# Disappointment Aversion Preferences, and the Credit Spread Puzzle[1]

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**Abstract**

Structural models of default are unable to generate measurable Baa-Aaa credit spreads, when these models are calibrated to realistic values for default rates and losses given default. Motivated by recent results in behavioral economics, this paper is the first to propose a consumption-based asset pricing model with disappointment aversion preferences in an attempt to resolve the credit spread puzzle. Simulation results suggest that as long as losses given default and default boundaries are countercyclical, then the disappointment model can explain Baa-Aaa credit spreads using preference parameter values that are consistent with experimental findings. Further, the disappointment aversion discount factor can match key moments for stock market returns, the aggregate price-dividend ratio, and the risk-free rate.

**keywords:** structural models of default, credit spreads, consumption-based asset pricing, disappointment aversion preferences, credit spread puzzle, recovery rates, default boundaries, stock market

*JEL classification:* D51, D53, D81, D91, E21, E44, G11, G12, G33

*ουκ αν λάβοις παρά του μη έχοντος*

***You cannot receive anything by someone who has nothing***

*"Dialogues of the Dead"*, Lucian ($125 - 175$ *A.D.*)

# 1 Introduction

When traditional structural models of default are calibrated to realistic values for default rates and losses given default, then these models are unable to generate measurable Baa-Aaa credit spreads, an empirical conundrum also known as the credit spread puzzle. Moreover, recent results[1] suggest that state-of-the-art consumption-based asset pricing models cannot rationalize Baa-Aaa bond spreads, even if they can successfully explain equity premia. Nevertheless, a universal stochastic discount that can resolve the equity premium puzzle should also be able to fit credit spreads in corporate bond markets.

Although behavioral theories have been extensively used to explain equity risk premia[2], this is the first paper to address the credit spread puzzle from a behavioral perspective. Towards this objective, I use a general equilibrium model of an endowment economy populated by disappointment averse investors in order to price zero-coupon corporate bonds subject to default. Disappointment aversion preferences were first introduced by Gul (1991), and are able to capture well documented patterns for risky choices, such as asymmetric marginal utility over gains and losses or reference-based evaluation of stochastic payoffs[3]. Moreover, disapppointment aversion preferences do not violate first-order stochastic dominance, transitivity of preferences or aggregation of investors. The disappointment aversion framework can therefore help us shed additional light on the link between credit-spreads and aggregate economic activity while maintaining investor rationality.

Disappointment averse investors are characterized by first-order risk aversion[4] preferences with

[1]Chen et al. 2009.

[2]Epstein and Zin (1990), Bernatzi and Thaler (1995), Barberis et al. (2001), Andries (2011), Piccioni (2011), Easley and Yang (2012), Delikouras (2013).

[3]Kahneman and Tversky (1979), Duncan (2010), Pope and Schweitzer (2011).

[4]Segal and Spivak (1990).

endogenous expectation-based reference points for gains and losses. Due to the linear homogeneity of these preferences, I am able to obtain approximate analytical solutions for the price-payout ratios (price-dividends, price-earinigs) in the economy. This is the first paper to derive analytical expressions for price-payout ratios when investors and disappointment averse and aggregate uncertainty is time-varying. Explicit solutions for price-payout ratios, in turn, facilitate model simulation, and provide valuable intuition. Price-payout ratios are log-linear functions of three state variables: consumption growth, consumption growth volatility, and consumption growth variance.

The main mechanism in place for disappointment aversion preferences is related to asymmetric marginal utility, and the fact that disappointment averse investors penalize losses below the endogenous reference level three times more than they do for losses above the reference level. The disappointment aversion model highlights the interaction between default rates and periods of worse-than-expected aggregate macroeconomic conditions. During these periods there is an upwards jump in marginal utility. Almeida and Philipon (2007) also document that distress costs are most likely to happen during times when marginal utility is high.

Figure I shows Baa-Aaa credit spreads, Baa default rates, and NBER recessions for the 1946-2011 period. Two things become immediately clear from Figure I. First, credit spreads are strongly countercyclical. Second, Baa default rates remain close to zero over long periods of time, and tend to spike up at or after a recession. Through first-order risk aversion, the disappointment model amplifies very small risks, such as the almost zero default risk for Baa firms, and is able to generate measurable Baa-Aaa credit spreads despite the very low default rates.

Although several consumption-based asset pricing models have proposed frameworks that generate credit spreads consistent with empirical observations, with the exception of the habit model in Chen et al. (2009), either preference parameters (eg. the risk aversion coefficient) in these models are much larger than those estimated in clinical experiments[5], or these models cannot perfectly match other asset pricing moments such as equity risk premia[6]. On the other hand, the disappoint-

[5]Chen (2010), p. 2190, assumes a risk aversion parameter equal to 6.5 and an EIS larger than 1.

[6]The equity premium in Bhamra et al. (2010) p. 682 is 3.19%, whereas the sample equity premium for the 1946-2011 period is around 6.5%.

ment model of this paper is calibrated to preference parameter values which are consistent with recent experimental results[7]: the risk aversion parameter is equal to 1.8, and the disappointment aversion coefficient is equal to 2.03.

This paper compliments a growing literature which argues that disappointment aversion preferences are able to address a variety of stylized facts in financial markets such as the equity premium puzzle (Routledge and Zin 2010, Bonomo et al. 2011), the cross-section of expected returns (Ostrovnaya et al. 2006, Delikouras 2013), or limited stock market participation (Ang et al. 2005, Khanapure 2012). Simulation results suggest that as long as losses given default and default boundaries are countercyclical, then the disappointment model can explain the credit spread puzzle, and generate expected Baa-Aaa credit spreads equal to 100 bps for four-year maturities. This is very close to the historical average of 103 bps in Huang and Huang (2012). In contrast, for the discrete time version of Merton's model (Merton 1974), the Baa-Aaa credit spread is equal to 51 bps. Nevertheless, the disappointment model seems to overpredict expected credit spreads for long maturities (15yr+).

Ever since Merton's model, most results on corporate bond pricing (Leland 1994, Leland and Toft1996, Goldstein et al. 2001, Bhamra et al. 2010) rely directly on risk-neutral probability measures for asset returns, while being silent on investor preferences and the stochastic discount factor. In contrast, this paper adds to recent works by Chen et al. (2009), and Chen (2010) who approach the equity premium and credit spread puzzles in a unified manner, explicitly using a universal consumption-based stochastic discount factor across all financial markets.

Taking a stance on the functional form of the stochastic discount factor is particularly important for two reasons. First, we can identify whether a particular set of preferences is able to generate plausible asset pricing moments across different markets. For instance, besides explaining the credit spread puzzle, the disappointment aversion discount factor in this paper matches moments for aggregate state variables, stock market returns, and the risk-free rate. Second, estimates for preference parameters can be compared to recent experimental findings for choices under

[7] Choi et al. (2007), Gill and Prowse (2012).

uncertainty in order to assess the empirical plausibility of the model.

There are many asset pricing models that can efficiently explain stylized facts in financial markets, yet these models usually explain asset prices one market at a time. The strategy of this paper is to impose more discipline on investor preferences, and provide solid micro-foundations for a universal discount factor across different markets by taking into account recent experimental results for choices under uncertainty. These results emphasize the importance of expectation-based reference-dependent utility. The use of disappointment aversion preferences is motivated by strong experimental and field evidence from aspects of economic life that are not directly related to finance[8]. This paper also adds to the relatively limited strand of literature that incorporates elements of behavioral economics into a consumption-based asset pricing model without violating key assumptions of the traditional general equilibrium framework.

# 2 The credit spread puzzle

## 2.1 Historical data

Average default rates for the 1970-2011 period[9] and recovery rates are from the Moody's 2012 annual report. Data on recovery rates start in 1982. Corporate bond yields are obtained from Datastream and the St. Louis Fed website for four different sets of indices: two Moody's indices[10], four Barclays indices[11], six BofA indices[12], and eighteen Thomson-Reuters corporate bond indices[13].

In terms of aggregate variables, personal consumption expenditures (PCE), and PCE index data

[8] Choi et al. (2007), Gill and Prowse (2012), Artstein-Avidan and Dillenberger (2011).

[9] Average default rates in the Moody's report are calculated for three different periods: 1920-2011, 1970-2011, and 1983-2011. Average default rates for the 1983-2011 sample are almost identical to the ones used in this study. However, average default rates for the 1920-2011 period are substantially higher than for the 1970-2011 or the 1983-2011 samples due to the inclusion of the Great Depression.

[10] Moody's Seasoned Aaa and Baa Corporate Bond Indices (1920-2011).

[11] US Agg. Corp. Intermediate Aaa and Baa Indices, US Agg. Corp. Long Aaa and Baa Indices (1974-2011).

[12] US Corp. 1-5y Aaa and Baa, US Corp. 7-10y Aaa and Baa, and US Corp. 15y+ Aaa and Baa Indices (2001-2011).

[13] US Corp. AAA and BBB Indices for maturities from 2yr up to 10yr (2003-2011). Even though BofA indices use S&P ratings (AAA, BBB), for the practical purposes of this study, BBB (AAA) and Baa (Aaa) ratings are considered equivalent. See also Cantor and Packer (1994).

are from the BEA. Per capita consumption expenditures are defined as services plus non-durables. Each component of aggregate consumption expenditures is deflated by its corresponding PCE price index (base year is 2004). Population data are from the U.S. Census Bureau. For consumption data, I follow the "beginning-of-period" convention as in Campbell (2003) and Yogo (2006) because beginning-of-period consumption growth is better aligned with asset returns. Recession dates are from the NBER. Interest rates are from Kenneth French's (whom I kindly thank) website. Market returns, dividends, and price-dividend ratios are obtained through the CRSP-WRDS database for the value weighted AMEX/NYSE/NASDAQ index.

Earnings are gross profits (item GP) from the merged CRSP-Compustat database. I use gross profits as a measure of earnings because Compustat EBIT (or EBITDA) growth rates are very volatile[14]. For earinings data, I also follow the "beginning-of-period" convention so that earnings are aligned with consumption. Earnings have been exponentially detrended due to the increasing number of firms in the Compustat sample over time. Stock market returns, dividend growth, earnings growth, and interest rates have been adjusted for inflation by subtracting the growth rate of the PCE price index[15]. Aggregate variables and market data are sampled for the 1946-2011 period, with the exception of earnings that start in 1950 and end in 2010 due to Compustat data availability. All variables have been sampled and simulated at the annual frequency.

## 2.2 A benchmark model for credit spreads

Consider a discrete-time, single-good, closed, endowment economy in which the aggregation problem has been solved. Implicit in the representative agent framework lies the assumption of complete markets. There is no productive activity, yet at each point in time the endowment of the economy is generated exogenously by $n$ "tree-assets" as in Lucas (1978). There are also markets where equity, debt, and claims on the total output of these "tree-assets" can be traded. In addition to rational expectations, I will also assume that there are no restrictions on individual asset holdings or trans-

[14]EBIT growth volatility in the compustat sample is around is around 12%. Earnings growth volatility from Shiller's website is around 30%.

[15]$R_{\text{real},t+1} = exp(logR_{\text{nom},t+1} - log\frac{PCE_{t+1}}{PCE_t})$.

action costs, that preferences over risky payoffs can be described by CRRA power utility, and that all agents can borrow and lend at the same risk-free rate. This paper focuses on zero-coupon bonds because, according to Chen et al. (2009) p. 3384, the inclusion of coupon payments does not really affect credit spreads.

Consider a $T$-period, zero-coupon bond written on firm's i's assets. This bond pays \$1 if the firm remains solvent at time $t+T$, and $\$(1-L) < \$1$ otherwise. According to Appendix A, expected yields for zero-coupon, corporate bonds are given by[16]

$$\mathbb{E}[y_{i,t,t+T}] = r_f - \frac{1}{T} log\Big[1 - LN\Big(N^{-1}(\pi^{\mathbb{P}}_{i,T}) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big]. \tag{1}$$

$y_{i,t,t+T}$ and $r_f$ are the continuously compounded yield-to-maturity and risk-free rate respectively, $L$ are losses given default, $N()$ is the standard normal c.d.f. and $N^{-1}()$ is the inverse of the standard normal c.d.f.. $\pi^{\mathbb{P}}_{i,T}$ is the physical probability of default, while $\tilde{\mu}_i$ and $\sigma_i$ are the expected value and standard deviation for asset log-returns. Expected corporate bond yields in (1) depend on Sharpe ratios ($\frac{\tilde{\mu}_i - r_f}{\sigma_i}$), physical probabilities of default ($\pi^{\mathbb{P}}_{i,T}$), losses given default ($L$), and bond maturity ($T$). In calibrating the model, I set the Sharpe ratio equal to 0.22 which is the Sharpe ratio for the median Baa firm in Chen et al. (2009)[17]. Losses given default $L$ are set equal to 54.9% to match the average recovery rate of 45.1% for senior unsecured bonds in the Moody's report[18]. Physical probabilities of default, $\pi^{\mathbb{P}}_{i,T}$ , are calibrated to average default probabilities for Aaa and Baa bonds during the 1970-2011 period, which are shown in Panel A of Table I.

Panel B in Table I shows average Baa-Aaa credit spreads estimated in previous studies, as well as average spreads for the four sets of bond indices (Moody's, Barclays, BofA, Thomson-Reuters)[19]. Following the credit spread puzzle literature, this paper focuses on Baa-Aaa spreads instead of Baa-$r_f$ spreads because Aaa yields seem to encompass parts of credit spreads such as liquidity, callability,

[16]This expression is identical to the continuous-time one in Chen et al. (2009) p. 3377. However, Appendix A derives the expression in (1) for a discrete-time economy with CRRA investors.

[17]The Sharpe ratio in (1) is the Sharpe ratio for the firm's assets in place, not the equity Sharpe ratio. However, because returns for assets in place are hard to measure, I follow Chen et al. (2009) p. 3375 who proxy asset Sharpe ratios with equity Sharpe ratios.

[18]Chen et al. (2009) use an average recovery rate of 44.1%.

[19]See subsection 2.1.

or tax issues which are unrelated to default risk, and are ignored by the model in (1)[20]. According to Panel B, the average Baa-Aaa spread in the Huang and Huang sample (2012) is around 103 bps for short maturities, and 131 bps for medium maturities. Expected credit spreads for the long maturity Barclays indices is 112 bps. In Duffee (1998), average credit spreads are low because the sample is short (1985-1995), and is heavily influenced by the 1990-1995 period which, according to Figure I, is characterized by very low spreads (around 50 bps). In contrast, average credit spreads for the BofA and Thomson-Reuters indices are high because average credit spreads for the these indices are calculated over a short sample (2001-2011), and average spreads are affected by extreme observations during the 2009 recession (Figure I).

For the rest of the paper, target expected credit spreads to be explained will be 103 bps for 4yr maturities and 131 bps for 10yr maturities from Huang and Huang (2012). These spreads are frequently cited in the literature, and have been calculated over a relatively long period (1973-1993). The 4yr credit spreads from Huang and Huang are very similar to the 4yr spreads in Chen et al. (2009) (107 bps for the 1970-2001 period), while 10yr expected credit spreads from Huang and Huang are very close to 10yr spreads in the Barclays sample (129 bps for the 1974-2011 period). Finally, target expected spreads for long maturities (15yrs+) are equal to 112 bps from the long-term Barclays indices. Average credit spreads for long-term Barclays indices, in turn, are similar to the Moody's sample (118 bps for the 1920-2011 period).

The second-to-last line in Panel B of Table I shows average Baa-Aaa credit spreads generated by the benchmark model in (1). Expected Baa bond yields were generated using default probabilities for Baa firms from Panel A, a Sharpe ratio of 0.22, and losses given default equal to 54.9%. Expected bond yields for Aaa bonds were estimated using the same values for the Sharpe ratio and losses given default as in the Baa case, but Aaa default probabilities were used instead. Expected Baa-Aaa spreads generated by the model in (1) are substantially smaller in magnitude than those observed in the data. For instance, model implied expected credit spreads for short maturites (4yr)

[20]Longstaff, Mithal and Neis (2005) find evidence in favor of a liquidity component in the spreads of corporate bonds over treasuries, while Ericsson and Renault (2006) suggest part of the spread over treasuries can also be attributed to taxes.

are almost half the average spreads observed in practice (51 bps vs. 103 bps in Huang and Huang 2012)[21].

The credit spread puzzle is clearly illustrated in Figure II. The dotted line shows expected credit spreads according to the expression in (1) accross different maturities. The scattered dots in Figure II are average Baa-Aaa spreads from Huang and Huang (2012), and the three sets of bond indices shown in Table I (Barclays, Thomson-Reuters, BofA). If the expression in (1) were able to fit expected credits spread reasonably well, then the scattered points should belong to the credit spread curve. According, to Figure II, the credit spread puzzle is particularly pronounced for short maturities up to 10 years. However, as maturity ($T$) increases, the term $\frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}$ in (1) becomes larger, and the benchmark model is able to fit credit spreads better.

Besides the implicit assumption of CRRA preferences, the model in (1) imposes three very important limitations that can explain its problematic empirical performance. First, even though time-variation in expected asset returns is considered a key mechanism for resolving a number of stylized facts in financial markets, asset returns in (1) are normally distributed with constant mean ($\tilde{\mu}_i$) and variance ($\sigma_i$). Ferson and Harvey (1991) emphasize the importance of time-varying expected returns, while Campbell and Cochrane (1999), Bansal and Yaron (2004), and Ostrovnaya et al. (2006) describe different mechanisms (habit, time-varying macroeconomic uncertainty, generalized disappointment aversion) which can generate time-variation in investors' risk attitudes, and, consequently, time-varying expected returns[22].

Second, recovery rates $(1 - L)$ in (1) are also constant. Table II shows OLS regression results for recovery rates and aggregate consumption growth during the 1982-2011 period. The regression coefficient is positive (4.461), and statistically significant ($t$-stat. 3.036, $R^2$ 24.767%), suggesting that recovery rates are most likely procyclical. Figure III also indicates that recovery rates decrease substantially during recessions[23]. Appendix B shows that if recovery rates comove with aggregate

[21] Nevertheless, the benchmark model in (1) is doing quite well in matching average credit spreads for the Duffee (1998) sample or credit spreads for long maturities.

[22] Besides constant moments, the normal distribution also appears to be a restrictive assumption. Nevertheless, Huang and Huang (2012) and Chet et al. (2009) show that introducing jumps or relaxing the normality assumption cannot resolve the credit spread puzzle.

[23] Evidence in favor of procyclical recovery rates can be found in Altman et al. (2005), and Acharya et al. (2007)

economic conditions (consumption growth) in a linear way[24]

$$1 - L_{t+T} = a_{rec,0} + a_{rec,c}\Delta c_{t+T-1,t+T}, \qquad (2)$$

then the benchmark model becomes

$$\mathbb{E}[y_{i,t,t+T}] = r_f - \frac{1}{T}log\Big[1 - \Big(\mathbb{E}[L_{t+T}] + \underbrace{a_{rec,c}\frac{\tilde{\mu}_m - r_f}{\rho_{m,c}\sigma_m}\sigma_c}_{\text{risk adjustment for } L_{t+T}}\Big)N\Big(N^{-1}(\pi^{\mathbb{P}}_{i,T}) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big]. \qquad (3)$$

$\frac{\tilde{\mu}_m - r_f}{\sigma_m}$ in (3) is the stock market Sharpe ratio (0.378 from Table VI), $\rho_{m,c}$ is the correlation coefficient between stock market returns and consumption growth (0.463 in Table VI), and $\sigma_c$ is consumption growth volatility (1.914% in Table IV)[25].

