Entrepreneurial Under-Diversification: Over Optimism and Overconfidence

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Abstract

Past research shows that entrepreneurs often invest a large share of their personal wealth in their company, exposing themselves to idiosyncratic risk. In this paper, we focus on a possible explanation for this costly exposure, based on two behavioral biases: overconfidence and over optimism. Both these biases, which we parameterized in our model, affect the fundamental variables of the risk-return analysis à la Markowitz and lead entrepreneurs to invest too much in their company holding an undiversified portfolio.

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1. Introduction

Several empirical findings show that entrepreneurs often invest a large share of their personal wealth in one company, exposing themselves to idiosyncratic risk: their stake in the company is higher than the stake that a risk-return analysis would suggest (Heaton and Lucas 2000; Moskowitz and Vissing-Jørgensen 2002; Müller 2011; Yazdipour 2011). This exposure to idiosyncratic risk is very costly (Kerins et al. 2004; Pattitoni et al. 2013).

Since some studies point out that entrepreneurs demand compensation for their exposure to idiosyncratic risk (Müller 2011), one possible explanation for this puzzling evidence – i.e., that entrepreneurs do not understand idiosyncratic risk – can be ruled out. Other justifications mostly rely on non-pecuniary benefits as benefits of control: entrepreneurs obtain substantial rewards from being their own boss and, thus, they are willing to accept a suboptimal risk-return trade-off (Moskowitz and Vissing-Jørgensen 2002; Müller 2011; Shefrin 2011). Despite these justifications, it is still debated why entrepreneurs overinvest in their private companies given the suboptimal risk-return trade-off.

In this paper, we propose a complementary story by suggesting that behavioral biases may help explain this phenomenon. We specifically focus on overconfidence and over optimism (Shefrin 2008). On one hand, overconfidence may lead the entrepreneur to undervalue the risk on the investment in her private company. On the other hand, over optimism may cause the entrepreneur to overvalue the return on the investment in her private company. Both these biases, which we include in our model as parameters, may affect risk-return analyses à la Markowitz (Markowitz 1952; Markowitz 1959).[1] Through this parameterization, we can measure the potential bias in the portfolio weights of over optimist and/or overconfident entrepreneurs.

[1] Our theoretical model relies on the Slovic and Olsen's notion of perceived risk (Olsen 2011; Slovic 1987; Slovic 2000) and the two-component total perceived risk formula proposed by Yazdipour (2011). Using (an extension of) their framework of analysis, we are able to distinguish between the objective and subjective components of both risk and expected return.

2 Theoretical setup

2.1 Overconfidence

2.1.1 Risk minimization

Consider an entrepreneur who holds a portfolio of two risky assets with weights $\boldsymbol{\omega} = [\omega_I, \omega_M]'$ and a risk free asset with weight ω_F. The asset I is the entrepreneur investment in her private company and the asset M is the entrepreneur investment in a well-diversified market portfolio. The excess return of the entrepreneur's portfolio can be expressed by $\mu_P = \boldsymbol{\omega}'\boldsymbol{\mu}$, where $\boldsymbol{\mu} = [\mu_I, \mu_M]'$ is the vector of the excess returns over the risk free rate r_F. The variance of this portfolio is given by $\sigma_P^2 = \boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega}$, where $\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_I^2 & \sigma_{IM} \\ \sigma_{IM} & \sigma_M^2 \end{bmatrix}$ represents the positive-definite variance-covariance matrix of the returns of the risky assets with $\det \boldsymbol{\Sigma} = \sigma_I^2 \sigma_M^2 - \sigma_{IM}^2 > 0$.

For a given value of portfolio expected excess return, $\mu_P = k$, the entrepreneur prefers the portfolio with the lowest variance. She faces the problem

$$\begin{cases} \min \dfrac{1}{2} \boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega} \\ \boldsymbol{\omega}'\boldsymbol{\mu} = k \end{cases} \tag{1}$$

Note that the constraint $\omega_I + \omega_M + \omega_F = 1$ is implicit in $\boldsymbol{\omega}'\boldsymbol{\mu} = k$.

