Bank Regulation Under Adverse Selection and the Cost of Capital

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Abstract

This paper studies a model of bank capital regulation whereby banks have private information regarding the value of their existing assets. Raising capital (e.g. equity) is costly for banks whose assets are undervalued by the market, leading them to forgo new investments when capital requirements are too high. Under this foundation, the regulator faces a tradeoff between minimizing the liability that bank failure imposes on society and inducing banks to invest in valuable projects. We show the existence of capital regulations that effectively screen the banks revealing their information to the market, resolving the underinvestment problem. We further show that pooling the bank's information though a simple capital requirement can still be socially optimal. The main result characterizes the optimal capital regulations as a function of the strength and opacity of the banking sector and the resulting policy implications.

Introduction

Since the financial crisis, policy makers have worked to enhance the regulatory framework in order to prevent future crises and their associated spill over effects. Yet, some critics suggest that the increase in bank capital requirements following the crisis were not sufficient to achieve this goal (see e.g. Admati and Hellwig (2013)). This begs the question of what keeps regulators

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from increasing capital requirements to higher levels and, more importantly, what is the theoretical foundation that drives such decisions?[1]

Higher capital requirements serve as a way to prevent bank failures by increasing the bank's ability to absorb unexpected losses before failing. Given that bank failures have shown to impose large negative externalities on society, this creates a well accepted rationale for higher capital requirements.[2] On the other hand, the social cost of higher capital requirements is less clear. Namely, while the increase in the cost of bank financing due to higher capital requirements has been well studied (see e.g., King (2009)), there are many difficulties in estimating how those costs spillover to society and therefore into the regulator's welfare function.[3] The contribution of this paper is to provide a foundation for the social cost of capital and to illustrate the resulting insights into the optimal design of prudential bank regulation.

We study a model whereby banks have private information about the value of their existing assets. In such a setting, raising capital is costly for banks whose assets are undervalued by the market which may lead them to forgo new projects when subject to high capital requirements. Bank equity issuance during the financial crisis provides strong evidence for this private information cost of raising capital. Namely, many of the largest (and most opaque) U.S. banks were reluctant to raise capital during the crisis, leading to government injections of equity through programs such as the Supervisory Capital Assessment Program and the Trouble Asset Relief Program (TARP). Yet, during the same time period, over $450 billion worth of bank equity was voluntarily issued

[1]Chapter 3 of Dewatripont, Rochet, and Tirole (2010) highlights that most of the motivations for the Basel I and II accords come from political pressure on policy makers by the banking industry, first to create a regulatory framework that avoided competitive distortions, then to allow the banks to use their superior information to decide the risk weighting of assets. Even today, there seems to be little consensus regarding the optimal design of bank regulations among policy makers. For example, only a few years after the Dodd-Frank act was signed into law, which introduced many new post crisis regulations in the U.S., the Financial Choice Act is close to being enacted which would repeal many of those regulations.

[2]The loss in output due to the financial crisis is estimated to be over $75 trillion for Basel committee member countries (Basel Committee (2015)).

[3]Typically papers treat the cost of capital as a black box, taking the cost to the bank as a proxy for the cost to society. The issue with this approach is that many of the costs of higher capital requirements to the bank act as transfers from the bank to other agents in society. For example, the administrative costs of issuing equity (e.g. investment bank fees) are direct transfers to the administrators. Similarly, if banks must issue equity at a price below its true value then this acts as a transfer from old shareholders to new shareholders leaving welfare unchanged.

without any government assistance (Black, et. al. (2016)). This can be explained by the fact that the bank's cost of raising equity varies with its private information so that the banks whose assets are not undervalued by the market face little to no cost of issuing equity, even during the crisis.[4]

In this context, the regulator faces a tradeoff between minimizing the losses that bank failure imposes on society and inducing banks to invest in valuable projects.[5] While it has been empirically documented that banks decrease lending in response to regulatory capital requirements (see e.g. Peek and Rosengren (1995), Gropp, et. al (2016), Fraisse, et. al. (2017)), this idea has yet to be incorporated into a model of banking regulation. The key insight that we develop is that capital requirements can be designed to credibly reveal the bank's private information to the market, effectively eliminating the bank's incentive to forgo productive investments. Yet, we show that in some cases pooling the bank's information can be socially optimal instead, highlighting how the social cost of capital depends not only on the level of bank capital requirements but also on the way that those capital requirements are implemented.