According to the expression in (3), risk averse individuals adjust (decrease) expected values for recovery rates $1 - \mathbb{E}[L_{t+T}]$ because these rates are procyclical ($a_{rec,c} > 0$ in 2). The risk adjustment term for recovery rates would normally depend on the risk aversion parameter in the CRRA power utility. However, Appendix B shows that we can use the Euler equation for stock market returns (eqn. 26 in Appendix F.1) in order to substitute the risk aversion parameter with the stock market Sharpe ratio $\frac{\tilde{\mu}_m - r_f}{\sigma_m}$ adjusted for the correlation ($\rho_{m,c}$) between stock market returns and consumption growth. Nevertheless, the last line in Panel B suggests that the addition of procyclical recovery rates leads to a small increase in credit spreads (10 bps across maturities) relative to the benchmark model in (1). The small improvement in credit spreads is either because recovery rates do not covary much with aggregate consumption (low $a_{rec,c}$ in 3), or because the standard CRRA power utility framework does not penalize enough recovery rate risk.

The third drawback of the benchmark model in (1) is related to the constant and exogenous default boundary. In the original Merton model, default boundaries are constant, and equal to the

among others. Shleifer and Vishny (1992) also provide theoretical arguments in favor of procyclical recovery rates.

[24] Throughout the paper, recovery rates do not change across all firms, even though they are allowed to vary through time.

[25] For comparison purposes with the disappointment model in subsection 4.2, values for $\frac{\tilde{\mu}_m - r_f}{\sigma_m}$, $\rho_{m,c}$, and $\sigma_c$ are from the simulated economy.

face value of debt. In (1), the default boundary is also assumed constant but not necessarily equal to the face value of debt, because a number of studies[26] suggest that default happens below the debt level. For instance, Chen et al. (2009) argue that since average recovery rates are around 45%, if default happened at the face value of debt, then default costs would amount to 55% of face value, which is an extremely large number. Contrary to the constant default case of the original Merton model, Chen et al. (2009) set an exogenous default boundary which comoves negatively with surplus consumption[27]. Chen (2010) and Bhama et al. (2010), on the other hand, endogenize default boundaries exploiting the smooth pasting conditions in a continous-time framework. Although default boundaries are hard to measure, it seems that time-variation in these boundaries is an important ingredient for resolving the credit spread puzzle.

In a continuous-time setting, the derivation of the benchmark models in (1) and (3) hinges on continuous trading so that, under the risk-neutral probability measure, expected returns ($\tilde{\mu}_i$) are replaced by the risk-free rate [28]. However, for discrete-time models, in which continuous trading is not an option, replacing the mean with the risk-free rate while preserving log-normality of asset returns necessarily requires that investor preferences are characterized by power utility[29]. Hence, in a discrete-time world, the models in (1) and (3) are essentially a statement about investor preferences.

The aim of this paper is to examine whether relaxing the CRRA power utility assumption, and introducing disappointment aversion preferences can help us resolve the credit-spread puzzle. Unfortunately, by introducing more complicated preference structures, we are no longer able to derive simple pricing formulas for corporate bond yields like the ones in (1) and (3).

[26]Leland (2004), Davydenko (2012).

[27]The assumption of countercyclical default boundaries in Chen et al. (2009) is necessary for positive comovement between default rates and credit spreads.

[28]Black and Scholes (1973).

[29]Brennan (1979) and Appendix F.1.

## 3 Recursive utility with disappointment aversion preferences

### 3.1 Disappointment aversion stochastic discount factor

For the benchmark disappointment model of this paper, I maintain the same assumptions as in th previous section, with the crucial difference that now the model economy is populated by disappointment averse, instead of CRRA, individuals. Disappointment aversion preferences are homothetic. Therefore, if all individuals have identical preferences, then a representative investor exists, and equilibrium prices are independent of the wealth distribution[30]. The expression for the disappointment aversion intertemporal marginal rate of substitution between periods $t$ and $t+1$[31] is given by

$$M_{t,t+1} = \underbrace{\underbrace{\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{(\rho-1)}}_{\text{time correction}} \quad \underbrace{\Big[\frac{V_{t+1}}{\mu_t\big(V_{t+1}\big)}\Big]^{-\alpha-\rho}}_{\text{second-order risk correction}}}_{\text{Epstein-Zin terms}} \quad \underbrace{\Big[\frac{1+\theta\mathbf{1}\{V_{t+1} \ < \ \delta\mu_t\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta \ > \ 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1} \ < \ \delta\mu_t\}]}\Big]}_{\text{disappointment (first-order risk) correction}}. \quad (4)$$

with

$$\mu_t(V_{t+1}) = \mathbb{E}_t\Big[\frac{V_{t+1}^{-\alpha}\ (1+\theta\mathbf{1}\{V_{t+1} \ < \ \delta\mu_t\})}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta \ > \ 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1} \ < \ \delta\mu_t\}]}\Big]^{-\frac{1}{\alpha}}. \quad (5)$$

The derivation of the disappointment aversion discount factor is shown in Appendix C.

$V_t$ is lifetime utility from time $t$ onwards. $\mu_t$ in equation (5) is the disappointment aversion certainty equivalent which generalizes the concept of expected value. $\mathbb{E}_t$ is the conditional expectation operator. The denominator in (5) is a normalization constant such that $\mu_t(\mu_t) = \mu_t$. $\mathbf{1}\{\}$ is the disappointment indicator function that overweighs bad states of the world (disappointment events). According to (5), disappointment events happen whenever lifetime utility $V_{t+1}$ is less than some multiple $\delta$ of its certainty equivalent $\mu_t$. The parameter $\delta$ is associated with the threshold below which disappointment events occur. In Gul (1991) $\delta$ is 1, whereas in Routlegde and Zin (2010),

[30]Chapter 1 in Duffie (2000), and Chapter 5 in Huang and Litzenberger (1989).

[31]See also Hansen et al. (2007), Routledge and Zin (2010), and Delikouras (2013).

disappointment events may happen below or above the certainty equivalent, $V_{t+1} < \delta\mu_t(V_{t+1})$, depending on whether the GDA parameter $\delta$ is lower or greater than one respectively. Here, I follow Gul (1991), and set $\delta$ equal to 1 for analytical tractability.

$\alpha \geq -1$ is the Pratt (1964) coefficient of second-order risk aversion which affects the smooth concavity of the objective function. $\theta \geq 0$ is the disappointment aversion parameter which characterizes the degree of asymmetry in marginal utility above and below the reference level. $\beta \in (0,1)$ is the rate of time preference. $\rho \leq 1$ characterizes the elasticity of intertemporal substitution (EIS$= \frac{1}{1-\rho}$) for consumption between two consecutive periods. In order to facilitate the derivation of analytical solutions, I set the EIS equal to unity ($\rho = 0$). For $\rho = 0$ and $\delta = 1$ in (4), the disappointment aversion stochastic discount factor becomes[32]

$$M_{t,t+1} = \underbrace{\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-1}}_{\text{time correction}} \underbrace{\Big(\frac{V_{t+1}}{\mu_t\big(V_{t+1}\big)}\Big)^{-\alpha}}_{\text{second-order risk correction}} \underbrace{\frac{1+\theta\ \mathbf{1}\{V_{t+1} < \ \mu_t(V_{t+1})\}}{\mathbb{E}_t[1+\theta\ \mathbf{1}\{V_{t+1} < \mu_t(V_{t+1})\}]}}_{\text{disappointment correction}}. \tag{6}$$

$M_{t,t+1}$ in (4) and (6) essentially corrects expected values by taking into account investor preferences over the timing and riskiness of stochastic payoffs. The first term in (4) and (6) corrects for the timing of uncertain payoffs (resolution of uncertainty) which happen at a future date. The second term adjusts future payoffs for investors' dislike towards risk (second-order risk aversion). When investors' preferences are time-additive, adjustments for time and risk are identical ($\alpha = \rho$), and the second term vanishes. The third term in equations (4) and (6) corrects future payoffs for investors' aversion towards disappointment events, defined as periods during which lifetime utility $V_{t+1}$ drops below its certainty equivalent $\mu_t$.

[32] The reader is referred to Delikouras (2013) for a more thorough analysis of the disappointment model.

### 3.2 Approximate analytical solutions for the disappointment aversion discount factor

Since lifetime utility $V_t$ in (6) is unobservable, it is hard to test the empirical performance of the disappointment model. The analysis will become much easier if we are able to express lifetime utility as a function of state variables.

Suppose that at each point in time, expected consumption growth is a function of a state variable $x_t$. For simplicity, I will assume that $x_t$ is equal to current consumption growth $\Delta c_{t-1,t}$. Suppose also that there is a second state variable $\sigma_t$ which drives aggregate economic uncertainty. Based on those two assumptions, our model economy is described by the following system of equations

$$\Delta c_{t,t+1} = \mu_c + \phi_c \Delta c_{t-1,t} + \sigma_t \epsilon_{c,t+1}, \tag{7}$$

$$\sigma_{t+1} = \mu_\sigma + \phi_\sigma \sigma_t + \nu_\sigma \epsilon_{\sigma,t+1}, \tag{8}$$

$$\Delta o_{m,t,t+1} = \mu_o + \phi_o \Delta c_{t-1,t} + \sigma_o \sigma_t \epsilon_{o,t+1}. \tag{9}$$

According to (7), consumption growth is an AR(1) process with time-varying volatility. $\phi_c \in (-1,1)$ is the first-order autocorrelation coefficient, $\mu_c$ is the constant term, and $\epsilon_{c,t+1}$ are i.i.d. $N(0,1)$ shocks. Although, the AR(1) model for consumption growth is quite common in the asset pricing literature (Mehra and Prescott 1985, Routledge and Zin 2010), a number of works (Campbell and Cochrane 1999, Cochrane 2001) suggest that consumption growth is i.i.d., and $\phi_c$ in (7) is zero.

Time-varying macroeconomic uncertainty[33] is captured by consumption growth volatility $\sigma_t$ which is stochastic. Following Chen et al. (2009), $\sigma_t$ is an AR(1) process in which $\epsilon_{\sigma,t+1}$ are i.i.d. $N(0,1)$ shocks, $\phi_\sigma \in (-1,1)$ is the first-order autocorrelation coefficient, $\mu_\sigma \in \mathbb{R}_{>0}$ is the constant term, and $\nu_\sigma \in \mathbb{R}_{>0}$ captures the conditional volatility in macroeconomic uncertainty. Bansal and Yaron (2004), Bansal et al. (2007), Lettau et al. (2007), and Bonomo et al. (2011) all use similar autoregressive models for macroeconomic uncertainty, although they consider consumption growth

[33] In addition to the asset pricing implications of stochastic volatility, Bloom (2009) and Bloom et al. (2012) propose a model in which stochastic second moments in TFP shocks are the single cause for business cycle fluctuations.

variance instead of consumption growth volatility. Because shocks in (8) are normally distributed, the probability of negative volatility is non-zero. However, consumption growth variance $\sigma_t^2$ is always positive[34].

The last equation describes the evolution of aggregate payout growth. Depending on the asset we want to price, $o_{m,t}$ represents different kinds of cashflows. For aggregate equity claims, the relevant payout is dividends ($o = d$). For the valuation of aggregate assets in place, the relevant payout is earnings ($o = e$). According to (9), expected payout growth depends on aggregate consumption growth $\Delta c_{t-1,t}$ through $\phi_o \in \mathbb{R}$. For $\phi_o > 1$, aggregate payout is a levered claim to consumption, whereas for $\phi_o = 0$, payout growth is i.i.d.. $\sigma_o \in \mathbb{R}_{>0}$ is the volatility parameter for payout growth. This specification for aggregate payout growth is very similar to the one in Bansal and Yaron (2004) where expected dividend growth depends on expected consumption growth (the long-run risk state variable). Finally, for algebraic convenience, I will assume that shocks to consumption growth, consumption growth volatility, and payout growth ($\epsilon_{c,t}, \epsilon_{\sigma,t}, \epsilon_{o,t}$), are mutually uncorrelated.

Using the system of equations in (7) and (8), and the log-linear structure of investor's lifetime utility, I can derive analytical solutions for the log utility-consumption ratio $v_t - c_t$ expressed in terms of consumption growth $\Delta c_{t-1,t}$ and aggregate uncertainty $\sigma_t$.

***Proposition 1:*** For $\rho = 0$, $\delta = 1$ in (4), and macroeconomic dynamics in (7) and (8), the log utility-consumption ratio, $v_t - c_t = log(V_t/C_t)$, is approximately affine in consumption growth, consumption growth volatility, and consumption growth variance: $v_t - c_t \approx A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2 \ \ \forall t$, where

- $A_1 = \frac{\beta\phi_c}{1-\beta\phi_c}$,
- $A_2 = \frac{\overbrace{-\theta\beta n(\bar{x})(A_1+1)(1+2\alpha A_3\nu_\sigma^2)}^{\text{disappointment aversion}} + 2\beta A_3 \mu_\sigma \phi_\sigma}{1+2\alpha A_3 \nu_\sigma^2 - \beta\phi_\sigma}$,
- $A_3 = \frac{-[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2]+\sqrt{[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2]^2-4\beta\alpha^2(A_1+1)^2\nu_\sigma^2}}{4\alpha\nu_\sigma^2}$,

[34]Hsu and Palomino (2011) resolve the issue of negative variance by assuming an autoregressive gamma process as in Gourieroux and Gasiak (2006).

- $A_0 = \frac{\beta}{1-\beta}[(A_1+1)\mu_c + \frac{1}{1+2\alpha A_3 \nu_\sigma^2}(A_2\mu_\sigma + A_3\mu_\sigma^2 - 0.5\alpha A_2^2\nu_\sigma^2) + \frac{1}{2\alpha}log(1+2\alpha A_3\nu_\sigma^2)]$,

and $n(.)$ is the standard normal p.d.f..

***Proof.*** *See Appendix F.4*

$A_1$ is the consumption growth multiplier. The sign and magnitude of $A_1$ depend on consumption growth autocorrelation $\phi_c$. If consumption growth is i.i.d. then $A_1$ is zero. $A_3$ is the multiplier for consumption growth variance $\sigma_t^2$. If the risk aversion coefficient $\alpha$ is positive, then $A_3$ is negative[35]. For $A_3$ to be real, we require that the terms inside the square root are positive, and that $\alpha$ is different than zero[36].

$A_2$ is the multiplier for consumption growth volatility $\sigma_t$. $A_2$ captures first-order risk aversion through the $\theta n(\bar{x})$ term. For $A_3$ negative and positive $\theta$, then $A_2$ is also negative. Finally, $A_0$ is the constant term in the log utility-consumption ratio. For $A_0$ to be well defined, we require $1+2\alpha A_3\nu_\sigma^2$ to be positive, and that $\alpha$ is non-zero. If consumption growth is positively autocorrelated ($\phi_c > 0$), and preference parameters ($\alpha, \theta > 0$) are also positive, then the log utility-consumption ratio is procyclical since $A_1$ is positive, and $A_2, A_3$ are negative.

An immediate consequence of *Proposition 1* is that we can express the disappointment aversion stochastic discount factor in (6) as a function of consumption growth $\Delta c_{t-1,t}$, and consumption growth volatility $\sigma_t$

$$M_{t,t+1} \approx \underbrace{e^{log\beta - \Delta c_{t,t+1}}}_{\text{time correction}} \underbrace{e^{-\alpha\left\{A_0(1-\frac{1}{\beta})+[(A_1+1)\Delta c_{t,t+1}-\frac{1}{\beta}A_1\Delta c_{t-1,t}]+A_2(\sigma_{t+1}-\frac{1}{\beta}\sigma_t)+A_3(\sigma_{t+1}^2-\frac{1}{\beta}\sigma_t^2)\right\}}}_{\text{second-order risk correction}} \times \underbrace{\frac{1+\theta\ \mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2 < \ \frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)\}}{\mathbb{E}_t[1+\theta\ \mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2 < \ \frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)\}]}}_{\text{disappointment correction}}. \quad (10)$$

$M_{t,t+1}$ in (10) corrects expected future payoffs for timing, risk and disappointment, much like the one in (6). The crucial difference between the two expressions is that in equation (10) unobservable

[35] Appendix F.4.

[36] A detailed discussion on parameter restrictions can be found in Appendix F.4. The requirement $\alpha \neq 0$, implies that preferences need to be non-separable across time, since $\rho$ is already assumed zero in *Proposition 1*.

lifetime utility $V_{t+1}$ is expressed in terms of state variables.

Armed with the expression for the stochastic discount factor, we can also solve for the one-period log risk-free rate (see Appendix F.5)

$$r_{f,t,t+1} \approx \underbrace{-log\beta + 1 \cdot \mu_c + 1 \cdot \phi_c \Delta c_{t-1,t}}_{\text{impatience and future prospects}} \underbrace{\underbrace{-0.5[2\alpha(A_1+1)+1]\sigma_t^2}_{\text{second-order risk aversion}} - \underbrace{\theta n(\bar{x})\sigma_t.}_{\text{disappointment aversion}}}_{\text{precautionary savings motive}} \tag{11}$$

Consumption growth terms $\big(\mu_c(1-\phi_c),\, \phi_c \Delta c_{t-1,t}\big)$ in (11) are multiplied by unity, since the EIS is assumed equal to one ($\rho$=0), and consumption growth moves one-for-one with interest rates. The last two terms in (11) reflect the precautionary motive for investors to save. This motive depends on both risk and disappointment aversion. Notice that second-order risk aversion terms depend on consumption growth variance ($\sigma_t^2$), while disappointment aversion terms depend on consumption growth volatility ($\sigma_t$) due to first-order risk aversion. For $\alpha, \theta$ positive, higher uncertainty (high values for $\sigma_t$ and $\sigma_t^2$) will force investors to save more in the risk-free technology, and therefore decrease interest rates.

Turning now to risky financial assets, let $R_{m,t}$ be the cum-payout, one-period, gross return for a claim on a stream of aggregate payments (dividends or earnings). If claims are traded in complete, frictionless markets, then the consumption-Euler equation implies that

$$\mathbb{E}_t\Big[M_{t,t+1} R_{m,t,t+1}\Big] = 1.$$

Using the results in Appendix D, aggregate log-returns $r_{m,t,t+1}$ can be written as a linear function of log price-payout ratios, and the Euler equation becomes

$$\mathbb{E}_t\Big[M_{t,t+1} e^{\kappa_{m,0}+\kappa_{m,0} z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1}}\Big] = 1,$$

where $\kappa_{m,0}$ and $\kappa_{m,1}$ are linearization constants, and $z_{m,t} = log\frac{P_{m,t}}{O_{m,t}}$ is the log price-payout ratio.

