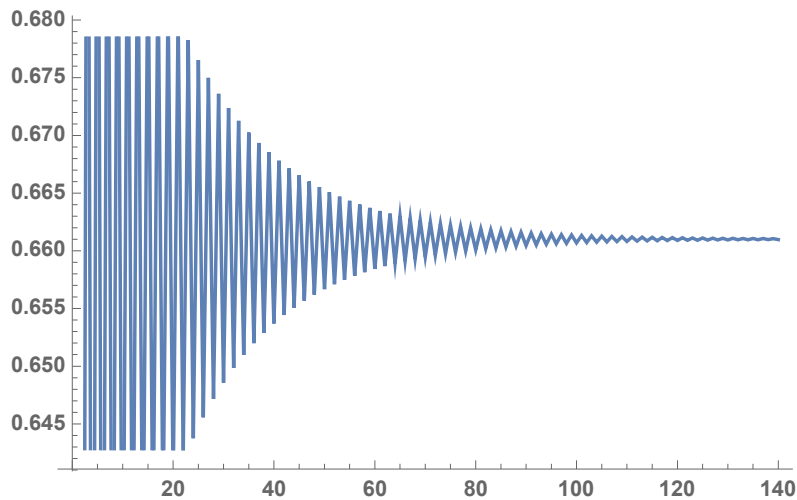
Figure 6

Figure 6: Local turbulence, r=2.95

As we can see, a further increase of $r$ as the barriers to accumulation grow in size and complexity progresses results in turbulent fluctuations that are, however, decreasing over time. This captures exactly the key insights of the first graph of Figure 5 where fluctuations eventually disappear. Importantly, the average value of the level of capital accumulation $x$ (or M in Figures 2 and 5) is almost double compared to the initial extended expansion phase mapped in Figure 4. This highlights the fact that even though local turbulence emerges, the accumulation rate is higher than before, hence this type of "local instability" does not yet provide a serious challenge to the accumulation path or the prevailing hegemon. "If the surpluses of capital and labour power exist within a given territory … and cannot be absorbed internally … then they must be sent elsewhere to find fresh terrain for their profitable realization if they are not devalued. … [But] … [t]he problem of overaccumulation is alleviated only in the short term." (Harvey, 2003, 116-117).

As the geographical expansion progresses further, financialization deepens and capital becomes ever more mobile further intensifying interstate competition. A higher $r$ leads to a behavior which captures the key insights of the systemic turbulence, where fluctuations are not temporary but are persistent over time with no sign of fading. The graph below is for $r = 3.3$.
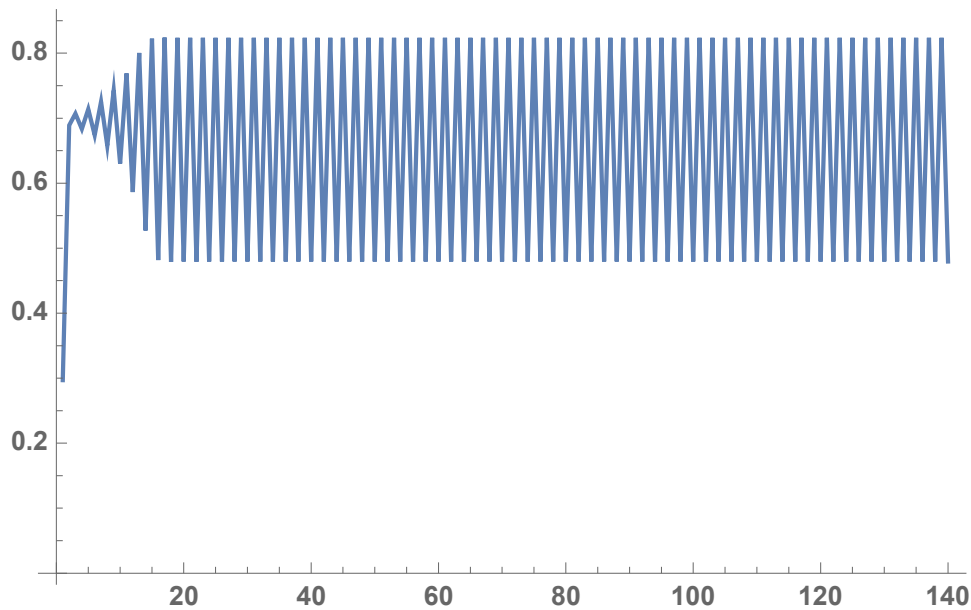
Figure 7