Our baseline model is similar to Myers and Majluf (1984) whereby the bank has private information regarding its existing assets and must decide whether to undertake a new project. We then introduce a regulator who has the ability to tax the bank and set *capital requirements* which dictate that a fraction of the bank's new investments must be financed through the sale of a restricted *capital security* (e.g. equity), also chosen by the regulator. After characterizing the equilibria of the capital raising game between the bank and the market given the regulator's mechanism, we then proceed to characterize the optimal regulations.

We show the existence of three optimal regulatory designs over the underlying parameter space. Under the first design (IRB-type), the regulator completely resolves the underinvestment problem by designing capital requirements that induce the banks to credibly reveal their private information to the market. This type of *mechanism* is similar to the Internal Ratings Based approach (IRB)

[4]This is consistent with the finding that banks with more opaque assets (measured by lower turnover, higher volatility, and higher bid-ask spreads) were more likely to issue equity using government programs as opposed to issuing to private investors over this period (Black, et. al. (2016)).

[5]Such losses can consist of spill over effects on the real economy due to the bank's failure (e.g. for systemic reasons) or losses from distortionary taxes utilized to fund the repayment of insured deposits.

introduced in Basel II whereby banks utilize their own internal risk models to provide the regulator with key statistics of their asset returns (e.g. probability of default, loss given default, etc.) that determine the bank's capital requirement. Although it is not clear whether the IRB approach was designed to act as a way for banks to credibly signal their private information to the market[6], we show that this is precisely the merit of allowing banks to utilize their own information to influence their capital requirements. In this sense, our results highlight a neglected benefit of the IRB approach whereby slightly augmenting the approach with a report specific (ex-ante) transfer can lead to a large welfare improvement.[7] That being said, the regulator must pay information rents to the banks (in the form of lower capital requirements) in order to induce them to reveal their private information under this framework which is why it is not always optimal over the underlying parameter space.

The second optimal design (SA-type) is one whereby the regulator sets a simple *pooling* capital requirement, independent of the bank's private information. Such a mechanism is similar to the Standardized Approach (SA) of Basel I-III whereby the bank's capital requirements are grouped by asset type and credit rating, but independent of any additional information the bank may posses about those assets. This is precisely the mechanism under which banks with good news will optimally forgo investments when capital requirements are set too high. Hence, under the SA-type mechanism, capital requirements are set as high as possible subject to inducing investment by the banks with good news.

Finally, it may be the case that the cost to society of lowering capital requirements — either to induce information revelation in the IRB-type design or to induce investment in the SA-type design — does not outweigh the benefit of the investments that these regulations induce (e.g. when the net present value of new investments is very low). In this case, the regulator utilizes a third under-investment (UI) design that sets high capital requirements, inducing an equilibrium whereby only

[6]Samuels et. al. (2012) survey bank investors and find that a majority lack confidence in banks' risk weighted asset reports and believe that the bank's discretion to choose internal models for the calculation of risk should be abandoned.

[7]An example of a mechanism that incentivizes credible information revelation is one whereby banks with good (bad) news face lower (higher) capital requirements on their new investments but a higher (lower) deposit insurance premium.

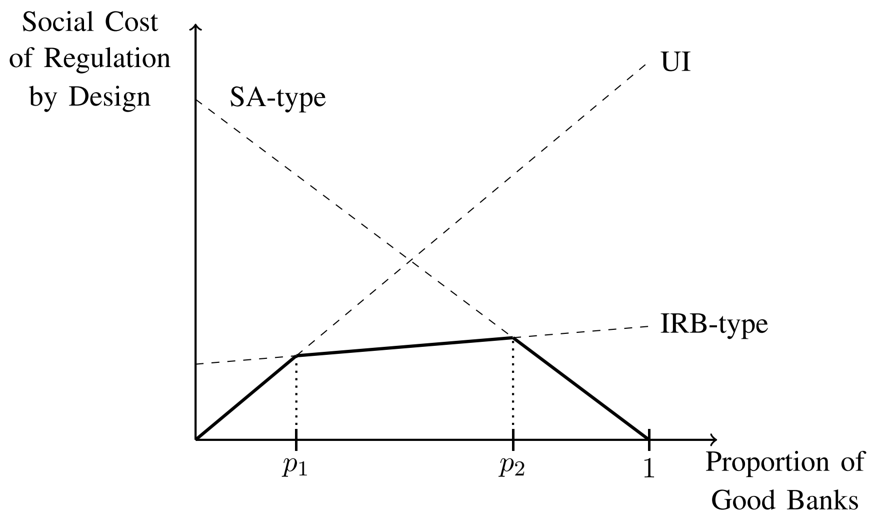


Figure 1: The Social Cost of Capital Requirements Under the Optimal Regulatory Design

banks with bad news invest in the new project. The main result of the paper is a characterization of the optimal regulatory mechanism which formalizes the optimal capital requirements, taxes, and securities utilized in the SA-type, IRB-type, and UI mechanism and conditions under which each respective mechanism is optimal.

