

# Moral Hazard and Time Inconsistency: Dynamic Contracting with Non-exponential Discounting

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## Abstract

We study dynamic contracting when one or both parties have time-inconsistent preferences arising from non-standard discounting in continuous time. Such inconsistencies are an immediate result of any non-exponential setup. Some prominent examples are a stochastic discount factor (arising from varying interest rates/short rates [Vasicek \(1977\)](#), [Cox et al. \(1985\)](#), [Gabaix \(2012\)](#)), or present biased preferences ([Harris and Laibson \(2012\)](#), [Jackson and Yariv \(2015\)](#)). Paper has two parts: in the first part we lay down the general framework and describe the principal optimal dynamic contract as a system of non-linear equations, rather than the classical Hamilton-Jacobi-Bellman equation. In the second part, we explore different applications to canonical continuous-time contracting models and deliver the contracts in closed form. Since the time inconsistencies we explore are arising from just non-standard discounting (potentially from changes outside their control) we take the stance that the parties are sophisticated about the time inconsistency. We show that such an inconsistency necessitates a game-theoretic framework, as now both parties need to take into account the changes in their preferences that will arise in the future. Optimal contract is characterized as

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a subgame perfect Nash equilibrium of the game played between the agent and the principals' future selves. We show that the time inconsistency of the agent only causes history dependence for the agent's continuation utility, but otherwise does not affect the standard contracting setups such as (Holmstrom and Milgrom (1987), Sannikov (2008), He (2011)). On the other, hand time inconsistency on the side of the principal changes the problem completely, and an equilibrium approach resulting in a system of non-linear equations becomes necessary. We show that the equilibrium system is equivalent to a coupled Backward Stochastic Differential Equation system which has a solution.

We demonstrate the applicability of the framework by solving two examples explicitly: one where the principal has Quasi-hyperbolic discounting and one where the principal has a stochastic discount rate. Our first example is motivated by contracting problems arising when the principal might have a "term" but has a chance to continue. Good examples of the first case would be a politician or a board member in an organization being reelected hence has higher valuations for returns during his secured term, but values future returns less as s/he might not be reelected. Our second example is motivated by contracting problems when the principal might be facing volatile and different interest rates. A good example of such a setting is when the principal is paid according to stocks and hence the stock fluctuations and short rates affect his/her valuations of future returns. These contracts show that there are both some similarities and some stark contrasts between the standard contracting setups and time-inconsistent ones. For example, the expected utility of the agent does not change whether he contracts with a time consistent or inconsistent principal. However, the actions the agent takes and the payments he receives vary over time with a time-inconsistent principal whereas those are constant in the time-consistent case. These variabilities in actions and payments take different forms based on the setup, for example, a "deadline effect" in quasi-hyperbolic discounting or a "bright future effect" in stochastic discounting.

**Note:** While we do not as yet have a paper to circulate, all the main results are in place and a working paper should be complete in a month.

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