library(rstan)

library(loo)

options(scipen=999)

D = list(T=252,

y=c(21.36,21.36,21.31,21.26,21.36,21.31,21.39,21.31,21.20,21.29,21.24,21.31,21.28,21.22,21.31,

21.40,21.33,21.23,21.30,21.44,21.52,21.45,21.30,21.27,21.38,21.39,21.39,21.34,21.33,21.39,21.46,

21.39,21.35,21.38,21.35,21.35,21.24,21.35,21.56,21.52,21.44,21.34,21.29,21.32,21.32,21.36,21.41,

21.31,21.23,21.39,21.29,21.88,21.27,21.51,21.34,21.28,21.33,21.24,21.32,21.26,21.27,21.37,21.34,

21.31,21.51,21.34,21.25,21.21,21.32,21.23,21.18,21.05,21.20,21.28,21.28,21.27,21.33,21.36,21.36,

21.35,21.41,21.29,21.49,21.49,21.44,21.36,21.37,21.42,21.47,21.39,21.28,21.34,21.42,21.58,21.48,

21.37,21.27,21.26,21.23,21.29,21.18,21.25,21.52,21.39,21.32,21.28,21.36,21.32,21.48,21.87,21.67,

21.45,21.39,21.34,21.86,21.40,21.67,21.60,21.50,22.15,21.49,21.50,21.45,21.60,21.42,20.82,21.35,

21.41,21.25,21.24,21.32,21.32,21.31,21.20,21.17,21.29,21.35,21.32,21.31,21.27,21.38,21.41,21.63,

21.34,21.36,21.32,21.47,21.47,21.37,21.25,21.37,21.43,21.51,21.31,21.25,21.18,21.31,21.44,21.40,

21.18,21.19,21.15,21.20,21.09,21.17,21.22,21.26,21.36,21.12,21.36,21.37,21.42,21.30,21.32,21.28,

21.39,21.32,21.76,21.35,21.32,21.44,21.30,21.22,21.44,21.38,21.44,21.33,21.40,21.40,21.40,21.39,

21.35,21.28,21.12,21.31,21.33,21.42,21.29,21.21,21.22,21.27,21.32,21.31,21.32,21.33,21.51,21.47,

21.60,21.42,21.44,21.44,21.54,21.51,21.50,21.51,21.47,21.53,21.40,21.40,21.41,21.37,21.42,21.41,

21.31,21.36,21.19,20.56,21.30,21.42,21.62,21.62,21.55,21.61,21.46,21.36,21.38,21.31,21.32,21.34,

21.38,21.41,21.98,21.49,21.45,21.35,21.32,21.15,20.99,21.07,20.99,21.18,21.03))

**#**

**# AR1**

**#**

AR1.stan <- "

data {

int<lower=0> T;

vector[T] y;

}

parameters {

real mu;

real phi;

real y0;

real<lower=0> sigma;

}

model {

sigma ~ normal(0, 1);

mu ~ normal(0, 20);

phi ~ normal(0, 1);

y0 ~ normal(0,10);

y[1] ~ normal(mu + phi \* y0, sigma);

for (n in 2:T)

y[n] ~ normal(mu + phi \* y[n-1], sigma);

}

generated quantities {

vector[T] log\_lik;

log\_lik[1] = normal\_lpdf(y[1] | mu+phi\*y0 , sigma);

for (t in 2:T) { log\_lik[t] = normal\_lpdf(y[t] | mu+phi\*y[t-1] , sigma);

}}

"

**# Estimation**

smAR <- stan\_model(model\_code=AR1.stan)

pars =c("mu","phi","sigma","y0","log\_lik")

fitAR <- sampling(smAR,data =D,pars=pars ,iter = 10000,warmup=1000,chains = 2,seed= 12345)

print(fitAR,digits=3)

**# Fit**

**LOO=loo(as.matrix(fitAR,pars="log\_lik"))**

loocase <- as.vector(LOO$pointwise[,3])

plot(loocase,ylab="Pointwise LOO-IC",xlab="Day")

**#**

**# random coefficient AR1 (RCAR)**

**#**

RCAR.stan <- "

data {

int<lower=0> T;

vector[T] y;

}

parameters {

real mu;

real eta[T];

real y0;

real mu\_phi;

real<lower=0> sigma;

real<lower=0> sigma\_phi;

}

transformed parameters {

vector[T] muy;

vector[T] phi;

phi[1] = mu\_phi+eta[1]\*sigma\_phi;

for (t in 2:T) {phi[t]=mu\_phi+eta[t]\*sigma\_phi;}

muy[1] =mu+(mu\_phi+eta[1]\*sigma\_phi)\*y0;

for (t in 2:T) {muy[t] =mu+(mu\_phi+eta[t]\*sigma\_phi)\*y[t-1];}

}

model {

sigma ~ normal(0, 1);

eta ~ normal(0,1);

mu ~ normal(0, 20);

mu\_phi ~ normal(0, 1);

y0 ~ normal(0,20);

for (t in 1:T) {y[t] ~ normal(muy[t], sigma);}

}

generated quantities {

vector[T] log\_lik;

for (t in 1:T) { log\_lik[t] = normal\_lpdf(y[t] | muy[t] , sigma); }

}

"