In order to provide valuable intuition, we can further express the log price-payout ratio $z_{m,t}$ as a linear function of the state variables $\Delta c_{t,t+1}$ and $\sigma_t$ using *Proposition 2.*

***Proposition 2:*** For $\rho = 0$, $\delta = 1$ in (4), and the dynamics in (7) - (9), the log price-payout ratio $z_{m,t} = log(P_{m,t}/O_{m,t})$ for a claim on a stream of aggregate payments is approximately affine in consumption growth, consumption growth volatility, and consumption growth variance: $z_{m,t} \approx A_{m,0} + A_{m,1}\Delta c_{t-1,t} + A_{m,2}\sigma_t + A_{m,3}\sigma_t^2 \ \ \forall t$, where

- $A_{m,1} = \frac{\phi_o - \phi_c}{1-\kappa_{m,1}\phi_c}$,
- $A_{m,2} \approx \frac{\overbrace{\theta n(\bar{x})(1-\kappa_{m,1}A_{m,1})}^{\text{disappointment aversion}} + 2\kappa_{m,1}A_{m,3}\mu_\sigma\phi_\sigma}{1-\kappa_{m,1}\phi_\sigma}$,
- $A_{m,3} \approx \frac{1}{2}\frac{(1-\kappa_{m,1}A_{m,1})^2 + 2\alpha(A_1+1)(1-\kappa_{m,1}A_{m,1}) + \sigma_o^2}{1-\kappa_{m,1}\phi_\sigma^2}$,
- $A_{m,0} \approx \frac{1}{1-\kappa_{m,1}}\left[log\beta + \kappa_{m,0} + \mu_o + (\kappa_{m,1}A_{m,1} - 1)\mu_c + \kappa_{m,1}A_{m,2}\mu_\sigma + \kappa_{m,1}A_{m,3}\mu_\sigma^2\right]$,

***Proof.*** *See Appendix F.6*

Note that the values for $A_{m,2}$, $A_{m,3}$ and $A_{m,0}$ above are approximations assuming that the variance for consumption growth volatility ($\nu_\sigma^2$) is a number close to zero. Exact solutions can be found in Appendix F.6. The above approximations preserve the intuition without the notational burden. However, for the simulation part of this study, I use the exact solutions. Moreover, the multipliers $A_{m,1}, A_{m,2}$, $A_{m,3}$ and $A_{m,0}$ for the price-dividend ratio, in which $o = d$, are different than the multipliers for the price-earnings ratio, in which $o = e$, because aggregate dividend growth dynamics are different than aggregate earnings growth dynamics ($\phi_d \neq \phi_e$ or $\mu_d \neq \mu_e$ or $\sigma_d \neq \sigma_e$).

As long as $\phi_o \neq \phi_c$, the multiplier for consumption growth $A_{m,1}$ will be non-zero, and the price-payout ratio $z_{m,t}$ will depend on consumption growth, even if $\phi_c = 0$ and consumption growth is i.i.d.. The sign of $A_{m,1}$ essentially depends on $\phi_o - \phi_c$ because $1 - \kappa_{m,1}\phi_c$ is always positive[37]. $A_{m,3}$ is the multiplier for $\sigma_t^2$, and depends on the risk aversion coefficient $\alpha$, as well as on the persistence of aggregate shocks through the terms $\phi_\sigma^2$ and $A_{m,1}$. $A_3$ in *Proposition 1* is always negative for

[37]For consumption growth to be stationary we require $\phi_c \in (-1, 1)$. Additionally, $\kappa_{m,1} < 1$ from (25), and thus $1 - \kappa_{m,1}\phi_c > 0$.

positive values of the risk aversion parameter $\alpha$. On the other hand, $A_{m,3}$ in *Proposition 2* can turn positive even if $\alpha$ is positive, provided that $\sigma_o$ is a large number. When $A_{m,3}$ is positive, an increase in consumption growth variance will increase the price-payout ratio.

$A_{m,2}$ is the stochastic volatility multiplier. If investors are not disappointment averse ($\theta = 0$) and $A_{m,3}$ is positive, then $A_{m,2}$ is also positive, and an increase in aggregate uncertainty will unambiguously lead to an increase in the price-payout ratio. However, for positive $\theta$, $A_{2,m}$ can be negative, even if $A_{m,3}$ is positive. In this case, the effects of aggregate uncertainty on the price-payout ratio operate in two different directions thought the first and second-order risk aversion channels. This is a subtle, but important, difference between disappointment aversion and the traditional Epstein-Zin (Epstein and Zin 1989) framework without any first-order risk aversion effects. Finally, $A_{0,m}$, the constant term in the price-divided ratio, is equal to the sum of the constant terms ($\mu_m,\ \mu_c$) from (7) and (9) adjusted for disappointment ($A_{m,2}\mu_\sigma$) and uncertainty ($A_{m,3}\mu_\sigma^2$).

The results in *Proposition 2* are particularly important, since we can use the price-payout identity in Appendix D to express asset log-returns as a function of the state variables

$$
\begin{aligned}
r_{m,t,t+1} &\approx \kappa_{m,0} + \kappa_{m,1} z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1},\ \forall t \qquad (12)\\
z_{m,t} &= A_{m,0} + A_{m,1}\Delta c_{t-1,t} + A_{m,2}\sigma_t + A_{m,3}\sigma_t^2.
\end{aligned}
$$

Asset returns in (12) correspond to aggregate claims. In order to describe firm-level asset returns, we need to introduce idiosyncratic shocks as follows

$$
r_{i,t,t+1} \approx \underbrace{\kappa_{m,0} + \kappa_{m,1} z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1}}_{\text{systematic component}} + \underbrace{\sigma_i \epsilon_{i,t+1}}_{\text{idiosyncratic part}}, \qquad (13)
$$

for cum-payout returns, and

$$
r^x_{i,t,t+1} = z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1} + \sigma_i \epsilon_{i,t+1}, \qquad (14)
$$

for ex-payout returns. $\epsilon_{i,t+1}$ are i.i.d. N(0,1) idiosyncratic shocks, orthogonal to the rest of the aggregate shocks in (7)-(9). The above specification for firm-level returns matches perfectly a long-standing concept in finance according to which asset returns can be decomposed into a systematic part, and an idiosyncratic one. For equity returns the relevant payout in (12) - (14) is dividends ($o = d$), whereas for assets in place returns the relevant payout is earnings ($o = e$).

# 4 Simulation results for the disappointment aversion discount factor

## 4.1 Preference parameters, and state variable moments for the simulated economy

The EIS and the GDA parameters for the disappointment aversion discount factor in (4) are assumed equal to one for analytical tractability. For the remaining parameters, I set the risk aversion coefficient $\alpha$ equal to 1.8 and the disappointment aversion parameter $\theta$ equal to 2.030 in Table III. These values are within the range of clinical estimates[38], and are very similar to those used in Bonomo et al. (2011). The value for $\theta$ implies that whenever lifetime utility is below its certainty equivalent (disappointment events), investors penalize losses 3 times more than during normal times. Finally, the rate of time preference $\beta$ is equal to 0.9955. In the deterministic steady-state of the economy, an additional \$1 of consumption tomorrow is worth \$0.9955 today.

In order to explain the market-wide equity premium, Routledge and Zin (2010) employ a constant consumption growth variance framework, and set $\theta$ equal to 9 with $\alpha$ equal to -1 (second-order risk neutrality). In Bonomo et al. (2011), consumption growth variance is stochastic, $\theta$ is 2.33, and $\alpha$ is 1.5. Choi et al. (2007) conduct clinical experiments on portfolio choice under uncertainty, and find disappointment aversion coefficients that range from 0 to 1.876, with a mean of 0.39. They also estimate second-order risk aversion parameters that range from -0.952 to 2.871, with a mean of 1.448.

[38] Gill and Prowse (2012).

Using experimental data on real effort provision, Gill and Prowse (2012) estimate disappointment aversion coefficients ranging from 1.260 to 2.070. Ostrovnaya et al. (2006) estimate disappointment aversion parameters from the cross-section of expected returns using market wide stock market returns as the explanatory variable, instead of consumption growth. Their estimates for $\theta$ range from 1.825 to 2.783. Finally, Delikouras (2013) assumes constant consumption growth volatility in a consumption-based model for the cross-section of expected returns, and provides $\theta$ estimates around 10, and risk aversion estimates around 1. Routledege and Zin (2010) also assume constant consumption growth volatility, and set $\theta$ equal to 9 and the risk aversion arameter equal to -1 (second-order risk neutrality) in order to explain the equity premium.

Table III also summarizes moment parameters for the state variable dynamics in (7)-(9). These values are carefully chosen so that simulated moments match those observed in real data. Many of these parameters have been used in previous studies. For instance, the consumption growth multipliers $\phi_d$ and $\phi_e$ in (9) are equal to 3 as in Bansal and Yaron (2004). Earnings are considered a levered claim to consumption ($\phi_e > 1$) because the endowment model ignores other claims to earnings such as salaries, depreciation, and taxes that need to be paid out before interest and dividends[39].

Volatility parameters for dividends and earnings growth ($\sigma_d = 7.166$ and $\sigma_e = 2.201$) are larger than one, because dividend and earnings growth are much more volatile than consumption growth. The autocorrelation parameter for aggregate consumption growth volatility is 0.971 because, according to prior works[40], aggregate uncertainty is a very persistent process. Idiosyncratic volatility $\sigma_i$ is set equal to 0.210 so as to match the Sharpe ratio for the median Baa firm which is 0.220 (Chen et al. 2009, p. 3377). Finally, the linearization constant $\bar{z}_m$ for log price-payout ratios in (25) is equal to 3, which is very close to the unconditional mean for the stock market log price-dividend ratio (Table VI).

[39]Also, for uncorrelated macroeconomic shocks in (7)-(9), letting $\phi_e$ ($\phi_d$) be larger than one is the only way to obtain realistic correlations between earnings (dividend) growth and consumption growth. Chen et al. (2009), p. 3404, set $\phi_d$ equal to 3.5 and $\phi_e$ equal to 2.7.

[40]Bansal and Yaron (2004), Bansal et al. (2007), Lettau et al. (2007), Chen et al. (2009), and Bonomo et al. (2011).

Despite the similarities with previous studies, there are a few very important differences in terms of parameter calibration. First, in Bansal and Yaron (2004) and Bansal et al. (2007), expected consumption growth is a very persistent process, whereas in Chen et al. (2009) and Bonomo et al. (2011) consumption growth is i.i.d. ($\phi_c$=0). Here, I set the autocorelation parameter $\phi_c$ equal to 0.5 to match the persistence in BEA consumption data.

Second, the volatility parameter $\mu_\sigma$ in Bansal and Yaron (2004) and Bonomo et al. (2011) is quite high. Their values for $\mu_\sigma$ imply that annual consumption growth volatility is approximately 3%, which is more than two times the volatility observed in the BEA sample (1.3% from Table IV). In this study, $\mu_\sigma$ is equal to a very small value (0.0004) so that consumption growth volatility remains low.

Table IV shows simulated and sample moments for all macroeconomic variables. Simulated values for the state variables are according to the system in (7)-(9), using parameter values from Table III[41]. Simulated moments for aggregate consumption growth are very close to actual ones (mean 1.834% vs. 1.838% in the data, autocorrelation 0.504 vs. 0.502), with the exception of consumption growth volatility which is higher for the simulated economy (1.914% vs. 1.346% in the data)[42].

Simulated moments for aggregate dividend growth are very realistic as well (mean 1.796% vs. 2.107%, volatility 13.232% vs. 13.079% in the data). However, the autocorrelation for the simulated aggregate dividend growth process is positive (0.093), whereas dividend growth in the data is a mean reverting process with negative autocorrelation (-0.278). Finally, the simulated dividend growth and consumption growth processes are positively correlated (0.218 vs. 0.286 in the data[43]).

Expected earnings growth for the simulated economy is positive (1.819%), and similar to the expected value for consumption and dividend growth. Even though, in the long-run, expected

[41] Because (8) admits negative volatility, if at some point volatility becomes negative, then the negative observation is replaced with the previous observation.

[42] In Chen et al. (2009) consumption growth volatility is around 1.5%. In Bansal and Yaron (2004) and Bonomo et al. (2011) consumption growth volatility is 3%, whereas consumption growth volatility in Shiller's data is 1.8%.

[43] For their habit model, Chen et al. (2009) p. 3377 assume that the correlation coefficient between aggregate dividends and aggregate consumption growth is equal to 0.60, more than twice the estimated value 0.286 in Table IV.

growth rates should be almost identical because dividends and earnings are cointegrated, Belo et al. (2012) explain how endogenous capital decisions can make dividends riskier than earnings in the short-run.

Expected earnings growth in the sample is negative (-3.831%) and approximately equal to expected inflation, because CRSP-Compustat nominal earnings have been exponentially detrended due to the increasing number of firms in the Compustat sample over time. Simulated earnings growth volatility is slightly lower than in the 1950-2010 sample (6.784% vs. 7.057%). Similarly, the simulated correlation coefficient between earnings growth and consumption growth is lower than in the sample (0.425 vs. 0.487)[44].

Macroeconomic uncertainty is hard to measure, and, therefore, there aren't any readily available data to benchmark simulation results for $\sigma_t$. Instead, simulations for $\sigma_t$ are compared against a discretized Ornstein-Uhlenbeck process with calibrated parameter values from Chen et al. (2006), which in turn is based on parameter values from Bansal and Yaron (2004). The key difference between the two stochastic volatility processes is that here the unconditional mean for $\sigma_t$ is lower than in Chen et al. (2006) (1.498% here vs. 2.697%) because in the latter paper, as well as in Bansal and Yaron (2004), consumption growth is much more volatile than the consumption growth process in this paper (around 3% as opposed to 1.8% here).

## 4.2 Simulation results for Baa-Aaa credit spreads

The main pricing equation used in this study is the unconditional Euler equation for zero-coupon, corporate bonds that are subject to default at the expiration date

$$\mathbb{E}[y_{i,t,t+T}] = \mathbb{E}\Big[-\frac{1}{T}log\mathbb{E}_t\big[\big(\prod_{j=1}^{T} M_{t+j-1,t+j}\big)\big(1 - L_{t+T}\mathbf{1}\{r^x_{i,t,t+T} < D_{i,t+T}\}\big)\big]\Big], \tag{15}$$

[44]The correlation coefficient between consumption growth and earnings growth in Chen et al. (2009) is 0.48 (p. 3377).

in which $M_{t,t+j}$ is the disappointment aversion stochastic discount factor from (10), $L_{t+T}$ are losses given default, $r^x_{i,t,t+T}$ are ex-payout, log-returns for assets in place according to (14), and $D_{i,t+T}$ is the default boundary. Credit spreads over the log risk-free rate are given by

$$\mathbb{E}\Big[-\frac{1}{T}log\mathbb{E}_t\big[\big(\prod_{j=1}^{T}M_{t+j-1,t+j}\big)\big(1-L_{t+T}\mathbf{1}\{r^x_{i,t,t+T}<D_{i,t+T}\}\big)\big]\Big]-\mathbb{E}\Big[-\frac{1}{T}log\mathbb{E}_t\big[\prod_{j=1}^{T}M_{t+j-1,t+j}\big]\Big], \quad (16)$$

while Baa-Aaa credit spreads are calculated according to

$$\mathbb{E}\Big[-\frac{1}{T}log\mathbb{E}_t\big[\big(\prod_{j=1}^{T}M_{t+j-1,t+j}\big)\big(1-L_{t+T}\mathbf{1}\{r^x_{Baa,t,t+T}<D_{Baa,t+T}\}\big)\big]\Big] \quad (17)$$

$$-\mathbb{E}\Big[-\frac{1}{T}log\mathbb{E}_t\big[\big(\prod_{j=1}^{T}M_{t+j-1,t+j}\big)\big(1-L_{t+T}\mathbf{1}\{r^x_{Aaa,t,t+T}<D_{Aaa,t+T}\}\big)\big]\Big]. \quad (18)$$

Although bond yields in Table I are measured in nominal terms, the model economy has been simulated in real terms, and thus, model implied spreads are inflation-free. To the extend that inflation risk premia are approximately equal for Baa and Aaa bonds, then nominal Baa-Aaa credit spreads should be very similar to real Baa-Aaa credit spreads. Unlike the model in (1), losses given default $L_{t+T}$ and default boundaries $D_{i,t+T}$ are allowed to vary over time, and also be functions of the state variables

$$1-L_{t+T}=a_{rec,0}+a_{rec,c}\Delta c_{t+T-1,t+T}, \quad (19)$$

$$D_{i,t+T}=a_{i,def,0}+a_{def,c}\big(\Delta c_{t+T-1,t+T}-\frac{\mu_c}{1-\phi_c}\big)+a_{def,\sigma}\big(\sigma_{t+T}-\frac{\mu_\sigma}{1-\phi_\sigma}\big). \quad (20)$$

Table V shows the main empirical results in this study which have been obtained through the simulation process discussed in Appendix E. Panel A in Table V specifies values for the Baa and Aaa default boundaries which are expressed in terms of asset log-returns. For example, the 4yr constant Baa default boundary is equal to -0.998 which means that the value of assets in place needs to decrease to $e^{-0.998}=36.861\%$ of initial value before a Baa firm defaults[45].

[45] Chen et al. 2009 also assume a similar constant default boundary for 4 year Baa bonds (p. 3384).

Simulated default probabilities in Panel B are practically indistinguishable from default rates in the Moody's report due to appropriately selecting default boundaries. The default rates in Panel B guarantee that the stochastic discount factor in (10) generates plausible credit spreads because investors severely penalize default states through the disappointment aversion mechanism, and not because default probabilities are abnormally high.

Panel C in Table V shows average credit spreads implied by the disappointment aversion discount factor in (10) with preference parameters from Table III, and aggregate state variable dynamics according to the system in (7)-(9). In order to address the shortcomings of the benchmark model in (1), I consider four different cases: 1) constant recovery rates and default boundaries, 2) procyclical recovery rates according to (19) and constant default boundaries, 3) constant recovery rates and countercyclical default boundaries according to (20), and 4) procyclical recovery rates and countercyclical default boundaries.

Expected credit spreads for the disappointment aversion discount factor in case 1 are larger than those for the benchmark model (average increase across maturities 15 bps) because disappointment averse investors heavily penalize periods during which lifetime utility is less than its certainty equivalent (disappointment events). During these periods, Baa defaults happen more often than defaults for Aaa firms, which are fairly acyclical. In other words, Baa corporate bonds expose the aggregate investor to more disappointment risk than Aaa bonds. Therefore, in order for Baa bonds to be part of the aggregate investor's portfolio, these claims should be discounted at higher rates than Aaa bonds.

Relative to the benchmark model in (1), case 1 in Panel C is different in two very important ways. First, as explained by *Lemma 1* in Appendix F.1, the benchmark model implicitly assumes CRRA preferences. Although concave CRRA utility functions overweigh unfavorable outcomes, they do not capture asymmetries in marginal utility, because CRRA preferences are isoelasic. On the other hand, the disappointment model relies heavily on investors penalizing losses that happen during disapppointment periods $1 + \theta$ times more than they do for loses happening during normal times. Second, disappointment aversion preference induce time-variation in risk attitudes. This time-variation is further amplified by stochastic consumption growth volatility in (8) so as to generate substantial time-variation in expected returns and Sharpe ratios. Recent results in asset pricing[46] suggest that time-variation in Sharpe ratios is almost a necessary condition for resolving a number of prominent asset pricing puzzles, including the credit spread puzzle.

Nevertheless, disappointment aversion alone cannot fully rationalize expected Baa-Aaa credit spreads, especially for very short maturities, since, according to Table V, 41 bps in expected credit spreads for 4yr bonds remain unexplained by the disappointment model. These results should not cast any doubt on the explanatory power of disappointment aversion. According to Chen et al. (2009), neither the habit, nor the long-run risk models can explain credit spreads[47], unless we assume time-varying recovery rates or stochastic default boundaries.

Table II provides evidence that recovery rates are procyclical. The assumption of constant recovery rates therefore ignores an important risk source for credit spreads. Case 2 in Table V relaxes this assumption, and, based on the results of Table II, assumes that losses given default $L_{t+T}$ are a linear function of aggregate consumption growth as in (19) in which $a_{rec,c}$ is set equal to 4.464 from Table II. The addition of procyclical recovery rates increases Baa-Aaa spreads implied by the disappointment model by 34 bps on average across maturities relatively to the benchmark model in (1), and by 23 bps relatively to the benchmark model with procyclical recovery rates in (3).