Setting up the Lagrangian and solving the problem (Pattitoni and Savioli 2011), the optimal portfolio weights are

$$\boldsymbol{\omega}(k) = \frac{k\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}} \tag{2}$$

The first element of $\boldsymbol{\omega}(k)$ represents the weight in the private company, namely

$$\omega_I(k) = \frac{k\left(\sigma_M^2 \mu_I - \rho_{IM}\sigma_I\sigma_M\mu_M\right)}{\sigma_M^2\mu_I^2 - 2\rho_{IM}\sigma_I\sigma_M\mu_I\mu_M + \sigma_I^2\mu_M^2} = \frac{k\sigma_M^2\alpha}{\sigma_M^2\mu_I^2 - 2\rho_{IM}\sigma_I\sigma_M\mu_I\mu_M + \sigma_I^2\mu_M^2} \tag{3}$$

where $\rho_{IM} = \sigma_{IM}/(\sigma_I\sigma_M)$ and α is the Jensen's alpha, i.e. $\alpha = \mu_I - \rho_{IM}(\sigma_I/\sigma_M)\mu_M$.

2.1.2 Overconfidence-driven under-diversification

Overconfidence causes the entrepreneur to undervalue the actual risk on the investment in her private company. In this case, the biased standard deviation, indicated by $\tilde{\sigma}_I$, is lower than the actual standard deviation, i.e., $\tilde{\sigma}_I < \sigma_I$.[2] We model $\tilde{\sigma}_I$ as

$$\tilde{\sigma}_I = \sigma_I(1-\delta_C), \quad \delta_C \in [0,1) \tag{4}$$

where δ_C is the overconfidence parameter, which ranges from 0 (no overconfidence) to 1 (maximum overconfidence).[3] When δ_C tends to 1, then $\tilde{\sigma}_I$ tends to zero.

In order to see the variation of the portfolio weight in her private company in case of overconfidence, we define $\tilde{\omega}_I(k)$ as the $\omega_I(k)$ of Equation (3) with $\tilde{\sigma}_I$ in place of σ_I. Two cases need to be considered.

Case 1 $\rho_{IM} = 0$

In this case, $\partial\tilde{\omega}_I(k)/\partial\delta_C > 0$. Thus, when $\rho_{IM} = 0$, the overconfident entrepreneur tends to overinvest in her private company and to be under-diversified. The overconfidence bias is

$$b_c = \tilde{\omega}_I(k) - \omega_I(k) > 0 \tag{5}$$

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[2] Since $\sigma_{IM} = \rho_{IM}\sigma_I\sigma_M$, overconfidence leads to a biased perception of σ_{IM}.

[3] Choosing $\delta_C \in (-\infty,1)$, we would allow for underconfidence.

Case 2 $\rho_{IM} \neq 0$

Using the definition of $\tilde{\omega}_I(k)$, we get the partial derivative

$$\frac{\partial \tilde{\omega}_I(k)}{\partial \delta_C} = \frac{k\sigma_I \sigma_M \mu_M \left[2\sigma_I(1-\delta_C)\sigma_M \mu_I \mu_M - \rho_{IM}\sigma_M^2 \mu_I^2 - \rho_{IM}\sigma_I^2 (1-\delta_C)^2 \mu_M^2\right]}{\left[\sigma_M^2 \mu_I^2 - 2\rho_{IM}\sigma_I(1-\delta_C)\sigma_M \mu_I \mu_M + \sigma_I^2(1-\delta_C)^2 \mu_M^2\right]^2} \tag{6}$$

Looking at Equation (6), we can conveniently divide our analysis in two subcases.

Case 2.1 $\rho_{IM} < 0$

We can see that $\partial \tilde{\omega}_I(k)/\partial \delta_C > 0$. Therefore, the result in Equation (5) continues to hold.