When characterizing which of the three aforementioned mechanisms is optimal, the key parameter is the proportion of banks with good news. As illustrated in Figure (1), when the proportion of good banks is high (greater than p_2), then the optimal mechanism mitigates underinvestment through the SA-type mechanism. This comes from the fact that, in this case, the cost of raising capital for the banks with good news is small as the market's (average) security price is close to the bank's true valuation. Hence, the regulator can set high capital requirements and still induce investment. If instead, the proportion of banks with good news is low (below p_1) the optimal (UI) mechanism sets high capital requirements, inducing underinvestment by the good banks. This is optimal as the cost to society of underinvestment by the good types diminishes when their proportion goes to zero.[8] Finally, if the proportion of good banks is intermediate (between p_1 and p_2) then inducing the banks to reveal their private information through the IRB-type mechanism is optimal.

[8]When a bank receives bad news this implies that its assets are overvalued by the market and therefore it receives a subsidy when raising new capital. For this reason, banks with bad news will never forgo new investments.

This is because the cost of inducing investment through the SA-type mechanism is too large as the good type's security is heavily undervalued by the market, yet the proportion of good banks is too high for underinvestment to be socially desirable through the UI mechanism.[9]

In this paper our results are characterized for general securities. We show that the SA-type mechanism optimally restricts banks to issue securities that are the least *informationally sensitive*: securities that minimize the difference in the value of the security with respect to the bank's private information. On the other hand, the IRB-type mechanism optimally restricts the banks with good news to issue the least informationally sensitive security while banks with bad news are required to issue the most informationally sensitive security. This lends support for the use of contingent convertible (CoCo) bonds for the financing of regulatory capital as (in standard cases) Cocos minimize the information sensitivity of the security similar to debt securities, but also have the desirable property of absorbing losses before the bank fails similar to equity.

The results of this paper allow us to characterize how capital requirements should be adjusted with respect to the strength of the banking industry. Briefly put, the regulator should resort to a uniform SA-type capital requirement when the banking sector is strong (i.e. most banks have good news) but should resort instead to IRB-type capital regulations that help to resolve the asymmetric information between banks and the market when the banking sector is faltering. If instead the banking sector is very weak then the regulator should focus more on recapitalizing the banks through the use of the UI mechanism as opposed to inducing investment. We further show how the value of new investments and the opacity of the bank's assets determine the optimal capital requirements in each respective mechanism. Important to note here is that if the regulator utilizes the SA-type mechanism and does not adjust capital requirements with respect to these variables when necessary, then this will lead to suboptimal underinvestment. Similarly, a static IRB-type mechanism can lead to incentive compatibility issues which can also result in underinvestment. This is an important insight to be gained, especially in the context of the current regulation which, for the

[9] As will be seen, the conditions to induce information revelation through the IRB-type are independent of the proportion of good banks. The only (minor) variation in the IRB-type capital requirements comes from the change in weights the regulator puts on each type.

most part, sets static capital requirements that do not adjust with the underlying fundamentals (e.g. NPV, risk, and opacity of new investments).

Aside from the adjustment of capital requirements over time, our results also contribute to the policy debate on current capital regulations. Namely, we discuss in Section 5 when the regulator should regulate the banks under either the IRB-type or SA-type mechanism depending on the opacity of the bank's balance sheet. Further, we note that the current discretion that banks have to choose whether they are regulated by the SA or IRB approach under Basel III should be removed as we show that such discretion will lead banks to choose the suboptimal framework when it is allowed. We then discuss other policy implications such as how our model provides insight into the new counter cyclical capital requirement (CCyB) of Basel III and stress testing. Finally, we discuss how our results provide insight into alternatives to government interventions during financial crises such as the TARP program utilized in the U.S. We show how our IRB-type mechanism can effectively offer a private solution to these programs by credibly screening banks to provide information to the market that allows them to correctly determine the quality of the bank's assets.

Related Literature

In this paper we study how capital requirements can lead to underinvestment when securities are issued to a less informed market, an idea inspired by Myers and Majluf (1984). Our general security design problem and capital raising game is similar to that studied in Nachman and Noe (1994) and Noe (1988). Nachman and Noe (1994) characterize conditions on the distribution of returns under which firms prefer to finance their assets with debt as opposed to equity. In contrast to these papers, our aim is not to characterize what security maximizes the value of the firm to existing shareholders, but rather to characterize the optimal securities for the use of prudential regulation.