**# Estimation**

smRCAR <- stan\_model(model\_code=RCAR.stan)

fitRCAR <- sampling(smRCAR,data =D,iter = 10000,warmup=1000,chains = 2,seed= 12345)

print(fitRCAR,digits=3)

loo(as.matrix(fitRCAR,pars="log\_lik"))

**#**

**# GARCH[1,1]**

**#**

GARCH.stan <- "

data { int<lower=1> T;

real y[T];

int<lower=0> T\_out;

}

parameters {

real mu;

real y0;

real<lower=0> alpha0;

real<lower=0> sigma0;

real<lower=0,upper=1> alpha1;

real<lower=0, upper=(1-alpha1)> beta1;

}

transformed parameters {

real<lower=0> sigma[T];

sigma[1] = sqrt(alpha0+alpha1\*square(y0-mu)+beta1\*square(sigma0));

for (t in 2:T) {sigma[t] = sqrt(alpha0+alpha1\*square(y[t-1]-mu)+beta1\*square(sigma[t - 1]));}

}

model {

sigma0 ~ normal(0, 1);

y0 ~ normal(0, 20);

y[1] ~ normal(mu,sigma[1]);

for (t in 2:T) {y[t] ~ normal(mu,sigma[t]);}

}

generated quantities {

real y\_out[T\_out];

real sigma\_out[T\_out];

vector[T] log\_lik;

sigma\_out[1] = sqrt(alpha0 + alpha1 \* square(y[T] - mu) + beta1 \* square(sigma[T]));

y\_out[1] = normal\_rng(mu, sigma\_out[1]);

for (t in 2:T\_out) {

sigma\_out[t] = sqrt(alpha0+alpha1\*square(y\_out[t-1]-mu)+beta1\*square(sigma\_out[t-1]));

y\_out[t] = normal\_rng(mu, sigma\_out[t]); }

log\_lik[1] = normal\_lpdf(y[1] | mu, sigma[1]);

for (t in 2:T) { log\_lik[t] = normal\_lpdf(y[t] | mu, sigma[t]); }

}

"

**# Estimation**

smGARCH <- stan\_model(model\_code=GARCH.stan)

pars =c("alpha0","alpha1","sigma","y0","y\_out","log\_lik","beta1")

D$T\_out=5

fitGARCH <- sampling(smGARCH,data =D,pars=pars ,iter = 10000,warmup=1000,chains = 2,seed= 12345)

print(fitGARCH,digits=4)

**# Plot of posterior mean sigma**

sigmasamps <- a**s.matrix(fitGARCH,pars="sigma")**

sigma.pmn=apply(sigmasamps,2,mean)

plot(sigma.pmn,xlab="Day",ylab="Mean Sigma")

**# Fit**

loo(as.matrix(fitGARCH,pars="log\_lik"))

**#**

**# AR1, GARCH[1,1]**

**#**

ARGARCH.stan <- "

data { int<lower=1> T;

real y[T];

int<lower=0> T\_out;

}

parameters {

real mu;

real phi;

real y0;

real<lower=0> alpha0;

real<lower=0> sigma0;

real<lower=0,upper=1> alpha1;

real<lower=0, upper=(1-alpha1)> beta1;

}

transformed parameters {

real<lower=0> sigma[T];

sigma[1] = sqrt(alpha0+alpha1\*square(y0-mu)+beta1\*square(sigma0));

for (t in 2:T) {sigma[t] = sqrt(alpha0+alpha1\*square(y[t-1]-mu)+beta1\*square(sigma[t - 1]));}

}

model {

sigma0 ~ normal(0, 1);

phi ~ normal(0, 1);

y0 ~ normal(0, 20);

y[1] ~ normal(mu+ phi \* y0,sigma[1]);

for (t in 2:T) {y[t] ~ normal(mu+ phi \* y[t-1],sigma[t]);}

}

generated quantities {

real y\_out[T\_out];

real sigma\_out[T\_out];

vector[T] log\_lik;

sigma\_out[1] = sqrt(alpha0 + alpha1 \* square(y[T] - mu) + beta1 \* square(sigma[T]));

y\_out[1] = normal\_rng(mu+phi\*y[T], sigma\_out[1]);

for (t in 2:T\_out) {

sigma\_out[t] = sqrt(alpha0+alpha1\*square(y\_out[t-1]-mu)+beta1\*square(sigma\_out[t-1]));

y\_out[t] = normal\_rng(mu+phi\*y\_out[t-1], sigma\_out[t]); }

log\_lik[1] = normal\_lpdf(y[1] | mu +phi\*y0, sigma[1]);

for (t in 2:T) { log\_lik[t] = normal\_lpdf(y[t] | mu +phi\*y[t-1], sigma[t]); }

}

"

**# Estimation**

smARGARCH <- stan\_model(model\_code=ARGARCH.stan)

pars =c("alpha0","alpha1","beta1","sigma","y0","y\_out","log\_lik","phi")

D$T\_out=5

fitARGARCH <- sampling(smARGARCH,data =D,pars=pars ,iter = 10000,warmup=1000,chains = 2,seed= 12345)

print(fitARGARCH,digits=4)

**# Fit**

**loo(as.matrix(fitARGARCH,pars="log\_lik"))**