Case 2.2 $\rho_{IM} > 0$

In this last subcase, the sign of $\partial \tilde{\omega}_I(k)/\partial \delta_C$ is not straightforward. Imposing the condition $\partial \tilde{\omega}_I(k)/\partial \delta_C = 0$, we find two stationary points. In the space $(\delta_C, \tilde{\omega}_I)$, the coordinates of these two points are

$$\begin{aligned}
(\delta_C^-, \tilde{\omega}_I^-) &= \left(1 - \frac{\mu_I}{\mu_I - \alpha}\left(1 + \sqrt{1-\rho_{IM}^2}\right), \frac{k}{2\mu_I}\frac{\rho_{IM}^2}{-\sqrt{1-\rho_{IM}^2} - \left(1-\rho_{IM}^2\right)}\right) \\
(\delta_C^+, \tilde{\omega}_I^+) &= \left(1 - \frac{\mu_I}{\mu_I - \alpha}\left(1 - \sqrt{1-\rho_{IM}^2}\right), \frac{k}{2\mu_I}\frac{\rho_{IM}^2}{\sqrt{1-\rho_{IM}^2} - \left(1-\rho_{IM}^2\right)}\right)
\end{aligned} \tag{7}$$

Since $(\delta_C^-, \tilde{\omega}_I^-)$ is a minimum and $(\delta_C^+, \tilde{\omega}_I^+)$ is a maximum, $\partial \tilde{\omega}_I(k)/\partial \delta_C > 0$ in the interval (δ_C^-, δ_C^+), and $\partial \tilde{\omega}_I(k)/\partial \delta_C \leq 0$ elsewhere.

$\alpha > 0$ is a sufficient condition for $\delta_C^- < 0$.[4] However, we assumed $\delta_C \in [0,1)$. Thus, $\tilde{\omega}_I$ reaches its minimum when $\delta_C = 0$ and $\tilde{\omega}_I = \omega_I$.

[4] $\alpha > 0$ is a prerequisite to justify investments in private companies.

When $\delta_C \in (\delta_C^+, 1)$, $\partial\tilde{\omega}_I(k)/\partial\delta_C < 0$. In this case, the entrepreneur invested so much in her private company that, to meet the constraints of the portfolio selection problem, the weight in the well-diversified market portfolio needs to be negative.[5] If we exclude this extreme case, there is no ambiguity on the sign of the derivative. Thus, we conclude that, in general, overconfidence leads to overinvestment and under-diversification.

When overconfidence approaches its limiting value, we find a particular weight

$$\lim_{\delta_C \to 1} \tilde{\omega}_I(k) = \frac{k}{\mu_I} \tag{8}$$

Figure 1 offers a graphical representation of all the aforementioned results.

Overconfidence implies suboptimal portfolio weights and a biased perception of portfolio risk. Since the perceived portfolio risk increases in the perceived private company risk (i.e., $\partial\tilde{\sigma}_P/\partial\tilde{\sigma}_I > 0$), the perceived private company risk decreases in the level of overconfidence (i.e., $\partial\tilde{\sigma}_I/\partial\delta_C = -\sigma_I < 0$), and $\partial\tilde{\omega}_I(k)/\partial\delta_C > 0$,[6] then it follows that $\partial\tilde{\omega}_I(k)/\partial\tilde{\sigma}_P = (\partial\tilde{\omega}_I(k)/\partial\delta_C)(\partial\delta_C/\partial\tilde{\sigma}_I)(\partial\tilde{\sigma}_I/\partial\tilde{\sigma}_P) < 0$. This result is presented in Figure 2, which shows the link between the perceived frontier of investments (dashed) and the weight in the private company. The first plot shows the shift in the frontier caused by overconfidence; the second plot projects this shift in the private company weight. Note that the slope of the curve in the second plot is determined by $\partial\tilde{\omega}_I(k)/\partial\tilde{\sigma}_P$.

2.2 Over optimism

It is well known that the portfolio optimization problem is dual: either the entrepreneur minimizes the risk for a given portfolio expected return, or she maximizes the return for a given portfolio risk.

Since overconfidence affects risk perception, in Section 2.1.1 we studied its effect on portfolio risk using a risk minimization approach, which keeps the expected return level fixed.