Figure 7: Systemic turbulence, r=3.3

Again, we observe an increase regarding the average value for the level of capital accumulation $x$ but also the variance of the fluctuations is much higher compared even to the maximum variance of the previous example.

These two examples of values for r demonstrate the correspondence between the accumulation dynamics derived through increasing values of the bifurcation parameter of the logistic map and the insights of the SCA process shown in Figure 5. This demonstrates the correspondence between the SCA dynamics and the logistic map.

## 4.3   Chaos

For Arrighi and Silver (1999, 21) hegemonic transition at the end of an SCA is through "a process of radical reorganization of the modern world system that changes substantively the nature of the system's components, the way in which these components relate to one another, and the way in which the system operates and reproduces itself." Based on Abu-Lughod (1989, 369) in such a situation "the 'same-cause-yields-same-effects' logic that underlies our thinking…is ill-equipped to apprehend this kind of change, and we should instead draw inspiration from 'chaos theory'" (ibid.).

While the relationship between chaotic dynamics and hegemonic transition is mentioned in Arrighi (2010), the concrete link is not analyzed. Furthermore, it is not clear whether a possible link would fit with the SCA framework. Despite the systemic turbulences that arise when a continuous expansion reaches a certain threshold, the limits to accumulation are pushed even further as capital encounters further barriers arising from the contradictions set in motion by its previous solution to the previous barriers. Considering that "[t]he entire credit system … rests on the necessity of expanding and leaping over the barriers to circulation and the sphere of exchange" (Marx, 1973; 416), credit facilitates the geographical

expansion of capital, speculation is fuelling more speculation and the limits to accumulation are pushed once again. This corresponds to a further increase in $r$.

However, if $r$ is increased even more (here $r = 4$), the accumulation process takes a chaotic form which is characterized by what looks like randomness (Figure 8). Small changes can lead to very different values of $x$. This means that it is impossible to predict $x_{t+1}$ even if all the previous values of $x$ up to some point in time t are known. This corresponds to Arrighi's characterization of "systemic chaos" as a total lack of organization (Arrighi, 2010, 31).
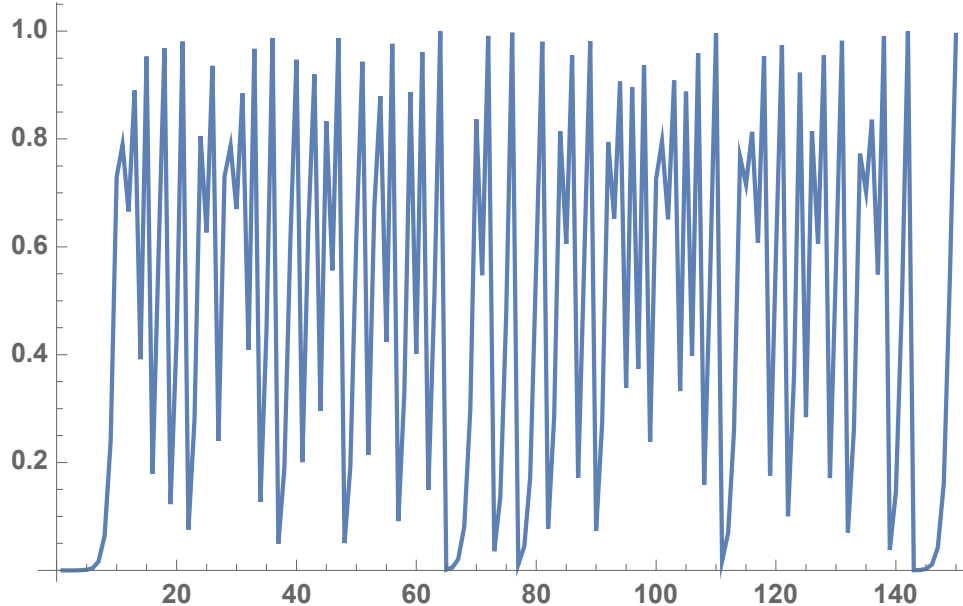
Figure 8