In the case of countercyclical losses given default, corporate bonds have to compensate disappointment averse investors for two sources of systematic risk. The first one is related to the fact that during economic downturns default frequencies for Baa firms increase more than default frequencies for Aaa bonds. The second source of systematic risk captures the fact that during disappointment periods recovery rates decrease. Moreover, disappointment aversion preferences punish the procyclicality of recovery rates more severely than CRRA power utility. First-order risk aversion amplifies recovery rate risk, despite the relatively low covariance between recovery rates

[46] Campbell and Cochrane (1999), Bansal and Yaron (2004), Verdelhan (2010), Routledge and Zin (2010), Bansal and Shaliastovich (2013).

[47] Chen et al. (2009) p. 3384 and p. 3405.

and consumption growth. However, despite the improvement relatively to the benchmark case in (3), even with countercyclical recovery rates, 26 bps in 4yr expected credit spreads (17 bps for 10yr maturities) cannot be explained by case 2 of the disappointment model.

Cases 3 and 4 in Table V assume stochastic default boundaries. Since these boundaries are hard to measure, parameters for the stochastic default boundary have been calibrated so that average default rates for the simulated economy match actual ones. Unlike Chen (2010) or Bhamra et al. (2010), but similar to Chen et al. (2009), default boundaries in this study are exogenous, even though they are functions of state variables. The calibrated values for default boundary parameters ($a_{def,c}$ and $a_{def,\sigma}$) in (20) imply that these boundaries are strongly countercyclical, since they comove negatively with consumption growth, and positively with macroeconomic uncertainty. In bad times, when consumption growth (volatility) is lower (higher) than its unconditional mean, default boundaries are low in absolute value, and thus managers find it easier to declare bankruptcy. In good times, when consumption growth (volatility) is higher (lower) than its mean, default boundaries are high in absolute value, and firms do not default as easily as in bad times.

Countercyclical default boundaries lead to a larger number of defaults during economic downturns, and fewer number of defaults during good times. However, unconditionally, average default rates are equal to the ones observed in actual data. Countercyclical default boundaries essentially imply that default events covary more with aggregate macroeconomic conditions relative to cases 1 and 2. The combination of disappointment aversion preferences with countercyclical default boundaries (case 3) improves the fit of the baseline disappointment model (case 1), and also increases model implied expected credit spreads by 25 bps across maturities relative to the benchmark model in (1). Nevertheless, the increase in credit spreads induced by stochastic default boundaries in case 3 is less than the increase due to procyclical recovery rates in case 2, and leaves 29 bps in 4yr expected credit spreads (25 bps in 10yr bonds) unexplained.

Case 4 of the disappointment model assumes procyclical recovery rates as well as countercyclical boundaries. Case 4 can almost perfectly fit average credit spreads for short (100 bps vs. 103 bps for 4yr spreads) and medium maturities (129 bps vs. 131 bps for 10yr spreads), but severely overestimates credit spreads at the long end of the term structure (148 bps vs. 112 bps in the data for 15yr bonds). Countercyclical default boundaries increase the frequency of defaults during bad times, while procyclical recovery rates imply that losses given default increase during periods of low economic growth. Because periods of high default rates and high losses given default are also associated with disappointment events (lifetime utility below its certainty equivalent), disappointment averse investors require larger compensation for holding Baa bonds than Aaa bonds.

Overall, results in Table V suggest that as long as we allow for procyclical recovery rates and countercyclical default boundaries, disappointment aversion preferences are able to resolve the credit spread puzzle using risk and disappointment aversion parameters that are consistent with recent experimental results. However, as shown in Figure IV, by fitting mean credit spreads for short and medium maturities, the disappointment model overestimates mean credit spreads for maturities longer than 15 years. The credit spread literature has almost exclusively considered 4yr or 10yr bonds, and does not provide any results on long maturities. Therefore, we cannot assess the relative performance of the disappointment aversion model for long maturities relative to other asset pricing models. Moreover, matching average credit spreads for very short maturities (1-3yr) still remains an open question for all models.[48].

Although, the goal of this paper is not a horse race between prominent asset pricing models, we need to highlight that the disappointment aversion mechanism is unique. First, disappointment aversion preferences fully encompass recent clinical and field evidence for behavior under uncertainty which emphasize the importance of expectation-based reference-dependent utility[49]. The key mechanism in disappointment aversion is asymmetric marginal utility over gains and losses. Gains and losses are, in turn, endogenously characterized by the forward-looking certainty equivalent for lifetime utility.

Asymmetric marginal utility is not present in the habit model, which assumes a backwards-

[48] According to Table I, default rates for 1 up to 3 years are almost zero. Because no asset pricing model can map zero default rates for short term bonds into measurable yields, the credit spread literature focuses on medium to long term maturities (4-10yr).

[49] Choi et al. (2007), Post et al. (2008), Doran (2010), Crawford and Meng (2011), Abeler et al. (2011), Gill and Prowse (2012).

looking unobservable habit process, and, according to Ljungqvist and Uhlig (2009), leads to policy inconsistencies for the central planner. Furthermore, in the habit model of Campbell and Cochrane (1999) consumption never drops below its habit, otherwise marginal utility becomes infinity. On the other hand, for disappointment aversion preferences it is precisely periods during which consumption growth falls below its certainty equivalent that are important for credit spreads. Asymmetric marginal utility is not captured by the long-run risk model either which assumes a highly persistent mean in expected consumption growth[50].

## 4.3 Equity premium, and the risk-free rate

By assuming extremely high risk premia, one could possibly improve the performance of consumption-based models in fitting credit spreads. However, extreme risk premia would also imply abnormally high expected returns for the stock market. In this section, I show that the disappointment aversion model in (10) can match moments for the equity premium, the price-dividend ratio, and the risk-free rate reasonably well, with the same preference parameters and state variable dynamics from Table III. Equity returns, the risk-free rate, and the price-dividend ratio, have been simulated according to the expressions in (12), (11), and *Proposition 2* respectively, while sample moments are calculated using the data described in subsection 2.1.

According to Table VI, simulated stock market returns for the disappointment aversion model have a high mean (6.653% vs. 6.581% in the data), are quite volatile (15.049% vs. 17.216% in the data), are i.i.d. $\big(\rho(r_{m,t,t+1}, r_{m,t-1,t}) = 0.035$ vs. -0.030 in the data $\big)$, and are positively correlated with consumption growth $\big(\rho(r_{m,t,t+1}, \Delta c_{t-1,t}) = 0.463$ vs. 0.503 in the data $\big)$. The disappointment model also predicts a highly autocorrelated (0.650 vs. 0.696 in the data) and low mean (0.962% vs. 0.928% in the data) risk-free rate, yet the variance of the simulated risk-free rate is substantially smaller than the sample estimate (1.163% vs. 2.727%). Finally, even though results for the price-dividend ratio are fairly accurate, especially in terms of persistence (0.891 vs. 0.950 in the data), the simulated price-dividend in the disappointment averse economy has lower mean (3.000 vs. 3.433),

[50] Beeler and Campbell (2012), Bonomo et al. (2011).

and is less volatile (0.227 vs. 0.467) than the one obtained from the CRSP database.

Traditional consumption-based asset pricing models with time-separable power utility need exorbitant values for the risk aversion coefficient, around 50 for annual data[51] and around 150 for quarterly data[52], in order to match expected stock market returns. Further, extremely large risk aversion parameters lead to very volatile risk-free rates[53]. Non-separable Epstein-Zin preferences without first-order risk aversion effects, also require large coefficients of risk aversion, around 30[54], to match expected stock market returns, unless we assume a very persistent process for expected consumption growth[55]. These empirical discrepancies are ingeniously concealed by the benchmark models in (1) and (3) or any other model that directly uses risk-neutral pricing because these models do not explicitly account for investor preferences. In contrast, the disappointment aversion discount factor in (10) can generate realistic asset pricing moments using parameter values that are consistent with clinical results for behavior under uncertainty.

### 4.4 Comparative results for alternative preference parameters

The main goal of the paper is to examine whether disappointment aversion preferences can explain asset prices across different financial markets with risk and disappointment aversion parameters calibrated to experimental findings. This section performs a sensitivity analysis on preference parameters for the disappointment aversion discount factor in (10). Comparative results focus on the two parameters that affect risky choices, the risk aversion parameters $\alpha$ and the disappointment aversion coefficient $\theta$. The rest of the parameters in Table III as well as model dynamics from (7)-(9) are kept constant.

The choice of alternative parameter values for the disappointment aversion model serves three purposes. First, alternative parameters need to be close to clinical estimates. Second, alternative parameter values should be able to identify the marginal importance of the first and second-order

[51] Mehra and Prescott (1985), Cochrane (2001), Yogo (2004), Liu et al. (2009), Routldege and Zin (2010).
[52] Aït-Sahalia et al. (2004), Yogo (2004).
[53] Weil (1989), Delikouras (2013).
[54] Routldege and Zin (2010), Delikouras (2013).
[55] Bonomo et al. (2011), Beeler and Campbell (2012), Delikouras (2013)

risk aversion channels. Finally, the choice of these alternative values ought to guarantee that the multipliers $\{A_0, A_3\}$, and $\{A_{m,0}, A_{m,3}\}$ in *Propositions 1 and 2* are well defined and real[56].

For the first alternative scenario, the risk aversion parameter is set equal to -1 (second-order risk neutrality), and the disappointment aversion parameter is equal to 3. By setting $\alpha$ equal to -1, we are essentially downgrading the importance of consumption growth variance $\sigma_t^2$ as a state variable. This is done through the parameter $A_3$ which significantly decreases in magnitude, and even turns positive due to second-order risk neutrality. For the baseline disappointment model in Table V and Table VI where $\alpha$ is positive, $A_3$ is large in absolute value and negative.

According to Table VII, if we turn off the risk aversion channel, and slightly increase the magnitude for the disappointment aversion parameter, then the expected risk-free rate decreases relative to the baseline scenario in Table VI (0.519% vs. 0.962%) because the first-order precautionary savings motive in (11) intensifies. In contrast, expected equity premia remain essentially the same relative to the baseline disappointment model (5.676% vs. 5.691% for the baseline model in Table VI).

Even though the reduction in stock market risk premia is almost zero, the decrease in expected credit spreads relative to the baseline scenario in Table V is quite impressive: approximately -29 bps for 4yr maturities across all four cases. Results in Table VII suggest that although equity premia are insensitive to the second-order risk aversion channel, credit-spreads are hugely affected by setting $\alpha$ equal to -1. Because Baa defaults are very rare events, even the slightest change in systematic risk can lead to substantial changes in credit spreads. On the other hand, equity premia are not sensitive to second-order risk-neutrality because stock market returns are not related to rare events.

For the second alternative scenario, the disappointment aversion channel is turned off ($\theta = 0$), and the risk aversion parameter is set equal to 5. Although 5 is a reasonable value in the asset

[56]The systems of equations in *Proposition 1* and *Proposition 2* impose constraints on the magnitude of the risk aversion parameter. For instance, if $\alpha$ is greater than 8.7, then the solutions to the quadratic equations for $A_3$ and $A_{m,3}$ are imaginary numbers, unless we specify different parameters for the state variable dynamics in (7)-(9). In contrast, there are no constraints imposed on $\theta$, because $A_2$ and $A_{m,2}$ are solutions to linear equations.

pricing literature, experimental results imply that $\alpha$ cannot be greater than 2.8[57]. In the absence of the first-order risk aversion mechanism, there is an important decrease in average credit spreads relative to the baseline calibration for the disappointment model in Table V: approximately -38 bps for 4yr maturities across all four cases. Furthermore, expected excess stock returns are almost zero, while the expected risk-free rate doubles in magnitude relative to the baseline scenario (2.000% vs. 0.962%), because, without disappointment aversion, the precautionary savings motive attenuates.

Essentially comparative results for preference parameters in Table VII highlight the importance of both first- and second-order risk aversion terms in generating measurable credit spreads. On the other hand, disappointment aversion preferences can generate realistic equity premia even if we turn off the effects of second-order risk aversion. Asset pricing models that do not include disappointment aversion preferences, usually substitute first-order risk aversion effects with highly persistent shocks to the stochastic discount factor through the habit or the long-run risk channels[58].

# 5 Related literature

Before concluding the discussion on the credit spread puzzle, I will briefly relate the disappointment framework to some key results in the corporate bond literature. Merton (1974) was one of the first authors to propose a unified framework for the valuation of corporate securities, bonds and equities, which are priced as contingent claims written on a firm's assets in place. Previous results on the inability of the Merton model to match credit spreads date back to Jones et al. (1984), while Huang and Huang (2012) show that the credit puzzle is robust to a variety of specifications for the risk-neutral dynamics of asset returns.

In Merton's early framework, there were no taxes, no bankruptcy costs, and capital structure choices were irrelevant. Leland (1994) and Leland and Toft (1996), extend Merton's framework to account for tax benefits of debt, bankruptcy costs, and optimal leverage decisions. Goldstein et al. (2001) also propose an asset pricing model for corporate bonds in which the government,

[57] Choi et al. (2007).

[58] Campbell and Conchrane (1999), Bansal and Yaron (2004).

bondholders, and equityholders all have stakes in the firm's EBIT-generating process. In the Goldstein et al. model, bond coupons, default, and leverage are all endogenous decisions. However, all these papers rely directly on risk-neutral dynamics, remain silent on investor preferences, and do not really focus on the empirical performance of these models across financial markets.

Bhamra et al. (2010) also propose a unified framework to explain the equity premium and the credit-spread puzzle. Even though they assume Epstein-Zin utility for the aggregate investor, they use risk-neutral dynamics, and provide a comprehensive model with endogenous capital structure and default decisions in order to resolve the equity premium and credit spread puzzles. Nevertheless, their model generates a credit spread of only 45 bps for 5yr maturities an 75 bps for 10yr maturities in contrast to 103 bps and 112 bps. Moreover, their model implies an equity risk premium of 3.19%, about half the one in the data (6.653%).

Chen et al. (2009) compare the habit model of Campbell and Cochrane (1999), and the long-run risk model of Bansal and Yaron (2004) for their ability to explain the credit spread puzzle while generating possible moments for the stock market. Although, both models can resolve the equity premium puzzle, the long-run risk model has difficulties in generating measurable credit-spreads, while the habit-model needs to be combined with countercyclical default boundaries or procyclical recovery rates in order to fit Baa-Aaa credit spreads. Finally, Chen (2010) provides a parsimonious general equilibrium model in order to resolve the credit spread and underleverage puzzles, while matching moments for equity risk premia. However, he focuses only on 10yr maturities, while he sets the risk aversion coefficient equal to 6.5, and the EIS equal to 1.5., even though a number of empirical results[59] suggest that the EIS cannot be larger than one.

Table VIII shows model implied credit spreads and expected equity premia for some recent works previous works in the bond pricing lietrature. Almost all results focus on 4yr or 10yr maturities and remain silent on longer maturities. This paper is the first one to examine the credit-spread puzzle for 15yr+ maturities, and the first one to impose a stochastic discount factor which is micro-founded on experimental evidence for behavior under risk.

[59] Hall (1988), Bonomo et al. (2011), and Beeler and Campbell (2012).

# 6 Conclusion

The aim of this paper is to examine whether disappointment aversion preferences can help us resolve the credit spread puzzle within a consumption-based asset pricing framework of an endowment economy. Given the relative success of first-order risk aversion preferences in explaining other stylized facts in financial markets, the disappointment aversion discount factor seems a natural candidate for correctly pricing corporate bonds. However, the first-order risk aversion mechanism implied by disappointment aversion is not powerful enough to map the low probabilities of Baa default into measurable Baa-Aaa credit spreads.

Only when the disappointment model is combined with countercyclical losses given default and default boundaries, can disappointment aversion preferences resolve the credit spread puzzle. This is in line with the conclusions in Chen et al. (2009), according to which neither the habit nor the long-run risk models can price Baa-Aaa credit spreads, unless we assume additional sources of risk such as procyclical recovery rates, countercyclical default boundaries or stochastic idiosyncratic volatility.

Furthermore, by fitting credit spreads for the short and medium term, the disappointment model tends to overestimate credit spreads for long maturities (15yr+). Traditional consumption-based asset pricing models (habit, long-run risk) have only been tested against 4yr or 10yr bond maturities. It would be interesting to examine the predictions of these models for longer maturities, as well as for the ultra short-run (up to 4 yrs).

Another direction for future research is to introduce disappointment aversion preferences in a world where capital structure choices matter so as to endogenize default decisions. Despite all the above, the disappointment model is quite successful in explaining both corporate bond prices, as well as key moments for stock market returns, the risk-free rate, and the price-dividend ratio using preference parameters that are consistent with experimental data for choices under uncertainty.

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# 7 Tables

**Table I Average default rates, and expected credit spreads for Baa and Aaa bonds**

**Panel A: average default rates for Aaa and Baa bonds (1970-2011)**

| | 1 year | 4 year | 10 year | 15 year | 20 year |
|---|---|---|---|---|---|
| **Aaa** | 0.000% | 0.035% | 0.476% | 0.884% | 1.045% |
| **Baa** | 0.181% | 1.379% | 4.649% | 8.632% | 12.315% |

**Panel B: average Baa-Aaa credit spreads (bps)**

| | **sample** | **maturity** | | |
|---|---|---|---|---|
| | **period** | **short** | **medium** | **long** |
| Moody's Baa-Aaa Corp. Bond Yield | 1920-2011 | | | 118 |
| Barclays US Agg. Corp. Baa-Aaa | 1974-2011 | | 128 | 112 |
| BofA US Corp. BBB-AAA | 2001-2011 | 155 | 128 | 102 |
| Thomson-Reuters US Corp. Baa-Aaa | 2003-2011 | 157 | 180 | |
| Duffee (1998) | 1985-1995 | 75 | 70 | 105 |
| Chen et al. (2009) | 1970-2001 | 109 | | |
| Huang and Huang (2012) | 1973-1993 | 103 | 131 | |
| benchmark model in (1) | | 51 | 77 | 97 |
| stochastic recovery rates in (3) | | 58 | 87 | 112 |

Average default rates for Baa and Aaa-rated firms in Panel A are from the Moody's 2012 annual report. Panel B summarizes sample average credit spreads used in previous studies, as well as expected credit spreads implied by the models in (1) and (3). In Duffee (1998), short maturity is 2yr-7yr, medium is 7yr-15yr, and long maturity is 15yr-30yr. Chen et al. (2009) consider 4yr maturities, while Huang and Huang (2012) consider 4yr and 10yr maturities. For the Moody's indices, long maturity is between 20yr and 30yr. For the Barclays indices, medium maturity is 1yr-10yr, and long maturity is 10yr+. For the BofA indices, short maturity is 1yr-5yr, medium is 7yr-10yr, and long maturity is 15yr+. For the Thomson-Reuters indices, short maturity is 4yr and medium maturity is 10yr. Finally, for the benchmark and stochastic recovery rates models in (1) and (3), short maturity is 4yr, medium maturity is 10yr, and long maturity is 15yr.

**Table II OLS regression of recovery rates on aggregate consumption growth (1982-2011)**

| | recovery rates $1 - L_t$ |
|---|---|
| $\Delta c_{t-1,t}$ | 4.461 |
| | (3.036) |
| $R^2$ | 24.767% |

Table II shows results for the OLS regression of recovery rates on contemporaneous consumption growth. Recovery rates for senior subordinate debt are from the Moody's 2012 report. $\hat{a}_{rec,c}$ is the OLS estimate with the $t$-statistic in parenthesis.