[5] In this situation, the entrepreneur sells short the market portfolio. This situation is of little interest from an economic point of view.

[6] Excluding case 2.2, when $\delta_C \in (\delta_C^+, 1)$.

Conversely, in the next section we analyze the effect of over optimism on portfolio return using a return maximization approach, which holds the objective risk constant.

Figure 3 shows the duality of the problem by representing the tangency conditions that identify the lower iso-risk (left plot) and the upper iso-return (right plot).

2.2.1 Return maximization

The duality of the problem allows us to consider return maximization as the solution for the entrepreneur optimization problem for a given value of portfolio risk, $\sigma_P^2 = s^2$.

In such a setting, the entrepreneur faces the problem

$$\begin{cases} \max 2\left(\boldsymbol{\omega}'\boldsymbol{\mu} + r_F\right) \\ \boldsymbol{\omega}'\boldsymbol{\Sigma}\boldsymbol{\omega} = s^2 \end{cases} \tag{9}$$

Setting up the Lagrangian and solving the problem, the optimal portfolio weights are[7]

$$\boldsymbol{\omega}(s) = \frac{s\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\left(\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}\right)^{\frac{1}{2}}} \tag{10}$$

The weight in the private company is

$$\begin{aligned} \omega_I(s) &= \frac{s\left(\sigma_M^2 \mu_I - \sigma_{IM}\mu_M\right)}{\left[\left(\sigma_I^2\sigma_M^2 - \sigma_{IM}^2\right)\left(\sigma_M^2\mu_I^2 - 2\sigma_{IM}\mu_I\mu_M + \sigma_I^2\mu_M^2\right)\right]^{\frac{1}{2}}} \\ &= \frac{s\sigma_M^2\alpha}{\left[\left(\sigma_I^2\sigma_M^2 - \sigma_{IM}^2\right)\left(\sigma_M^2\mu_I^2 - 2\sigma_{IM}\mu_I\mu_M + \sigma_I^2\mu_M^2\right)\right]^{\frac{1}{2}}} \end{aligned} \tag{11}$$

[7] As the problem is quadratic, we also obtain a second solution with weights equal to minus those of Equation (10). We discard them since they are dominated ($\alpha > 0 \Rightarrow \omega_I(s) > 0$).

2.2.2 Over optimism-driven under-diversification

Over optimism causes the entrepreneur to overvalue the actual return of the investment in her private company. In this case, the biased expected return, indicated by $\tilde{\mu}_I$, is larger than the actual expected return, i.e., $\tilde{\mu}_I > \mu_I$.[8] We model $\tilde{\mu}_I$ as

$$\tilde{\mu}_I = \frac{\mu_I}{1-\delta_O}, \quad \delta_O \in [0,1) \tag{12}$$

where δ_O is the over optimism parameter, which ranges from 0 (no over optimism) to 1 (maximum over optimism). When δ_O tends to 1, then $\tilde{\mu}_I$ tends to infinity.[9]

In order to see the variation of the portfolio weight in her private company in case of over optimism, we define $\tilde{\omega}_I(s)$ as the $\omega_I(s)$ of Equation (11) with $\tilde{\mu}_I$ in place of μ_I. Using this definition, we get the partial derivative

$$\frac{\partial \tilde{\omega}_I(s)}{\partial \delta_O} = \frac{s \dfrac{\mu_I}{(1-\delta_O)^2} \mu_M^2 \left(\sigma_I^2 \sigma_M^2 - \sigma_{IM}^2\right)^{\frac{1}{2}}}{\left[\sigma_M^2 \dfrac{\mu_I^2}{(1-\delta_O)^2} - 2\sigma_{IM} \dfrac{\mu_I}{(1-\delta_O)} \mu_M + \sigma_I^2 \mu_M^2\right]^{\frac{3}{2}}} \tag{13}$$

Since $\partial \tilde{\omega}_I(s) / \partial \delta_O > 0$, the over optimist entrepreneur tends to overinvest in her private company and to be under-diversified.