Figure 8: Chaos, r=4

This shows the link between extended expansion and chaos when the SCA is represented by a logistic map. Thus, the use of a logistic map to express the accumulation process underlying SCA along with the theoretical understanding of the fact that spatio-temporal fixes (and switching crises) are only a temporary solution hence there is a continuous need for pushing towards higher accumulation limits. This also means a continuous increase of the value of bifurcation parameter. As chaos appears after the bifurcation parameter crosses a threshold, it

means that chaos is an endogenously created sub-phase following the preceding sequence of stable expansion, local and systemic turbulence.

Our analysis leads to a reformulation of the original SCA graph where the accumulation process is divided into two phases A and B. Using our insight, the B-phase includes two sub-phases. The first corresponds to the original insights of Arrighi (2010) where a stable extended expansion is followed by local and then systemic turbulence; while the second sub-phase corresponds to chaos and means that the hegemon has lost its power and the transition is under way.

The different parts of the B-phase can be presented through the bifurcation diagram of the logistic map below which in this case shows the behavior of the accumulation process for different values of the limits of accumulation bifurcation parameter.
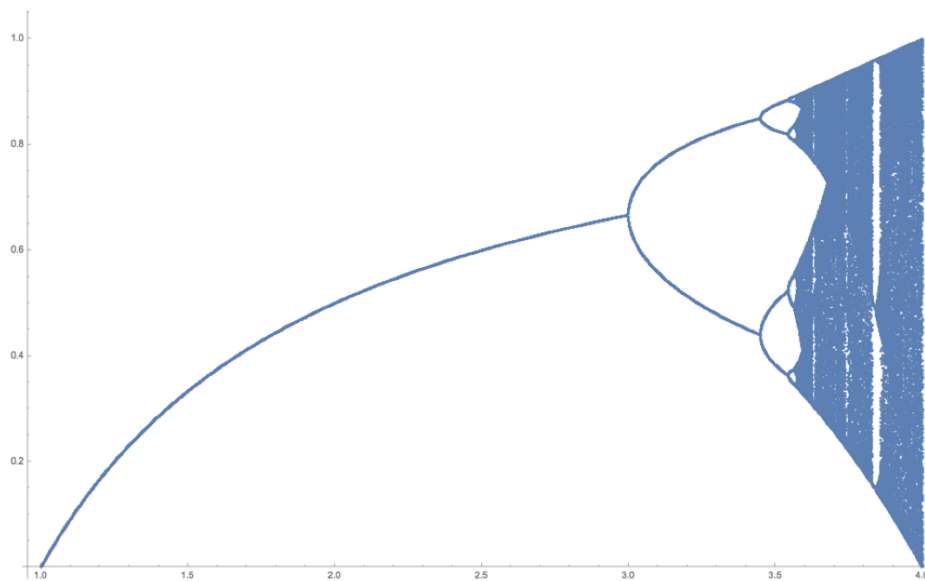
Figure 9



Figure 9: Bifurcation diagram of the logistic map

The horizontal axis represents different values for the bifurcation parameter and the vertical axis is the values for $x$. The blue line in figure 9 shows the values that $x$ takes (or converges to), in the long run for given values of $r$. Note that, given that as we have discussed above $r$, increases over time, the blue line also captures the accumulation process over time. As we

can note, the initial part corresponds to the second part of the logistic (B-phase), where accumulation is slowing down. Up to the point where the first bifurcation occurs (for $r$ around 3), the maximum (or long run) level of accumulation is an increasing function of $r$. This first (smooth) part of the graph captures the dynamics in figures 3, 4 and also 6.