**Table III Preference parameters and state variable moments for the baseline disappointment model**

| variable | variable description | variable value |
|---|---|---|
| $EIS$ | elasticity of intetemporal substitution | 1 |
| $\delta$ | generalized disappointment aversion | 1 |
| $\beta$ | rate of time preference | 0.9955 |
| $\alpha$ | risk aversion | 1.8000 |
| $\theta$ | disappointment aversion | 2.0303 |
| $\mu_c$ | consumption growth constant | 0.0091 |
| $\phi_c$ | consumption growth autocorrelation | 0.5026 |
| $\mu_\sigma$ | volatility constant | 0.0004 |
| $\phi_\sigma$ | volatility autocorrelation | 0.9715 |
| $\nu_\sigma$ | volatility of volatility | 0.0017 |
| $\mu_d$ | dividend growth constant | -0.0367 |
| $\phi_d$ | sensitivity dividend growth to consumption growth | 3 |
| $\sigma_d$ | volatility parameter for dividend growth | 7.1664 |
| $\mu_e$ | earnings growth constant | -0.0367 |
| $\phi_e$ | sensitivity of earnings growth to consumption growth | 3 |
| $\sigma_e$ | volatility parameter for earnings growth | 2.2011 |
| $\sigma_i$ | idiosyncratic volatility | 0.2100 |
| $\bar{z}_m$ | linearization constant for the price-payout ratio in (25) | 3 |
| $\bar{x}$ | linearization constant for the normal c.d.f. in (30) | 0 |

Table III summarizes preference coefficients for the disappointment aversion stochastic discount factor in (10), as well as moment parameters for aggregate state dynamics in equations (7)-(9).

## Table IV Simulation results for aggregate state variables

| | **1946-2011** | **simulated economy** |
|---|---|---|
| $\mathbb{E}[\Delta c_{t,t+1}]$ | 1.838% | 1.834% |
| $\mathbf{Vol}(\Delta c_{t,t+1})$ | 1.346% | 1.914% |
| $\rho(\Delta c_{t-1,t}, \Delta c_{t,t+1})$ | 0.502 | 0.504 |
| $\mathbb{E}[\Delta d_{m,t,t+1}]$ | 2.107% | 1.796% |
| $\mathbf{Vol}(\Delta d_{m,t,t+1})$ | 13.079% | 13.232% |
| $\rho(\Delta d_{m,t-1,t}, \Delta d_{m,t,t+1})$ | -0.278 | 0.093 |
| $\rho(\Delta d_{m,t,t+1}, \Delta c_{m,t,t+1})$ | 0.286 | 0.218 |
| $\mathbb{E}[\Delta e_{m,t,t+1}]$ | -3.831% | 1.819% |
| $\mathbf{Vol}(\Delta e_{m,t,t+1})$ | 7.057% | 6.784% |
| $\rho(\Delta e_{m,t-1,t}, \Delta e_{m,t,t+1})$ | 0.114 | 0.360 |
| $\rho(\Delta e_{m,t,t+1}, \Delta c_{m,t,t+1})$ | 0.487 | 0.425 |
| $\mathbb{E}[\sigma_t]$ | 2.697% | 1.498% |
| $\mathbf{Vol}(\sigma_t)$ | 0.458% | 0.691% |
| $\rho(\sigma_t)$ | 0.922 | 0.967 |

Table IV shows sample and simulated moments for aggregate state variables. $\mathbb{E}$ is expected value, **Vol** is volatility, and $\rho$ is the correlation coefficient. $\Delta c_{t-1,t}$, $\Delta d_{m,t-1,t}$, and $\Delta e_{m,t-1,t}$ are real consumption, real dividend, and real earnings growth respectively. $\sigma_t$ is consumption growth volatility. Simulated values for the state variables are according to the system in (7)-(9), using parameter values from Table III. Sample moments are calculated using the data described in subsection 2.1. Sample results for consumption growth volatility $\sigma_t$ are according to the volatility process in Chen et al. (2006) p. 31. All variables have been simulated for 100,000 years.

## Table V Default boundaries, average default rates, and expected Baa-Aaa credit spreads for the disappointment model

**Panel A: default boundaries for the simulated economy**

**cases 1 & 2: constant default boundary**

| | **4 yr** | | **10 yr** | | **15 yr** | |
|---|---|---|---|---|---|---|
| | Baa | Aaa | Baa | Aaa | Baa | Aaa |
| $a_{def,0}$ | -0.998 | -1.600 | -1.108 | -1.832 | -1.007 | -1.970 |

**cases 3 & 4: time-varying default boundary**

| | **4 yr** | | **10 yr** | | **15 yr** | |
|---|---|---|---|---|---|---|
| | Baa | Aaa | Baa | Aaa | Baa | Aaa |
| $a_{i,def,0}$ | -1.085 | -1.790 | -1.150 | -1.920 | -1.032 | -2.040 |
| $a_{def,c}$ | -4 | -4 | -4 | -4 | -4 | -4 |
| $a_{def,\sigma}$ | 4 | 4 | 4 | 4 | 4 | 4 |

**Panel B: average default rates for the simulated economy**

| | **4 year** | | **10 year** | | **15 year** | |
|---|---|---|---|---|---|---|
| | | Aaa | Baa | Aaa | Baa | Aaa |
| **case 1** | 1.379% | 0.036% | 4.655% | 0.469% | 8.626% | 0.888% |
| **case 2** | 1.378% | 0.035% | 4.665% | 0.471% | 8.627% | 0.891% |
| **case 3** | 1.374% | 0.035% | 4.666% | 0.469% | 8.640% | 0.881% |
| **case 4** | 1.385% | 0.036% | 4.651% | 0.472% | 8.663% | 0.869% |
| **1970-2011 sample** | 1.375% | 0.035% | 4.649% | 0.476% | 8.632% | 0.884% |

**Panel C: average simulated Baa-Aaa credit spreads according to the disappointment model**

| | **4 year** | | | **10 year** | | | **15 year** | | |
|---|---|---|---|---|---|---|---|---|---|
| | Baa-$r_f$ | Aaa-$r_f$ | Baa-Aaa | Baa-$r_f$ | Aaa-$r_f$ | Baa-Aaa | Baa-$r_f$ | Aaa-$r_f$ | Baa-Aaa |
| **case 1** | 65 | 3 | 62 | 122 | 27 | 95 | 153 | 40 | 113 |
| **case 2** | 81 | 4 | 77 | 151 | 37 | 114 | 191 | 55 | 136 |
| **case 3** | 78 | 4 | 74 | 139 | 33 | 106 | 167 | 47 | 120 |
| **case 4** | 106 | 6 | 100 | 176 | 47 | 129 | 215 | 67 | 148 |
| **eq.** (1) | | | 51 | | | 77 | | | 97 |
| **eq.** (3) | | | 58 | | | 87 | | | 112 |
| **sample** | | | 103 | | | 131 | | | 112 |

Default boundaries for the simulated economy (Panel A) are expressed in terms of asset log-returns. I consider four different cases for the disappointment aversion discount factor: 1) constant recovery rates and default boundaries, 2) procyclical recovery rates according to (19) and constant default boundaries, 3) constant recovery rates and countercyclical default boundaries according to (20), and 4) procyclical recovery rates and countercyclical default boundaries. $a_{i,def,0}$ is a constant, and $a_{def,c}$, $a_{def,\sigma}$ are the loadings on consumption growth and consumption growth volatility in the expression for default boundaries (20). Panel B shows average default rates for the simulated data as well as for the Moody's sample. Finally, Panel C shows expected Baa-Aaa credit spreads (bps) for the simulated disappointment model according to (16) and (17). Benchmark expected credit spreads are from the models in (1) and (3). Sample average credit spreads are from Huang and Huang (2012) for 4yr and 10yr bonds, and from the Barclays corporate indices for long maturity bonds.

## Table VI Simulation results for the stock market and the risk-free rate according to the disappointment model

| | **1946-2011** | **simulated economy** |
|---|---|---|
| $\mathbb{E}[r_{m,t,t+1}]$ | 6.581% | 6.653% |
| $\mathbf{Vol}(r_{m,t,t+1})$ | 17.216% | 15.049% |
| $\rho(r_{m,t-1,t}, r_{m,t,t+1})$ | -0.030 | 0.035 |
| $\rho(r_{m,t,t+1}, c_{t,t+1})$ | 0.503 | 0.463 |
| Sharpe | 0.328 | 0.378 |
| $\mathbb{E}[r_{f,t,t+1}]$ | 0.928% | 0.962% |
| $\mathbf{Vol}(r_{f,t,t+1})$ | 2.727% | 1.163% |
| $\rho(r_{f,t-1,t}, r_{f,t,t+1})$ | 0.696 | 0.650 |
| $\mathbb{E}[z_{m,t}]$ | 3.433 | 3.000 |
| $\mathbf{Vol}(z_{m,t})$ | 0.427 | 0.227 |
| $\rho(z_{m,t}, z_{m,t-1})$ | 0.950 | 0.891 |
| Baa Sharpe ratio | 0.220 | 0.218 |

Table VI shows sample and simulated moments for the stock market and the risk-free rate. $r_{m,t,t+1}$ are real stock market returns, $r_{f,t,t+1}$ is the one-year real risk-free rate, $z_{m,t}$ is the aggregate price-dividend ratio, and Baa Sharpe ratio is the equity Sharpe ratio for the median Baa firm according to Chen et al. (2009). Equity returns, the risk-free rate, and the price-dividend ratio, have been simulated according to the expressions in (12), (11), and *Proposition 2* respectively, while sample moments are calculated using the data described in subsection 2.1.

**Table VII Simulation results for alternative preference parameters in the disappointment aversion model**

| | **baseline** $\theta = 2.03, \alpha = 1.8$ | **scenario I** $\theta = 3, \alpha = -1$ | **scenario II** $\theta = 0, \alpha = 5$ |
|---|---|---|---|
| **case 1** Baa-Aaa 4yr | 62 | 43 | 33 |
| **case 2** Baa-Aaa 4yr | 77 | 48 | 39 |
| **case 3** Baa-Aaa 4yr | 74 | 43 | 38 |
| **case 4** Baa-Aaa 4yr | 100 | 61 | 51 |
| $\mathbb{E}[r_{m,t,t+1} - r_{f,t,t+1}]$ | 5.691% | 5.676% | 0.000% |
| **Vol**$(r_{m,t,t+1})$ | 15.049% | 16.275% | 14.367% |
| $\mathbb{E}[r_{f,t,t+1}]$ | 0.962% | 0.519% | 2.000% |
| **Vol**$(r_{f,t,t+1})$ | 1.163% | 1.247% | 0.987% |

Table VII shows simulation results for expected Baa-Aaa credits spreads (bps) and the stock market when the disappointment aversion discount factor from (10) is calibrated to alternative preference parameters. In the baseline case of Table V and Table VI, $\theta = 2.03$ and $\alpha = 1.8$. For the first alternative scenario, $\theta$ is 3 and $\alpha$ is -1 (second-order risk neutrality). In the second alternative scenario, $\theta$ is zero (no disappointment aversion effect) and $\alpha$ is 5.

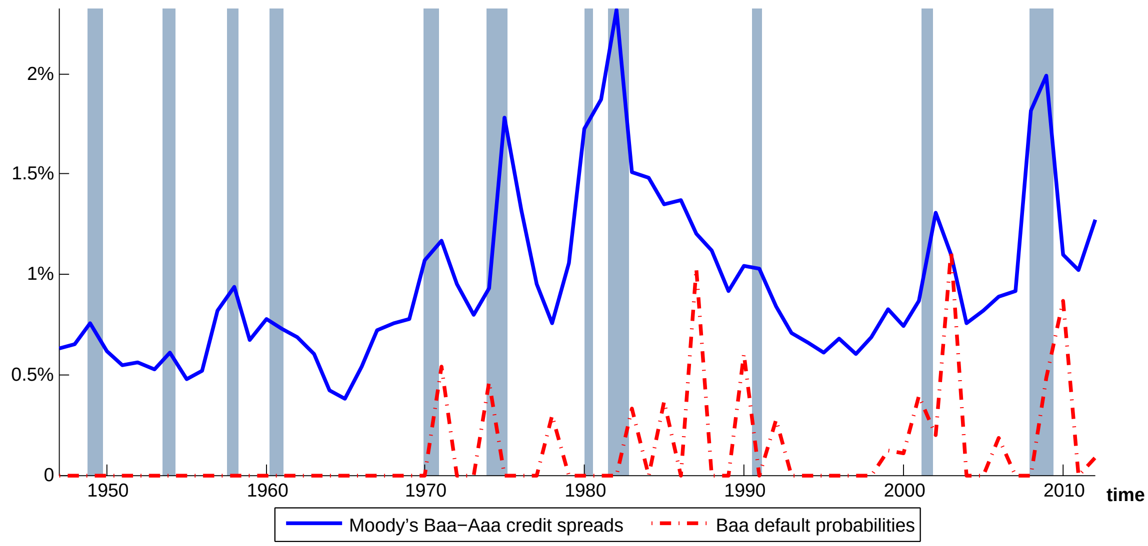
**Table VIII Model implied expected credit spreads and expected equity risk premia in the previous literature**

| | model characteristics | maturity 4yr | 10yr | 15yr | $\mathbb{E}[r_{m,t,t+1} - r_{f,t,t+1}]$ |
|---|---|---|---|---|---|
| Chen et al. (2009) | habit, $\alpha = 2.45$ countercyclical boundaries | 107 | 123 | | 7.30% |
| Chen et al. (2009) | long-run risk, EIS=2, $\alpha = 7.5$ | 52 | | | 7.40% |
| Chen (2010) | endogenous default, EIS=1.5, $\alpha = 6.5$ | | 105 | | 6.71% |
| Bhamra et al. (2010) | endogenous default, no preferences | 45 (5yr) | 75 | | 3.19% |
| Huang & Huang (2012) | Goldstein et al. (2001) model | 31 | 40 | | |
| case 4 in Table V | EIS=1, $\alpha = 1.8$, $\theta = 2.03$, countercyclical boundaries & losses given default | 100 | 129 | 148 | 6.65% |

Table VIII shows model implied expected credit spreads (bps) and equity risk premia calculated in prior works. "no preferences" implies that expected credit spreads have been calculated using risk-neutral measures, without modeling investor preferences. $\alpha$ is the risk aversion parameter, EIS is the elasticity of intertemporal substitution, and $\theta$ is the disappointment aversion parameter. $\mathbb{E}[r_{m,t,t+1} - r_{f,t,t+1}]$ is the equity risk premium.

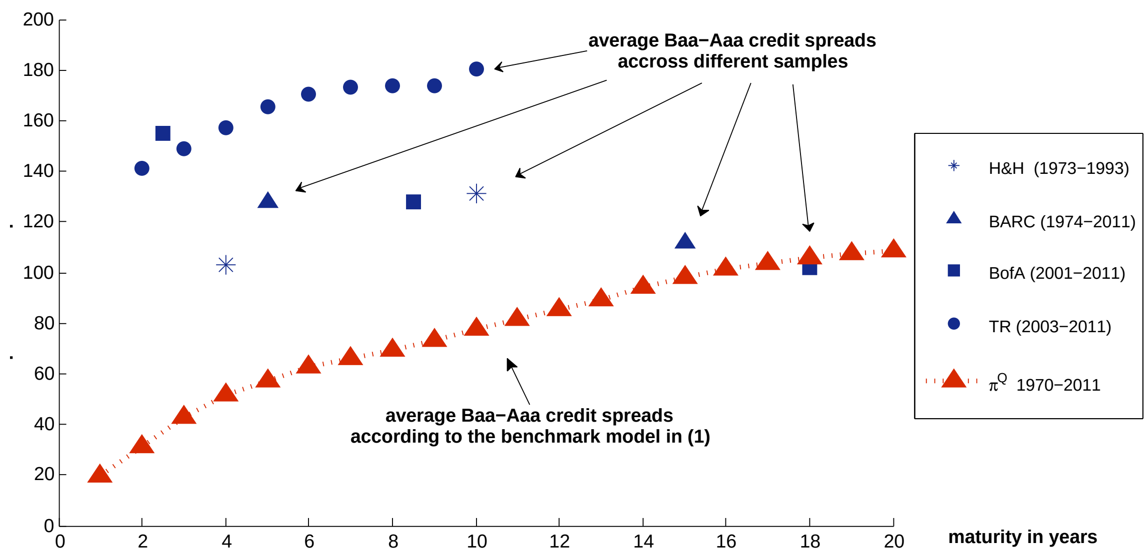
# 8 Figures

## Figure I Baa-Aaa credit spreads, and Baa default rates for the 1946-2011 period




The solid line in Figure I shows Baa-Aaa credit credit spreads for the Moody's Seasoned Aaa and Baa Corporate Bond Indices. The dashed line shows annual Baa default rates from the Moody's 2012 report. Shaded areas are NBER recessions.

**Figure II Sample and fitted expected Baa-Aaa credit spreads according to the benchmark model in (1)**




The dotted line in Figure II shows expected credit spreads (bps) according to the benchmark model in (1) for maturities from 1 up to 20 years. The scattered points are mean Baa-Aaa credit spreads for the three sets of corporate bond indices (Barclays, BofA, and Thomson-Reuters) and the Huang and Huang (2012) sample. If the benchmark model in (1) could succefully explain expected returns, then the scattered points should be part of the credit spread term structure.

**Figure III Recovery rates for senior subordinate bonds during the 1982-2011 period**

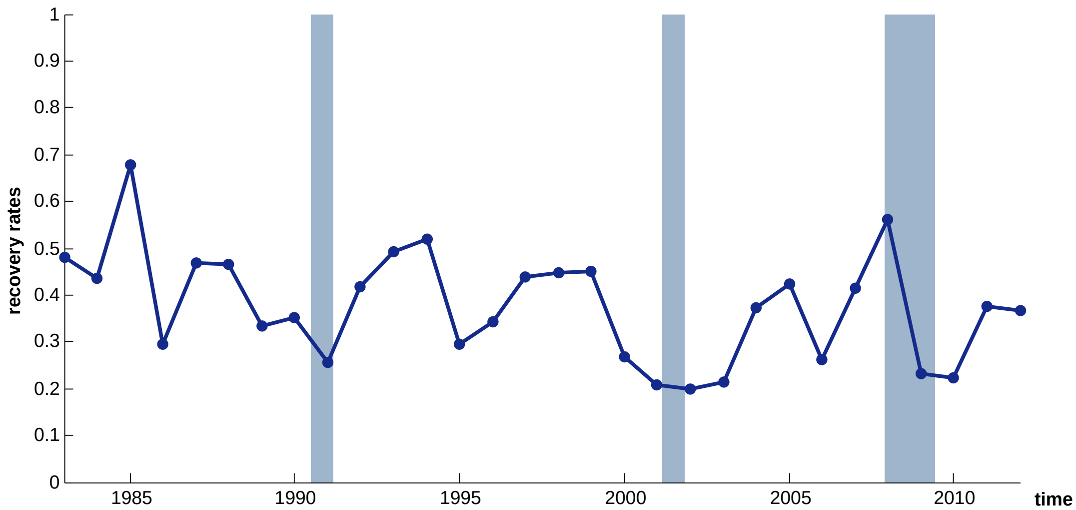



Figure III shows recovery rates for senior subordinate bonds from the Moody's 2012 report. Shaded areas are NBER recessions.