The over optimism bias is

$$b_o = \tilde{\omega}_I(s) - \omega_I(s) > 0 \tag{14}$$

[8] The justifications of entrepreneur's under-diversification based on non-pecuniary benefits as benefits of control can be modeled by varying μ_I as well. In that case, the "biased" μ_I would incorporate the value of non-pecuniary benefits.

[9] Choosing $\delta_O \in (-\infty, 1)$, we would allow for under optimism.

The limit case for over optimism identifies a particular weight

$$\lim_{\delta_O \to 1} \tilde{\omega}_I(s) = \frac{s\sigma_M}{\left(\sigma_I^2 \sigma_M^2 - \sigma_{IM}^2\right)^{\frac{1}{2}}} \tag{15}$$

Over optimism implies suboptimal portfolio weights and a biased perception of portfolio return. Since the perceived portfolio return increases in the perceived private company return (i.e., $\partial\tilde{\mu}_P / \partial\tilde{\mu}_I > 0$), the perceived private company return increases in the level of over optimism (i.e., $\partial\tilde{\mu}_I / \partial\delta_O = \mu_I / (1-\delta_O)^2 > 0$), and $\partial\tilde{\omega}_I(s)/\partial\delta_O > 0$, then it follows that $\partial\tilde{\omega}_I(s)/\partial\tilde{\mu}_p = (\partial\tilde{\omega}_I(s)/\partial\delta_O)(\partial\delta_O / \partial\tilde{\mu}_I)(\partial\tilde{\mu}_I / \partial\tilde{\mu}_P) > 0$. This result is presented in Figure 4, which shows the link between the perceived frontier of investments (dashed) and the weight in the private company. The plot on the right shows the shift in the frontier caused by over optimism; the plot on the left projects this shift on the private company weight. Note that the slope of the curve in the plot on the left is determined by $\partial\tilde{\omega}_I(s)/\partial\tilde{\mu}_p$.

3. Conclusions

Several empirical findings show that entrepreneurs often invest a large share of their personal wealth in their own company, exposing themselves to idiosyncratic risk. In this paper, we propose a possible explanation for this costly exposure that complements other explanations which rely on non-pecuniary benefits as benefits of control and that is based on two behavioral biases: overconfidence and over optimism. In particular, we show that both these biases affect the fundamental variables of the risk-return analysis à la Markowitz and may lead the entrepreneur to choose suboptimal portfolio weights in her private company and hold an under-diversified portfolio.

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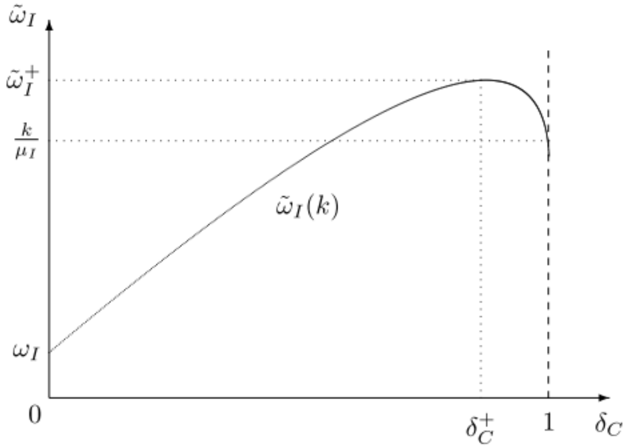