The next part where $r$ is between 3 and roughly 3.45 there is a bifurcation which corresponds to oscillations between two points of the type shown in the graphs representing the turbulent dynamics. The divergence of the two lines in this sub-phase represents the divergence of the points between which $x$ oscillates. Put differently, as $r$ increases the variations also increase. The second sub-phase roughly corresponds to $r$ >3.56 where chaotic dynamics appear and is represented by the blue surface. The two subphases are connected by an intermediate phase where $x$ oscillates between four and eight points respectively.[12] This intermediate phase is short and while the oscillations are non-cyclical they are not formally chaotic.

In the last stage, when $r$ crosses a new threshold, it becomes impossible to predict the value of $x$ and we get chaos which according to SCA signals a hegemonic transition. As Silver and Arrighi (1999, 21-22) point out drawing on Henri Poincaré who coined the term "bifurcation", "the order that will eventually emerge out of the present turbulence… is not inscribed in the parameters of the order that has broken down. But … there is an order within chaos". Chaos is itself the result of an order but the order that follows is not predetermined by that chaos. Cycles of accumulation follow a pattern but the characteristics of the new hegemon cannot be predicted from the preceding one.

# 5 Conclusion

The aim of this paper has been to show that by expressing the dynamics of the global accumulation process at the basis of the SCA through the use of a logistic map, we are able to explicitly place the endogenous emergence of chaos within the SCA phases. Our starting point has been the well-known fact that for low values of its bifurcation parameter the logistic map corresponds to a logistic (sigmoid) graph. Hence, given that the accumulation process underlying the SCA is described by a sigmoid, as the logistic map, for certain parameter values is also a sigmoid, this is a suitable formal representaion. Our analysis was based on three further key observations.

The first is related to two observations the bifurcation parameter. On the one hand higher values of the bifurcation parameter allow for higher levels of accumulation, while on the other there is a constant need for increasing its value, as new accumulation barriers appear. This means that for a capitalist economy where the accumulation process is described by a logistic curve there will be an endogenous push to increase the limits of accumulation and the value of the bifurcation parameter. This increase corresponds to geographic expansion and

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[12] Even if it is not visible here, we know from bifurcation theory that after eight points, the oscillations are between sixteen points.

intensification, hence shows that the parameter has not only a socioeconomic but also a spatial interpretation.

The second observation is related to the exact mapping of the different stages of the SCA with the various dynamics portrayed by the logistic map as the bifurcation parameter crosses certain threshold values. Not only as the parameter increases, we are able to replicate the key insights of the SCA but chaos also follows the turbulent last phase of the SCA.

Third, in order for the hegemon of the world capitalist system to be able to overcome accumulation limits, a geographical expansion and organizational complexification takes place. This provides a spatio-temporal fix to the accumulation barriers, however this fix is only temporary which means that there is a continuous need for an increase of the bifurcation parameter, which highlights how the process that leads to different phases of the SCA framework is endogenous within the capitalist accumulation process.

Hence our analysis shows that exactly because the logistic map can represent the SCA and the spatio-temporal fixes are only temporal, chaos is actually inherent in the accumulation process. Through this approach we have 'extended' the (B-phase of the) SCA framework by explicitly including the chaotic dynamics which are related to hegemonic transition as a 'natural' next step in the accumulation process. The theoretical framework we established consists of a consolidation of Harvey's (1981, 2003) spatio-temporal fix and switching crisis concepts with Arrighi's (2010) and Arrighi and Silver's (1999) framework of Systemic Cycles of Accumulation.

Adding the missing link in this paper leads to new research questions. Possible extensions of the logistic map include for example the explicit incorporation of the different fixes to overaccumulation crises and/or allowing for the explicit role of hegemony and power in stabilizing the accumulation process. Combining the two would be particularly interesting, and would also allow to systematically analyze the role of hegemony and power in stabilizing general expanded reproduction, not just the special case of an acute overaccumulation crisis.

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