**Figure IV Sample and fitted expected Baa-Aaa credit spreads according to the disappointment model in (15)**

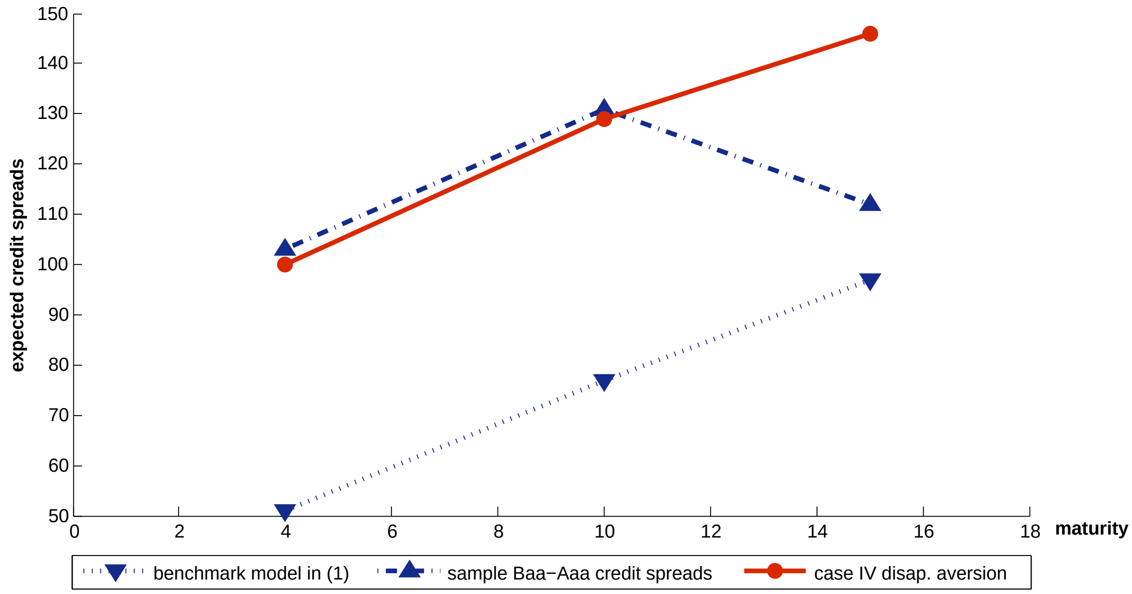



Figure IV shows fitted expected Baa-Aaa credit spreads across maturities according to the benchmark model in (1) and case 4 of the disappointment model in (17). Sample expected credit spreads are from Huang and Huang (2012) for 4yr and 10yr maturities. Sample credit spreads for 15yr maturities are from the Barclays corporate indices.

# Appendix

## Appendix A Bond yields according to the benchmark model in (1)

Suppose that single-period, cum-payout, asset log-returns for firm $i$ $r_{i,t,t+1}$ are i.i.d. normal random variables with constant mean $\tilde{\mu}_i - \frac{1}{2}\sigma_i^2 \in \mathbb{R}$, and volatility $\sigma_i \in \mathbb{R}_{>0}$. Let $\Delta_i$ be the constant log-payout yield: $\Delta_i = log\big(1 + \frac{O_{i,t+1}}{P_{i,t+1}}\big)$. $O_{i,t+1}$ is the payout, and $P_{i,t+1}$ is the price of assets in place[60]. Ex-payout log-returns $r^x_{i,t,t+1}$ are equal to cum-payout log-returns minus the log-payout yield ($r^x_{i,t,t+1} = r_{i,t,t+1} - \Delta_i$). Hence, $r^x_{i,t,t+1}$ are also normal random variables, and, in a discrete-time setting, can be expressed as

$$r^x_{i,t,t+1} = \tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2 + \sigma_i \epsilon_{i,t+1},$$

with $\epsilon_{i,t+1}$ i.i.d. $N(0,1)$ shocks. Moreover, $T$-period, ex-payout returns are also i.i.d. normal random variables with mean $(\tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2)T$ and volatility $\sigma_i\sqrt{T}$.

Suppose that the single-period, log risk-free rate is constant and equal to $r_f$. Assume also that there are no taxes, and that default boundaries $D_{i,T}$ as well as losses given default $L$ are constant. Let $\pi^{\mathbb{P}}_{i,t,t+T}$ be the physical probability of default for a $T$-period, zero-coupon bond

$$\pi^{\mathbb{P}}_{i,t,t+T} = \mathbb{P}_t\Big(P_{i,t+T} < D_{i,T}\Big).$$

Similar to the original Merton model, default can only happen at the expiration date $t+T$, but unlike the Merton model, the default boundary is not necessarily equal to the face value of debt. Normalizing current period firm value $P_{i,t}$ to one, the physical probability of default $\pi^{\mathbb{P}}_{i,t,t+T}$ can be expressed in terms of asset log-returns $r^x_{i,t,t+1}$

$$\pi^{\mathbb{P}}_{i,t,t+T} = N\Big(\frac{logD_{i,T} - (\tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}\Big), \tag{21}$$

in which $N()$ is the standard normal c.d.f.. Because asset log-returns are i.i.d. with constant mean and standard deviation, $\pi^{\mathbb{P}}_{i,t,t+T}$ depends only on maturity $T$, hence $\pi^{\mathbb{P}}_{i,t,t+T} = \pi^{\mathbb{P}}_{i,T}$. Finally, using the inverse of the normal c.d.f. $N^{-1}()$, we can express the log-default boundary $logD_{i,T}$ from (21) as a function of the physical probability of default $\pi^{\mathbb{P}}_{i,T}$, expected returns for assets in place $\tilde{\mu}_i$, and asset return volatility $\sigma_i$

$$logD_{i,T} = (\tilde{\mu}_i - \Delta_i - \frac{1}{2}\sigma_i^2)T + N^{-1}\big(\pi^{\mathbb{P}}_{i,T}\big)\sigma_i\sqrt{T}. \tag{22}$$

The continuous-time framework in Black and Scholes (1973) allows for frictionless trading and hedging between underlying and derivative securities. An immediate consequence of continuous trading is that if asset returns under the physical measure are normally distributed with constant mean and volatility, then asset returns under the risk-neutral measure are also normally distributed

[60] In the benchmark case, the price-payout ratio is constant.

with the same variance, and mean equal to the risk-free rate.

In a discrete-time setting, continuous trading is not possible. However, according to *Lemma 1* in Appendix F.1, the risk-neutral density for asset returns is normal, provided that aggregate preferences over consumption are described by a CRRA utility function, and that aggregate consumption growth is a log-normal random variable. Hence, assuming that all conditions for *Lemma 1* hold, $T$-period, ex-payout asset log-returns $r^x_{i,t,t+1}$ under the risk-neutral measure are normally distributed with mean $(r_f - \Delta_i - \frac{1}{2}\sigma_i^2)T$, and volatility $\sigma_i\sqrt{T}$.

Let $y_{i,t,t+T}$ be the continuously compounded yield to maturity for a $T$-period, zero-coupon bond written on firm $i$ at time $t$. Then, under the risk-neutral measure

$$e^{-Ty_{i,t,t+T}} = e^{-Tr_f}\Big(1 - LN\big(\frac{logD_{i,T} - (r_f - \Delta_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}\big)\Big). \tag{23}$$

Taking logs in (23), and substituting $logD_{i,T}$ with the expression from (22), we get that

$$y_{i,t,t+T} - r_f = -\frac{1}{T}log\Big[1 - LN\Big(N^{-1}\big(\pi^{\mathbb{P}}_{i,T}\big) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big].$$

Since the right-hand side above and the risk-free rate are constants, we conclude that

$$\mathbb{E}[y_{i,t,t+T} - r_f] = -\frac{1}{T}log\Big[1 - LN\Big(N^{-1}\big(\pi^{\mathbb{P}}_{i,T}\big) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big].$$

## Appendix B Bond yields according to the model in (3) with time-varying recovery rates

Suppose that recovery rates are the same across all bonds, and depend only on consumption growth

$$1 - L_{t+T} = a_{rec,0} + a_{rec,c}\Delta c_{t+T-1,t+T}.$$

Suppose also that all the assumptions in Appendix A hold. Then, the yield-to-maturity for a zero-coupon, T-period bond is given by[61]

$$e^{-Ty_{i,t,t+T}} = e^{-Tr_f}\mathbb{E}^{\mathbb{Q}}_t\Big[\mathbb{E}^{\mathbb{Q}}_t\big[1 - (1 - a_{rec,0} - a_{rec,c}\Delta c_{t+T-1,t+T})\mathbf{1}\big\{r_{i,t,t+T} < logD_{i,T}\big\}|\Delta c_{t+T-1,t+T}\big]\Big],$$

in which $\mathbb{E}^{\mathbb{Q}}_t$ is the expectation under the risk-neutral measure. Further algebra implies that

$$e^{-Ty_{i,t,t+T}} = e^{-Tr_f}\mathbb{E}^{\mathbb{Q}}_t\Big[1 - (1 - a_{rec,0} - a_{rec,c}\Delta c_{t+T-1,t+T})N\big(\frac{logD_{i,T} - (r_f - \Delta_i - \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}}\big)\Big]$$

According to Appendix F.3, under the risk neutral measure, log-consumption growth is a normal random variable with volatility $\sigma_c$, and mean $\tilde{\mu}_c - \frac{\tilde{\mu}_m - r_f}{\rho_{m,c}\sigma_m}\sigma_c$. $\frac{\tilde{\mu}_m - r_f}{\sigma_m}$ is the stock market Sharpe ratio, and $\rho_{m,c}$ is the correlation between stock market returns and consumption growth. Using

[61]Under the risk neutral measure $\mathbb{Q}$, asset returns $r_{i,t,t+1}$ and consumption growth are independent.

the expression for the default boundary $logD_{i,T}$ from (22), we obtain

$$e^{-T(y_{i,t,t+T}+r_f)} = \Big[1 - \Big(\underbrace{1 - a_{rec,0} - a_{rec,c}\tilde{\mu}_c}_{\mathbb{E}[L_{t+T}]} + a_{rec,c}\frac{\tilde{\mu}_m - r_f}{\rho_{m,c}\sigma_m}\sigma_c\Big)N\Big(N^{-1}\big(\pi^{\mathbb{P}}_{i,T}\big) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big].$$

Since the right-hand side and the risk-free rate are constants, we conclude that

$$\mathbb{E}[y_{i,t,t+T} - r_f] = -\frac{1}{T}log\Big[1 - \Big(\mathbb{E}[L_{t+T}] + a_{rec,c}\frac{\tilde{\mu}_m - r_f}{\rho_{m,c}\sigma_m}\sigma_c\Big)N\Big(N^{-1}\big(\pi^{\mathbb{P}}_{i,T}\big) + \frac{\tilde{\mu}_i - r_f}{\sigma_i}\sqrt{T}\Big)\Big].$$

## Appendix C Intertemporal marginal rate of substitution for disappointment aversion preferences

Along an optimal consumption path, the Bellman equation for the representative investor's consumption-investment problem implies that

$$V_t = \big[(1-\beta)C_t^{\rho} + \beta\mu_t\big(V_{t+1}\big)^{\rho}\big]^{\frac{1}{\rho}},$$

where $\mu_t$ is the disappointment aversion certainty equivalent from (5). The expression for the stochastic discount factor is given by

$$M_{t,t+1} = \frac{\partial V_t/\partial C_{t+1}}{\partial V_t/\partial C_t},$$

in which

$$\partial V_t/\partial C_t = \frac{1}{\rho}V_t^{1-\rho}(1-\beta)\rho C_t^{\rho-1},$$

and

$$\begin{aligned}\partial V_t/\partial C_{t+1} = &\frac{1}{\rho}V_t^{1-\rho}\beta\rho\mu_t\big(V_{t+1}\big)^{\rho-1} \times \\ &(-\frac{1}{\alpha})\mathbb{E}_t\Big[\frac{V_{t+1}^{-\alpha}\ (1+\theta\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\})}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta\ >\ 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}]}\Big]^{-\frac{1}{\alpha}-1} \times \\ &(-\alpha)V_{t+}^{-\alpha-1}\frac{1+\theta\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\})}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta\ >\ 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}]}\frac{1}{\rho}V_{t+1}^{1-\rho}(1-\beta)\rho C_{t+1}^{\rho-1},\end{aligned}$$

to conclude that

$$M_{t,t+1} = \beta\Big(\frac{C_{t+1}}{C_t}\Big)^{(\rho-1)}\Big[\frac{V_{t+1}}{\mu_t\big(V_{t+1}\big)}\Big]^{-\alpha-\rho}\Big[\frac{1+\theta\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta\ >\ 1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1}\ <\ \delta\mu_t\}]}\Big].$$

## Appendix D  Asset returns and the price-payout ratio

Let $P_{m,t}$, $O_{m,t}$, $Z_{m,t} = (P/O)_{m,t}$ be the price, payout, and price-payout ratio of a generic financial claim $m$ written on a stream of aggregate payments. Depending on the asset we want to price, payouts can be aggregate dividends (equity), aggregate earnings (assets in place), or even aggregate consumption (claim on aggregate consumption). Let $R_{m,t+1}$ be the cum-payout, gross return for claim $m$, then

$$R_{m,t+1} = \frac{P_{m,t+1} + O_{m,t+1}}{P_{m,t}}.$$

Dividing and multiplying the numerator with $O_{m,t+1}$, the denominator with $O_{m,t}$, and taking logs, we can express log-returns $r_{m,t,t+1}$ in terms of log price-payout ratios $z_{m,t}$

$$r_{m,t,t+1} = log[e^{z_{m,t+1}} + 1] - z_{m,t} + \Delta o_{m,t,t+1}.$$

Using a first-order Taylor series approximation for $log[e^{z_{m,t+1}} + 1]$ around the point $z_{m,t+1} = \bar{z}_m$, asset returns can be expressed as

$$r_{m,t,t+1} \approx \kappa_{m,0} + \kappa_{m,1} z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1}, \tag{24}$$

where

$$\kappa_{m,1} = \frac{e^{\bar{z}_m}}{e^{\bar{z}_m} + 1} \in (0,1), \text{ and } \kappa_{m,0} = log[e^{\bar{z}_m} + 1] - \frac{e^{\bar{z}_m}}{e^{\bar{z}_m} + 1} \bar{z}_m. \tag{25}$$

Following a similar line of arguments[62], ex-payout, asset log-returns are given by

$$r^x_{m,t,t+1} = z_{m,t+1} - z_{m,t} + \Delta o_{m,t,t+1}.$$

## Appendix E  Simulation

### Appendix E.1  Simulation methodology

The consumption-Euler equation for a $T$-period, zero-coupon bond written on firm's $i$ assets reads

$$e^{-Ty_{i,t,t+T}} = \mathbb{E}_t\Big[\Big(\prod_{j=1}^{T} M_{t+j-1,t+j}\Big)\Big(\mathbf{1}\{r^x_{i,t,t+T} \geq D_{i,t+T}\} + (1 - L_{t+T})\mathbf{1}\{r^x_{i,t,t+T} < D_{i,t+T}\}\Big)\Big].$$

[62] For ex-dividend returns, no linearization is needed, since $r^x_{m,t,t+1} = log\Big(\frac{P_{m,t+1}/O_{m,t+1}}{P_{m,t}/O_{m,t}} \frac{O_{m,t+1}}{O_{m,t}}\Big)$.

Unlike the model in (1), default boundaries $D_{i,t+T}$ and losses given default $L_{t+T}$ are allowed to vary over time, and be functions of the state variables

$$D_{i,t+T} = a_{i,def,0} + a_{def,c}\big(\Delta c_{t+T-1,t+T} - \frac{\mu_c}{1-\phi_c}\big) + a_{def,\sigma}\big(\sigma_{t+T} - \frac{\mu_\sigma}{1-\phi_\sigma}\big),$$

and

$$1 - L_{t+T} = a_{rec,0} + a_{rec,c}\Delta c_{t+T-1,t+T}.$$

The first step in the simulation exercise is to discretize the consumption growth and consumption growth volatility space into $N_{\Delta c} = 20$ and $N_\sigma = 20$ equidistant points with a pace of $d_{\Delta c}$ and $d_\sigma$ respectively. The consumption growth space is truncated from above and below by $\hat{\mathbb{E}}[\Delta c_{t-1,t}] \pm 3\widehat{\mathbf{Vol}}(\Delta c_{t-1,t})$, whereas the volatility space is truncated from above and below by $\hat{\mathbb{E}}[\sigma_t] \pm 1.9\widehat{\mathbf{Vol}}(\sigma_t)$. The lower bound for the volatility space guarantees that initial values for volatility are always positive. $\hat{\mathbb{E}}[]$ and $\widehat{\mathbf{Vol}}()$ are the simulated unconditional mean and standard deviation from Table IV.

The second step is to choose starting values for consumption growth and consumption growth volatility. To do so, I iterate though all possible pairs of $\{\Delta c_l, \sigma_k\}$, $l = 1, 2, ..., N_{\Delta c}$, $k = 1, 2, ..., N_\sigma$. For each pair of starting values, I simulate $N = 10,000$ [63] paths for consumption growth, consumption growth volatility, and aggregate payout growth according to the system in (7)-(9), as well as idiosyncratic volatility shocks. Each path contains $T$ nodes, as many nodes as the life of of the zero-coupon security. Negative volatility observations are replaced with the lowest positive observation $\big(\hat{\mathbb{E}}[\sigma_t] - 1.9\widehat{\mathbf{Vol}}(\sigma_t)\big)$ from the initial grid.

At each node of the simulated paths for $\Delta c_{t-1,t}$ and $\sigma_t$, I can obtain values for the stochastic discount factor $M_{t+j-1,t+j}$ from (10), price-payout ratios according to *Proposition 2*, one-period, ex-payout asset log-returns for the median firm from (14), as well as losses given default and default boundaries according to (19) and (20). $T$-period, ex-payout, asset log-returns are simply given by the sum of single-period returns $r^x_{i,t,t+T} = \sum_{j=1}^T r^x_{i,t,t+j}$. Finally, for each simulated path, the discounted cashflow of a zero-coupon corporate bond is $\Big(\prod_{j=1}^T M_{t+j-1,t+j}\Big)\Big(\mathbf{1}\{r^x_{i,t,t+T} \geq D_{i,t+T}\} + (1-L_{t+T})\mathbf{1}\{r^x_{i,t,t+T} < D_{i,t+T}\}\Big)$. Averaging across all $N$ simulated paths, we obtain a value for the yield to maturity given the initial values for $\Delta c_{t-1,t}$ and $\sigma_t$

$$\hat{y}_{i,t,t+T}(\Delta c_l, \sigma_k) \approx -\frac{1}{T} log\Big[\frac{1}{n}\sum_{n=1}^N \Big(\prod_{j=1}^T M^{(n)}_{t+j-1,t+j}\Big)\Big(\mathbf{1}\{r^{x\ (n)}_{i,t,t+T} \geq D^{(n)}_{i,t+T}\} + (1-L^{(n)}_{t+T})\mathbf{1}\{r^{x\ (n)}_{i,t,t+T} < D^{(n)}_{i,t+T}\}\Big)\Big].$$

The objective is to match unconditional first moments for credit spreads. We therefore need to calculate unconditional expected values over the grid of starting values for consumption growth

[63] Simulation results are not affected by the number of simulation paths $N$ or the number of grid points ($N_{\delta c}$, $N_\sigma$), provided of course that these numbers are relatively large.

and consumption growth volatility using the p.d.f's. for $\Delta c_{t-1,t}$, $\sigma_t$, and $\sigma_{t-1}$

$$\mathbb{E}[\hat{y}_{i,t,t+T}(\Delta c_l, \sigma_k)] \approx \sum_{j=1}^{N_\sigma} \Big\{ \sum_{k=1}^{N_\sigma} \Big[ \sum_{l=1}^{N_{\Delta c}} \hat{y}_{i,t,t+T}(\Delta c_l, \sigma_k) f(\Delta c_l|\sigma_k, \sigma_j) d'_{\Delta c} \Big] f(\sigma_k|\sigma_j) d'_\sigma \Big\} f(\sigma_j) d''_\sigma,$$

where $f(\Delta c_l|\sigma_k, \sigma_j)$, $f(\sigma_k|\sigma_j)$, and $f(\sigma_j)$ are the p.d.f.'s for $\Delta c_{t-1,t}$, $\sigma_t$, and $\sigma_{t-1}$, while $d'_{\Delta c}$, $d'_\sigma$ and $d''_\sigma$ are constants such that $\sum_{l=1}^{N_{\Delta c}} f(\Delta c_l|\sigma_k, \sigma_j) d'_{\Delta c} = 1$, $\sum_{k=1}^{N_\sigma} f(\sigma_k|\sigma_j) d'_\sigma = 1$, and $\sum_{j=1}^{N_\sigma} f(\sigma_j) d''_\sigma = 1$. The p.d.f.'s for $\Delta c_{t-1,t}$, $\sigma_t$, and $\sigma_{t-1}$ are derived in Appendix E.2.