Figure 1. Private company weight under overconfidence

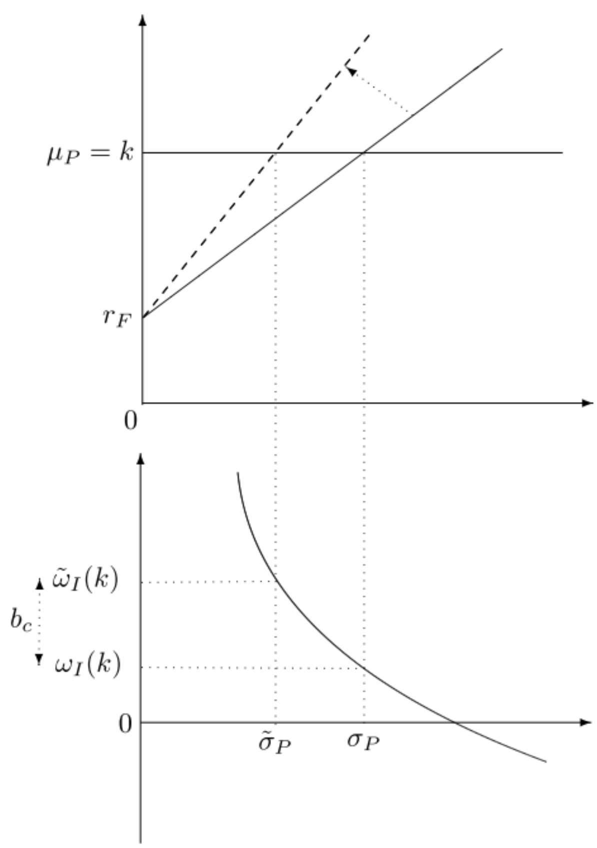

Figure 2. Frontier shift and overconfidence bias

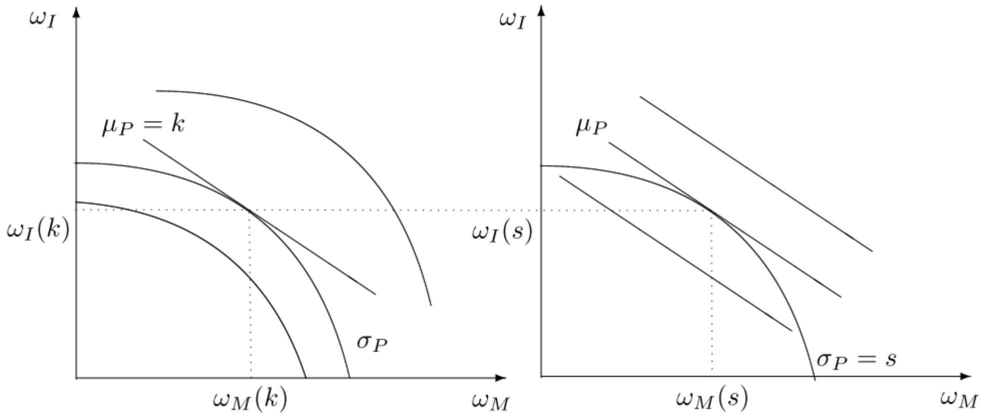

Figure 3. Duality in portfolio optimization

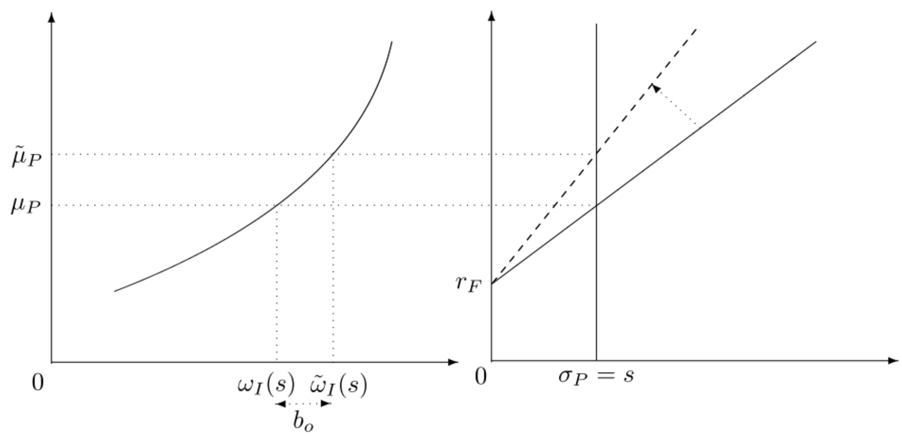

Figure 4. Frontier shift and over optimism bias.