### Appendix E.2 Unconditional p.d.f.'s for consumption growth, and consumption growth volatility

According to (8), consumption growth volatility $\sigma_{t-1}$ is unconditionally normally distributed with mean $\mu_\sigma/(1-\phi_\sigma)$ and variance $\nu_\sigma^2/(1-\phi_\sigma^2)$. According to (7), conditional on $\sigma_{t-1}$, $\Delta c_t$ is normally distributed with long-run mean

$$\mathbb{E}[\Delta c_{t-1,t}|\sigma_{t-1}] = \frac{\mu_c}{1-\phi_c},$$

and long-run variance

$$\mathbf{Var}(\Delta c_{t-1,t}|\sigma_{t-1}) = \frac{\sigma_{t-1}^2}{1-\phi_c^2}.$$

Using the above results and equations (7)-(8), we conclude that the long-run p.d.f. for $\sigma_{t-1}$ is equal to

$$f(\sigma_{t-1}) = \frac{1}{\sqrt{2\pi}(\nu_\sigma/\sqrt{1-\phi_\sigma^2})} e^{-\frac{(\sigma_{t-1} - \frac{\mu_\sigma}{1-\phi_\sigma})^2}{2\nu_\sigma^2/(1-\phi_\sigma^2)}}.$$

The p.d.f. for $\sigma_t|\sigma_{t-1}$ is equal to

$$f(\sigma_t|\sigma_{t-1}) = \frac{1}{\sqrt{2\pi}\nu_\sigma} e^{-\frac{(\sigma_t - \mu_\sigma - \phi_\sigma \sigma_{t-1})^2}{2\nu_\sigma^2}}.$$

The long-run p.d.f for $\Delta c_{t-1,t}$ conditional on $\sigma_t$ and $\sigma_{t-1}$ is equal to

$$f(\Delta c_{t-1,t}|\sigma_t, \sigma_{t-1}) = \frac{1}{\sqrt{2\pi}(\sigma_{t-1}/\sqrt{1-\phi_c^2})} e^{-\frac{(\Delta c_t - \mu_c/(1-\phi_c))^2}{2\sigma_{t-1}^2/(1-\phi_c^2)}}.$$

The joint p.d.f. for $\Delta c_{t-1,t}$, $\sigma_t$ and $\sigma_{t-1}$ is therefore equal to

$$f(\Delta c_{t-1,t},\sigma_t,\sigma_{t-1}) = f(\Delta c_{t-1,t}|\sigma_t,\sigma_{t-1})f(\sigma_t|\sigma_{t-1})f(\sigma_{t-1}) \Leftrightarrow$$

$$f(\Delta c_{t-1,t},\sigma_t,\sigma_{t-1}) = \frac{1}{\sqrt{2\pi}(\nu_\sigma/\sqrt{1-\phi_\sigma^2})}\frac{1}{\sqrt{2\pi}\nu_\sigma}\frac{1}{\sqrt{2\pi}(\sigma_{t-1}/\sqrt{1-\phi_c^2})}e^{-\frac{(\Delta c_{t-1,t}-\mu_c/(1-\phi_c))^2}{2\sigma_{t-1}^2/(1-\phi_c^2)}} \times$$
$$e^{-\frac{(\sigma_t-\mu_\sigma-\phi_\sigma\sigma_{t-1})^2}{2\nu_\sigma^2}}e^{-\frac{(\sigma_{t-1}-\mu_\sigma/(1-\phi_\sigma))^2}{2\nu_\sigma^2/(1-\phi_\sigma^2)}}$$

# Appendix F Proofs

## Appendix F.1 Lemma 1

*Lemma 1:* Suppose that one-period, cum-dividend, asset log-returns $r_{i,t,t+1}$ are i.i.d. normal random variables with constant mean $\tilde{\mu}_i - \frac{1}{2}\sigma_i^2$ and volatility $\sigma_i$. Suppose also that financial markets are complete, that there exists a representative investor with CRRA power utility defined over consumption[64], that log-consumption growth $\Delta c_{t,t+1}$ is a normal random variable with constant mean $\tilde{\mu}_c$ and constant volatility $\sigma_c$, and that the correlation coefficient between $r_{i,t,t+1}$ and $\Delta c_{t,t+1}$ is $\rho_{i,c}$. Then, the log risk-free rate $r_f$ is constant, and cum-payout asset log-returns under the risk-neutral measure $\mathbb{Q}$ are i.i.d. normal random variables with constant mean $r_f - \frac{1}{2}\sigma_i^2$ and volatility $\sigma_i$.

*Proof:*

In equilibrium, the consumption-Euler equation for asset log-returns implies that

$$\mathbb{E}_t\left[\beta e^{-\alpha\Delta c_{t,t+1}}e^{r_{i,t,t+1}}\right] = 1 \Leftrightarrow \tilde{\mu}_i + log\beta - \alpha\tilde{\mu}_c + \frac{1}{2}\alpha^2\sigma_c^2 - \alpha\rho_{i,c}\sigma_c\sigma_1 = 0. \quad (26)$$

in which $\beta \in (0,1)$ is the rate of time-preference, and $\alpha \geq -1$ is the risk aversion parameter in the CRRA power utility function. Similarly, for the log risk-free rate

$$r_f + log\beta - \alpha\tilde{\mu}_c + \frac{1}{2}\alpha^2\sigma_c^2 = 0. \quad (27)$$

which is constant since $\mu_c$ and $\sigma_c$ are also constant.

We can rewrite the consumption-Euler equation in (26) using the p.d.f. for $\Delta c_{t+1}$ conditional on $r_{i,t,t+1}$

$$\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma_i}e^{log\beta}e^{r_{i,t,t+1}}e^{-\frac{(r_{i,t,t+1}-\tilde{\mu}_i+0.5\sigma_i^2)^2}{2\sigma_i^2}}e^{-\alpha[\tilde{\mu}_c+\rho_{i,c}\frac{\sigma_c}{\sigma_i}(r_{i,t,t+1}-\tilde{\mu}_i+0.5\sigma_i^2)]+\frac{1}{2}\alpha^2(1-\rho_{i,c}^2)\sigma_c^2}dr_{i,t,t+1} = 1.$$

[64] More on the aggregation properties of the CRRA utility function can be found in Chapter 1 of Duffie (2000), and Chapter 5 in Huang and Litzenberger (1989).

Exploiting the consumption-Euler conditions in (26) and (27), we obtain

$$e^{-r_f}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma_i}e^{r_{i,t,t+1}}e^{-\frac{(r_{i,t,t+1}-r_f+0.5\sigma_i^2)^2+(\alpha\rho_{i,c}\sigma_i\sigma_c)^2-2(r_{i,t,t+1}-r_f+0.5\sigma_i^2)\alpha\rho_{i,c}\sigma_i\sigma_c}{2\sigma_i^2}}\times$$
$$e^{-\alpha\rho_{i,c}\frac{\sigma_c}{\sigma_i}(r_{i,t,t+1}-\tilde{\mu}_i+0.5\sigma_i^2)}e^{-\frac{1}{2}\alpha^2\rho_{i,c}^2\sigma_c^2}dr_{i,t,t+1}=1.$$

Further algebra yields

$$e^{-r_f}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\sigma_i}e^{r_{i,t,t+1}}e^{-\frac{(r_{i,t,t+1}-r_f+0.5\sigma_i^2)^2}{\sigma_i}}e^{-\frac{1}{2}\alpha^2\rho_{i,c}^2\sigma_c^2+(r_{i,t,t+1}-r_f+0.5\sigma_i^2)\alpha\rho_{i,c}\frac{\sigma_c}{\sigma_i}}\times$$
$$e^{-\alpha\rho_{i,c}\frac{\sigma_c}{\sigma_i}(r_{i,t,t+1}-r_f-\alpha\rho_{i,c}\sigma_i\sigma_c+0.5\sigma_i^2)}e^{-\frac{1}{2}\alpha^2\rho_{i,c}^2\sigma_c^2}dr_{i,t,t+1}=1.$$

Cancelling out terms, we conclude that

$$e^{-r_f}\int_{-\infty}^{+\infty}e^{r_{i,t,t+1}}\underbrace{\frac{1}{\sqrt{2\pi}\sigma_i}e^{-\frac{(r_{i,t,t+1}-r_f+0.5\sigma_i^2)^2}{\sigma_i}}}_{\text{risk-neutral c.d.f. for } r_{i,t,t+1}}dr_{i,t,t+1}=1.$$

### Appendix F.2 Lemma 2

*Lemma 2:* Let $x$ be a normal random variable with mean $\mu\in\mathbb{R}$ and standard deviation $\sigma\in\mathbb{R}_{>0}$. Let $A$ and $B$ two real numbers with $B>-\frac{1}{2\sigma^2}$, then

$$\mathbb{E}\Big[e^{-Ax-Bx^2}\Big]=e^{\frac{0.5A^2\sigma^2-A\mu-B\mu^2}{1+2B\sigma^2}}\frac{1}{\sqrt{1+2B\sigma^2}}. \tag{28}$$

*Proof:*

$$\mathbb{E}\Big[e^{-Ax-Bx^2}\Big]=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty}e^{\frac{-2A\sigma^2x-2\sigma^2Bx^2-x^2-\mu^2+2\mu x}{2\sigma^2}}dx.$$

Completing the square in the right-hand side

$$\mathbb{E}\Big[e^{-Ax-Bx^2}\Big]=e^{\frac{\left(\frac{\mu-A\sigma^2}{\sqrt{1+2B\sigma^2}}\right)^2-\mu^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{+\infty}e^{\frac{-(1+2B\sigma^2)x^2+2\frac{\mu-A\sigma^2}{\sqrt{1+2B\sigma^2}}\sqrt{1+2B\sigma^2}x-\left(\frac{\mu-A\sigma^2}{\sqrt{1+2B\sigma^2}}\right)^2}{2\sigma^2}}dx.$$

After a change of variables $\tilde{x}=\sqrt{1+2B\sigma^2}x$, we conclude that

$$\mathbb{E}\Big[e^{-Ax-Bx^2}\Big]=e^{\frac{0.5A^2\sigma^2-A\mu-B\mu^2}{1+2B\sigma^2}}\frac{1}{\sqrt{1+2B\sigma^2}}.$$

### Appendix F.3 Risk-neutral density for consumption growth under CRRA preferences

Following *Lemma 1* in Appendix F.1, assume that consumption growth is log-normally distributed with constant mean $\tilde{\mu}_c$ and volatility $\sigma_c$, and that aggregate investor preferences can be described by a CRRA power utility function. Let $f_t(\Delta c_{t,t+1})$ be the normal p.d.f. for log-consumption growth, then the risk-neutral density $f_t^{\mathbb{Q}}(\Delta c_{t,t+1})$ is given by

$$f_t^{\mathbb{Q}}(\Delta c_{t,t+1}) = \frac{M_{t,t+1}^{CRRA}}{\mathbb{E}_t[M_{t,t+1}^{CRRA}]} f_t(\Delta c_{t,t+1}).$$

Following a similar line of arguments as in *Lemma 1*, we obtain

$$f_t^{\mathbb{Q}}(\Delta c_{t,t+1}) = \frac{1}{\sqrt{2\pi}\sigma_c} e^{-\frac{\left(\Delta c_{t,t+1} - (\tilde{\mu}_c - \alpha\sigma_c^2)\right)^2}{2\sigma_c^2}}.$$

Exploiting the consumption-Euler equations for stock market returns and the risk-free rate in (26) and (27), we can substitute out the term $\alpha\sigma_c^2$ with the stock market Sharpe ratio adjusted for the correlation between the stock market and consumption growth

$$\alpha\sigma_c^2 = \frac{\tilde{\mu}_m - r_f}{\sigma_m \rho_{m,c}} \sigma_c,$$

to conclude that

$$f_t^{\mathbb{Q}}(\Delta c_{t,t+1}) = \frac{1}{\sqrt{2\pi}\sigma_c} e^{-\frac{\left(\Delta c_{t,t+1} - (\tilde{\mu}_c - \frac{\tilde{\mu}_m - r_f}{\sigma_m \rho_{m,c}} \sigma_c)\right)^2}{2\sigma_c^2}}.$$

### Appendix F.4 Proof of Proposition 1

For $\rho = 0$, the Bellman recursion for the aggregate investor's consumption problem becomes

$$V_t = C_t^{1-\beta} \mu_t \Big(V_{t+1}\Big)^{\beta}. \tag{29}$$

$\mu_t$ is the disappointment aversion certainty equivalent from (5) with $\delta = 1$. Suppose that $log \frac{V_t}{C_t} = v_t - c_t = A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2$. Then, the Bellman equation reads

$$\begin{aligned}
&exp\Big[\frac{1}{\beta}(A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2)\Big] = \\
&\mathbb{E}_t\Big\{exp\Big[-\alpha[A_0 + (A_1+1)\Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma_{t+1}^2]\Big] \times \\
&\frac{1 + \theta \mathbf{1}\{A_0 + (A_1+1)\Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2)\}}{1 + \theta \mathbb{P}_t\{A_0 + (A_1+1)\Delta c_{t,t+1} + A_2 \sigma_{t+1} + A_3 \sigma_{t+1}^2 < \frac{1}{\beta}(A_0 + A_1 \Delta c_{t-1,t} + A_2 \sigma_t + A_3 \sigma_t^2)\}}\Big\}^{-\frac{1}{\alpha}}.
\end{aligned}$$

Dividing both parts by the left-hand side,

$$\mathbb{E}_t\Big\{exp\Big[-\alpha(A_0-\frac{1}{\beta}A_0)-\alpha[(A_1+1)\Delta c_{t,t+1}-\frac{1}{\beta}A_1\Delta c_{t-1,t}]$$
$$-\alpha(A_2\sigma_{t+1}-\frac{1}{\beta}A_2\sigma_t)-\alpha(A_3\sigma_{t+1}^2-\frac{1}{\beta}A_3\sigma_t^2)\Big]\times$$
$$\frac{1+\theta\mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2<\frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)\}}{1+\theta\mathbb{P}_t\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2<\frac{1}{\beta}(A_0+A_1\Delta c_{t-1t}+A_2\sigma_t+A_3\sigma_t^2)\}}\Big\}^{-\frac{1}{\alpha}}=1.$$

Recall that $\epsilon_{c,t+1}$ and $\epsilon_{\sigma,t+1}$ from (7) and (8) are independent. We can use the law of total expectation to rewrite the above expression as

$$\mathbb{E}_t\Big\{\mathbb{E}_t\Big\{exp\Big[-\alpha(A_0-\frac{1}{\beta}A_0)-\alpha[(A_1+1)\Delta c_{t,t+1}-\frac{1}{\beta}A_1\Delta c_{t-1,t}]$$
$$-\alpha(A_2\sigma_{t+1}-\frac{1}{\beta}A_2\sigma_t)-\alpha(A_3\sigma_{t+1}^2-\frac{1}{\beta}A_3\sigma_t^2)\Big]\times$$
$$\frac{1+\theta\mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2<\frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)\}}{1+\theta\mathbb{P}_t\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2<\frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)\}}|\epsilon_{\sigma,t+1}\Big\}\Big\}=1.$$

Using the dynamics of consumption growth $\Delta c_{t,t+1}$ in (7), and partial moments for the normal distribution, the above expression becomes

$$\mathbb{E}_t\Big\{exp\Big[-\alpha(A_0-\frac{1}{\beta}A_0)-\alpha[(A_1+1)(\mu_c+\phi_c\Delta c_{t-1,t})-\frac{1}{\beta}A_1\Delta c_{t-1,t}]+\frac{1}{2}\alpha^2(A_1+1)^2\sigma_t^2$$
$$-\alpha(A_2\sigma_{t+1}-\frac{1}{\beta}A_2\sigma_t)-\alpha(A_3\sigma_{t+1}^2-\frac{1}{\beta}A_3\sigma_t^2)\Big]\times$$
$$\frac{1+\theta N\Big(\frac{\frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)-A_0-(A_1+1)\mu_c-(A_1+1)\phi_c\Delta c_{t-1,t}-A_2\sigma_{t+1}-A_3\sigma_{t+1}^2}{(A_1+1)\sigma_t}+\alpha(A_1+1)\sigma_t\Big)}{1+\theta N\Big(\frac{\frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)-A_0-(A_1+1)\mu_c-(A_1+1)\phi_c\Delta c_{t-1,t}-A_2\sigma_{t+1}-A_3\sigma_{t+1}^2}{(A_1+1)\sigma_t}\Big)}\Big\}=1,$$

where $N()$ is the standard normal c.d.f.. For $\theta=2$ and $N()$ a small number[65], we can use the following approximation $1+\theta N(y)\approx e^{\theta N(y)}$ to get

$$\mathbb{E}_t\Big\{exp\Big[-\alpha(A_0-\frac{1}{\beta}A_0)-\alpha[(A_1+1)(\mu_c+\phi_c\Delta c_{t-1,t})-\frac{1}{\beta}A_1\Delta c_{t-1,t}]+\frac{1}{2}\alpha^2(A_1+1)^2\sigma_t^2$$
$$-\alpha(A_2\sigma_{t+1}-\frac{1}{\beta}A_2\sigma_t)-\alpha(A_3\sigma_{t+1}^2-\frac{1}{\beta}A_3\sigma_t^2)\Big]\times$$
$$e^{\theta N\Big(\frac{\frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)-A_0-(A_1+1)\mu_c-(A_1+1)\phi_c\Delta c_{t-1,t}-A_2\sigma_{t+1}-A_3\sigma_{t+1}^2}{(A_1+1)\sigma_t}+\alpha(A_1+1)\sigma_t\Big)}\times$$
$$e^{-\theta N\Big(\frac{\frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)-A_0-(A_1+1)\mu_c-(A_1+1)\phi_c\Delta c_{t-1,t}-A_2\sigma_{t+1}-A_3\sigma_{t+1}^2}{(A_1+1)\sigma_t}\Big)}\Big\}=1,$$

[65]In simulations, the probability of disappointment events is less than 0.5

Further, we can use a first-order linear approximation for the difference of the two standard normal c.d.f.'s in the above equation, provided that this difference is small[66],

$$N(x) - N(y) \approx n(\bar{x})(x - y),$$

to obtain

$$\begin{aligned}&exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)(\mu_c + \phi_c \Delta c_{t-1,t}) - \frac{1}{\beta}A_1 \Delta c_{t-1,t}] + \frac{1}{2}\alpha^2 (A_1+1)^2 \sigma_t^2 + \\ &\alpha\theta n(\bar{x})(A_1+1)\sigma_t + \alpha\frac{1}{\beta}A_2\sigma_t + \alpha\frac{1}{\beta}A_3\sigma_t^2\Big]\mathbb{E}_t\Big\{exp\Big[-\alpha A_2 \sigma_{t+1} - \alpha A_3 \sigma_{t+1}^2\Big]\Big\} = 1,\end{aligned} \tag{30}$$

in which $n()$ is the standard normal p.d.f..

Combining the dynamics for aggregate uncertainty $\sigma_{t+1}$ in (8) with *Lemma 2* from Appendix F.2, the Bellman equation becomes

$$\begin{aligned}&exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)(\mu_c + \phi_c \Delta c_{t-1,t}) - \frac{1}{\beta}A_1 \Delta c_{t-1,t}] + \frac{1}{2}\alpha^2 (A_1+1)^2 \sigma_t^2 + \\ &\alpha\theta n(\bar{x})(A_1+1)\sigma_t + \alpha\frac{1}{\beta}A_2\sigma_t + \alpha\frac{1}{\beta}A_3\sigma_t^2\Big] \times \\ &exp\Big[\frac{0.5\alpha^2 A_2^2 \nu_\sigma^2 - \alpha A_2 \mu_\sigma - \alpha A_2 \phi_\sigma \sigma_t - \alpha A_3 \mu_\sigma^2 - \alpha A_3 \phi_\sigma^2 \sigma_t^2 - 2\alpha A_3 \mu_\sigma \phi_\sigma \sigma_t}{1 + 2\alpha A_3 \nu_\sigma^2}\Big]\frac{1}{\sqrt{1+2\alpha A_3 \nu_\sigma^2}} = e^0.\end{aligned} \tag{31}$$

We can now solve for $A_0$, $A_1$, $A_2$, and $A_3$ using the method of undetermined coefficients. We first collect $\Delta c_{t-1,t}$ terms to get

$$A_1 = \frac{\beta\phi_c}{1 - \beta\phi_c}. \tag{32}$$

Note that for $\beta \in (0,1)$ and $\phi_c \in (-1,1)$, then $A_1 + 1$ is positive. Also, for $\beta \in (0,1)$, the sign of $A_1$ depends only on the sign of $\phi_c$.

Similarly, collecting $\sigma_t^2$ terms yields

$$2\alpha\nu_\sigma^2 A_3^2 + [1 - \beta\phi_\sigma^2 + \beta\alpha^2(A_1+1)^2\nu_\sigma^2]A_3 + \frac{1}{2}\beta\alpha(A_1+1)^2 = 0. \tag{33}$$

For $\alpha \neq 0$, the solution for to the quadratic equation is

$$A_3 = \frac{-[1 - \beta\phi_\sigma^2 + \beta\alpha^2(A_1+1)^2\nu_\sigma^2] \pm \sqrt{[1 - \beta\phi_\sigma^2 + \beta\alpha^2(A_1+1)^2\nu_\sigma^2]^2 - 4\beta\alpha^2(A_1+1)^2\nu_\sigma^2}}{4\alpha\nu_\sigma^2}. \tag{34}$$

The ratio of the constant term over the quadratic coefficient in the above quadratic equation is a positive number $\big(\beta(A_1+1)^2/4\nu_\sigma^2\big)$. Hence, the roots of the quadratic equation will be of the same sign. Furthermore, since $\beta \in (0,1)$ and $\phi_\sigma \in (-1,1)$, then $1 - \beta\phi_\sigma^2$ is positive, $-[1 - \beta\phi_\sigma^2 + \beta\alpha^2(A_1+1)^2\nu_\sigma^2]$ is negative, and the solutions to the quadratic equation are therefore negative. We will pick the largest negative root so that the quadratic solution in (34) is very close to the linear

[66]Essentially we require that $\frac{\alpha}{1-\beta\phi_c}\sigma_t$ to be small.

approximation in (35) below.

For $A_3$ to be a real number, we require that

$$[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2]^2-4\beta\alpha^2(A_1+1)^2\nu_\sigma^2>0.$$

We cannot really examine whether the above inequality holds without having picked parameter values. However, $\nu_\sigma^2$ is a very small number close to zero ($0.00177^2$), and for $\nu_\sigma^2\approx 0$ the determinant in (34) is approximately equal to

$$\lim_{\nu_\sigma^2\downarrow 0}[1-\beta\phi_\sigma^2+\beta\alpha^2(A_1+1)^2\nu_\sigma^2]^2-4\beta\alpha^2(A_1+1)^2\nu_\sigma^2\approx[1-\beta\phi_\sigma^2]^2>0.$$

The restriction that $\nu_\sigma$ is a very small number is associated with higher consumption growth moments being well defined. Parameter values for the simulated economy ensure that the determinant in (34) is real, and that $1+2\alpha A_3\nu_\sigma^2>0$ as required by *Lemma 2* in Appendix F.2. Finally, for $\nu_\sigma^2\approx 0$, equation (33) becomes linear yielding an approximate solution for $A_3$

$$A_3\approx-\frac{1}{2}\frac{\beta\alpha(A_1+1)^2}{1-\beta\phi_\sigma^2}. \tag{35}$$

Collecting $\sigma_t$ terms in (31), we obtain the solution for $A_2$

$$A_2=\frac{-\theta\beta n(\bar{x})(A_1+1)(1+2\alpha A_3\nu_\sigma^2)+2\beta A_3\mu_\sigma\phi_\sigma}{1+2\alpha A_3\nu_\sigma^2-\beta\phi_\sigma}, \tag{36}$$

where $1+2\alpha A_3\nu_\sigma^2>0$ as required by *Lemma 2* in Appendix F.2. It is easy to verify that for negative $A_3$, then $A_2$ is also negative. Furthermore, as $\nu_\sigma^2\downarrow 0$, an approximate solution for $A_2$ reads

$$A_2\approx\frac{-\theta\beta n(\bar{x})(A_1+1)+2\beta A_3\mu_\sigma\phi_\sigma}{1-\beta\phi_\sigma}. \tag{37}$$

Finally, the remaining constant terms in (31) are grouped under $A_0$

$$A_0=\frac{\beta}{1-\beta}[(A_1+1)\mu_c+\frac{1}{1+2\alpha A_3\nu_\sigma^2}(A_2\mu_\sigma+A_3\mu_\sigma^2-0.5\alpha A_2^2\nu_\sigma^2)+\frac{1}{2\alpha}log(1+2\alpha A_3\nu_\sigma^2)], \tag{38}$$

with the approximation for $\nu_\sigma^2\downarrow 0$

$$A_0\approx\frac{\beta}{1-\beta}[(A_1+1)\mu_c+A_2\mu_\sigma+A_3\mu_\sigma^2]. \tag{39}$$

### Appendix F.5 The log risk-free rate

The Euler condition for the log risk-free rate reads

$$e^{-r_{f,t,t+1}}=\mathbb{E}_t\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-1}\Big(\frac{V_{t+1}}{\mu_t(V_{t+1})}\Big)^{-\alpha}\frac{1+\theta\ \mathbf{1}\{V_{t+1}<\ \mu_t(V_{t+1})\}}{\mathbb{E}_t[1+\theta\ \mathbf{1}\{V_{t+1}<\mu_t(V_{t+1})\}]}\Big].$$

Repeating all the steps that lead to equation (30) in Appendix F.4, we obtain

$$e^{-r_{f,t,t+1}} = exp\Big[log\beta - \alpha(A_0 - \frac{1}{\beta}A_0) - \big[[\alpha(A_1+1)+1](\mu_c + \phi_c\Delta c_{t-1,t}) - \frac{1}{\beta}A_1\Delta c_t\big] +$$
$$\frac{1}{2}[\alpha(A_1+1)+1]^2\sigma_t^2 + \theta n(\bar{x})[\alpha(A_1+1)+1]\sigma_t + \alpha\frac{1}{\beta}A_2\sigma_t + \alpha\frac{1}{\beta}A_3\sigma_t^2\Big]\mathbb{E}_t\Big\{exp\Big[-\alpha A_2\sigma_{t+1} - \alpha A_3\sigma_{t+1}^2\Big]\Big\}.$$

But from (30) we know that

$$exp\Big[-\alpha(A_0 - \frac{1}{\beta}A_0) - \alpha[(A_1+1)(\mu_c + \phi_c\Delta c_{t-1,t}) - \frac{1}{\beta}A_1\Delta c_{t-1,t}] + \frac{1}{2}\alpha^2(A_1+1)^2\sigma_t^2 +$$
$$\alpha\theta n(\bar{x})(A_1+1)\sigma_t + \alpha\frac{1}{\beta}A_2\sigma_t + \alpha\frac{1}{\beta}A_3\sigma_t^2\Big]\mathbb{E}_t\Big\{exp\Big[-\alpha A_2\sigma_{t+1} - \alpha A_3\sigma_{t+1}^2\Big]\Big\} = 1.$$

Therefore, the log risk-free rate must be approximately equal to

$$r_{f,t,t+1} \approx -log\beta + \mu_c + \phi_c\Delta c_{t-1,t} - \frac{1}{2}[2\alpha(A_1+1)+1]\sigma_t^2 - \theta n(\bar{x})\sigma_t$$

## Appendix F.6 Proof of Proposition 2

We conjecture that the log price-payout ratio $z_{m,t}$ for a financial claim on a stream of aggregate payments (dividends or earnings) is an affine function of the state variables $\Delta c_{t-1,t}$, $\sigma_t$, $\sigma_t^2$

$$z_{m,t} = A_{m,0} + A_{m,1}\Delta c_{t-1,t} + A_{m,1}\sigma_t + A_{m,2}\sigma_t^2.$$

Combining equation (24) with our conjecture about $z_{m,t}$, the Euler equation for asset returns becomes

$$\mathbb{E}_t\left[M_{t,t+1}e^{\kappa_{m,0}+\kappa_{m,1}(A_{m,0}+A_{m,1}\Delta c_{t,t+1}+A_{m,2}\sigma_{t+1}+A_{m,3}\sigma_{t+1}^2)-(A_{m,0}+A_{m,1}\Delta c_{t-1,t}+A_{m,2}\sigma_t+A_{m,3}\sigma_t^2)+\Delta o_{m,t,t+1}}\right] = 1.$$

Substituting the result for the disappointment aversion discount factor $M_{t,t+1}$ from (10), we can re-write the Euler equation as

$$\mathbb{E}_t\Big[e^{\log\beta - \Delta c_{t,t+1}}e^{-\alpha\big\{A_0(1-\frac{1}{\beta})+[(A_1+1)\Delta c_{t,t+1}-\frac{1}{\beta}A_1\Delta c_{t-1,t}]+A_2(\sigma_{t+1}-\frac{1}{\beta}\sigma_t)+A_3(\sigma_{t+1}^2-\frac{1}{\beta}\sigma_t^2)\big\}} \times \tag{40}$$
$$\frac{1+\theta\ \mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2 < \ \frac{1}{\beta}(A_0+A_1\Delta c_{t-1,t}+A_2\sigma_t+A_3\sigma_t^2)\}}{\mathbb{E}_t[1+\theta\ \mathbf{1}\{A_0+(A_1+1)\Delta c_{t,t+1}+A_2\sigma_{t+1}+A_3\sigma_{t+1}^2 < \ \frac{1}{\beta}(A_0+A_1\Delta c_t+A_2\sigma_t+A_3\sigma_t^2)\}]} \times$$
$$e^{\kappa_{m,0}+\kappa_{m,1}(A_{m,0}+A_{m,1}\Delta c_{t,t+1}+A_{m,2}\sigma_{t+1}+A_{m,3}\sigma_{t+1}^2)-(A_{m,0}+A_{m,1}\Delta c_{t-1,t}+A_{m,2}\sigma_t+A_{m,3}\sigma_t^2)+\Delta o_{m,t,t+1}}\Big] = 1.$$

Following the same line of arguments as in Appendix F.4, the Euler equation becomes

$$exp\Big[log(\beta) - \alpha(A_0 - \frac{1}{\beta}A_0) - \big[[\alpha(A_1+1) + 1 - \kappa_{m,1}A_{m,1}](\mu_c + \phi_c\Delta c_{t-1,t}) + \alpha\frac{1}{\beta}A_1\Delta c_{t-1,t}\big] \tag{41}$$

$$+\frac{1}{2}[\alpha(A_1+1) + 1 - \kappa_{m,1}A_{m,1}]^2\sigma_t^2 + \theta n(\bar{x})[\alpha(A_1+1) + 1 - \kappa_{m,1}A_{m,1}]\sigma_t + \alpha\frac{1}{\beta}A_2\sigma_t + \alpha\frac{1}{\beta}A_3\sigma_t^2$$

$$+\kappa_{m,0} + A_{m,0}(\kappa_{m,1} - 1) - A_{m,1}\Delta c_{t-1,t} - A_{m,2}\sigma_t - A_{m,3}\sigma_t^2 + \mu_m + \phi_m\Delta c_{t-1,t} + \frac{1}{2}\sigma_m^2\sigma_t^2\Big] \times$$

$$exp\Big[\frac{0.5(\alpha A_2 - \kappa_{m,1}A_{m,2})\nu_\sigma^2 - (\alpha A_2 - \kappa_{m,1}A_{m,2})\mu_\sigma - (\alpha A_2 - \kappa_{m,1}A_{m,2})\phi_\sigma\sigma_t}{1 + 2(\alpha A_3 - \kappa_{m,1}A_{m,3})\nu_\sigma^2}\Big] \times$$

$$exp\Big[\frac{-(\alpha A_3 - \kappa_{m,1}A_{m,3})\mu_\sigma^2 - (\alpha A_3 - \kappa_{m,1}A_{m,3})\phi_\sigma^2\sigma_t^2 - 2(\alpha A_3 - \kappa_{m,1}A_{m,3})\mu_\sigma\phi_\sigma\sigma_t}{1 + 2(\alpha A_3 - \kappa_{m,1}A_{m,3})\nu_\sigma^2}\Big] \times$$

$$\frac{1}{\sqrt{1 + 2(\alpha A_3 - \kappa_{m,1}A_{m,3})\nu_\sigma^2}} = e^0.$$

We are now able to solve for $A_{m,0}$, $A_{m,1}$, $A_{m,2}$, and $A_{m,3}$ using the method of undetermined coefficients. Specifically, for $A_{m,1}$ we get

$$-\phi_c - \alpha(A_1+1)\phi_c + \frac{1}{\beta}\alpha A_1 + \kappa_{m,1}A_{m,1}\phi_c - A_{m,1} + \phi_m = 0.$$

Using the expression for $A_1$ from (32), we conclude that

$$A_{m,1} = \frac{\phi_m - \phi_c}{1 - \kappa_{m,1}\phi_c}. \tag{42}$$

Collecting $\sigma_t^2$ terms from (40), $A_{m,3}$ must satisfy the quadratic equation

$$\frac{1}{2}\beta\big[[\alpha(A_1+1) + 1 - \kappa_{m,1}A_{m,1}]^2 + \sigma_m^2\big][1 + 2(\alpha A_3 - \kappa_{m,1}A_{m,3})\nu_\sigma^2] + \alpha A_3(1 + 2\alpha A_3\nu_\sigma^2 - \beta\phi_\sigma^2)$$

$$-2\alpha A_3\kappa_{m,1}A_{m,3}\nu_\sigma^2 - \beta A_{m,3} - 2\alpha A_3\nu_\sigma^2\beta A_{m,3} + 2\beta\kappa_{m,1}\nu_\sigma^2 A_{m,3}^2 + \beta\kappa_{m,1}\phi_\sigma^2 A_{m,3} = 0.$$

After tedious algebra, the solution for $A_{m,3}$ is equal to

$$A_{m,3} = \frac{-\tilde{b} \pm \sqrt{\tilde{b}^2 - 4\tilde{a}\tilde{c}}}{2\tilde{a}}, \tag{43}$$

with

$$\tilde{a} = 2\beta\kappa_{m,1}\nu_\sigma^2,$$
$$\tilde{b} = -\beta + \beta\kappa_{m,1}\phi_\sigma^2 - 2\alpha A_3\kappa_{m,1}\nu_\sigma^2 - 2\alpha\beta A_3\nu_\sigma^2,$$
$$\tilde{c} = \frac{1}{2}\beta\big[[\alpha(A_1+1) + 1 - \kappa_{m,1}A_{m,1}]^2 + \sigma_m^2\big](1 + 2\alpha A_3\nu_\sigma^2) + \alpha A_3(1 + 2\alpha A_3\nu_\sigma^2 - \beta\phi_\sigma^2).$$

We will pick the largest negative root so that the quadratic solution in (43) is very close to the linear approximation in (44) below. As in Appendix F.4, we need to make sure that $1 + 2(\alpha A_3 -$

$\kappa_{m,1}A_{m,3})\nu_\sigma^2$ is positive, and that the determinant in (43) is real. Both conditions are satisfied for very small $\nu_\sigma^2$, and reasonable values for the risk aversion coefficient $\alpha$. Finally, since $\nu_\sigma^2$ is a small number close to zero, we can obtain an approximate solution for $A_{m,3}$ using equation (35) for $A_3$

$$A_{m,3} \approx \frac{1}{2}\frac{[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}]^2+\sigma_m^2-\alpha^2(A_1+1)^2}{1-\kappa_{m,1}\phi_\sigma^2}. \tag{44}$$

Collecting $\sigma_t$ terms from (40), the solution for $A_{2,m}$ is given by

$$\begin{aligned} A_{m,2} = & \frac{\theta\beta n(\bar{x})[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}][1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2]}{\beta+2\beta(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2-\beta\kappa_{m,1}\phi_\sigma} \\ & \frac{+\alpha A_2[1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2-\beta\phi_\sigma]-2\beta(\alpha A_3-\kappa_{m,1}A_{m,3})\mu_\sigma\phi_\sigma}{\beta+2\beta(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2-\beta\kappa_{m,1}\phi_\sigma}. \end{aligned} \tag{45}$$

For $\nu_\sigma^2 \approx 0$, and the approximate expressions for $A_3$ and $A_2$ in (35) and (37) respectively, we conclude that

$$A_{m,2} \approx \frac{\theta n(\bar{x})(1-\kappa_{m,1}A_{m,1})+2\kappa_{m,1}A_{m,3}\mu_\sigma\phi_\sigma}{1-\kappa_{m,1}\phi_\sigma}. \tag{46}$$

Finally, collecting all the constant terms in (41), we get

$$\begin{aligned} A_{m,0} = & \frac{1}{1-\kappa_{m,1}}\Big[log\beta+\kappa_{m,0}+\mu_m-\alpha A_0\frac{\beta-1}{\beta}-[\alpha(A_1+1)+1-\kappa_{m,1}A_{m,1}]\mu_c \\ & -\frac{(\alpha A_2-\kappa_{m,1}A_{m,2})\mu_\sigma+(\alpha A_3-\kappa_{m,1}A_{m,3})\mu_\sigma^2-0.5(\alpha A_2-\kappa_{m,1}A_{m,2})^2\nu_\sigma^2}{1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2} \\ & -0.5log\big(1+2(\alpha A_3-\kappa_{m,1}A_{m,3})\nu_\sigma^2\big)\Big]. \end{aligned} \tag{47}$$

Exploiting the fact that $\nu_\sigma^2 \approx 0$, and the expression for $A_0$ in (39), an approximation for $A_{m,0}$ is

$$A_{m,0} \approx \frac{1}{1-\kappa_{m,1}}\big[log\beta+\kappa_{m,0}+\mu_m+(\kappa_{m,1}A_{m,1}-1)\mu_c+\kappa_{m,1}A_{m,2}\mu_\sigma+\kappa_{m,1}A_{m,3}\mu_\sigma^2\big]. \tag{48}$$