In our model, high capital requirements lead to credit rationing but this is not the only reason for credit rationing due to asymmetric information. Stiglitz and Weiss (1981) develop a model where banks ration credit due to the adverse selection problem that exists between the bank and its privately informed loan applicants as opposed to the bank and its equity investors (as in our model). Thakor (1996) shows that higher capital requirements can exacerbate this credit rationing problem. In Thakor (1996), the credit rationing effect of higher capital requirements relies on the assumption that higher capital requirements lead to a higher cost of financing, justified by the Myers and Majluf (1984) insight. What we show in this paper is that the regulator has to potential to eliminate this cost of capital financing by designing capital regulations that resolve the asymmetric information between the bank and the market. Extending our model to include adverse selection by the bank's borrowers would therefore increase the value of this information revelation.

A large portion of the banking literature studies the moral hazard problem of capital regulation. Besanko and Kanatas (1996) show that capital requirements may lead to greater risk taking due to agency problems in the spirit of Jensen and Meckling (1976). A key assumption that drives their results is that raising capital dilutes the value of *insider* equity hence decreasing the benefit of costly effort to improve asset quality. We show that when capital is raised voluntarily in order to invest in new projects, then banks will forgo those new projects precisely when raising the required amount of capital leads to a dilution of the existing shareholder's equity. Therefore, the moral hazard problem of Besanko and Kanatas (1996) disappears. This highlights the importance of using capital requirements on future investments to voluntarily recapitalize the banking industry as opposed to forced recapitalizations.[10]

From a mechanism design perspective, the closest related paper is Giammarino, et al. (1993). They consider the problem of combined moral hazard and adverse selection and study the optimal design of incentive compatible capital requirements and deposit insurance premia. In their model, they assume that equity is dilutive (as in Besanko and Kanatas (1996)) and bears an exogenous cost driven by investors "preference for liquidity". While Giammarino, et. al. (1993) study incentive compatible mechanisms, as in this paper, they see no need for information revelation to markets due to the fact that the cost of equity is driven exogenously and therefore cannot be influenced. This

[10]Understandably, in some cases it may be optimal for the regulator to force bank recapitalizations in times of distress, but then the key insight of Besanko and Kanatas (1996) is that such a forced recapitalization should come with heavy monitoring to prevent excessive risk taking.

highlights the gains of providing a proper micro-foundation for the cost of capital and insights that can otherwise be lost.

As mentioned above, the IRB-type mechanism that we propose is similar to the IRB approach introduced in Basel II. It is important to note here that this paper is not claiming that IRB is optimal as in practice insurance premiums/taxes are not linked to IRB reports. From a theoretical perspective, strategic underreporting of bank risk via IRB has been studied in papers such as Prescott (2004), Leitner and Yilmaz (2018), and Colliard (2017).[11] In particular, Colliard (2017) shows that when the bank's internal risk estimates are private information, costly auditing leads to less risk-sensitive capital requirements in order to counteract the bank's incentive to choose risk models that underreport their true risk. Blum (2008) studies incentive compatibility issues with the IRB approach and finds that if the regulator has limited scope to sanction banks when they detect misreporting of risk ex-post, then a leverage ratio can improve welfare. The contribution of this paper to this literature is to show how, when properly designed, the IRB mechanism can serve to resolve information asymmetries between the bank and the market. In this case, not only will banks have the correct incentives to truthfully report their risk but the regulator will effectively resolve the underinvestment problem stemming from the banks' private information.

Our results also complement the literature on stress testing and information disclosure. Leitner and Williams (2017) show how the regulator faces a trade off between keeping its stress testing model secret to prevent gaming and revealing the model to prevent suboptimal underinvestment (the key cost of capital in our model). Goldstein and Leitner (2018) study the optimal information disclosure policy of the regulator's stress test. They show that in some cases disclosure can eliminate risk sharing opportunities for the bank but that in other cases it is necessary to facilitate such opportunities. This paper compliments this literature by studying information disclosure through the design of capital requirements. Namely, stress testing may not be necessary when the optimal capital regulations take the form of the IRB-type which reveals the bank's private information to

[11] There is further empirical evidence that IRB is not incentive compatible along some dimensions (e.g. Plosser and Santos (2018)).

the market. In contrast, in Section 5 we discuss how stress testing can complement the results of this paper when the level of opacity of the banks' existing assets is large.

Finally, our IRB-type mechanism bears some similarity to that of optimal interventions as studied by Philippon and Skreta (2012) and Tirole (2012). Both of these papers consider optimal interventions to restore lending and investment in the face of adverse selection. Philippon and Schnabl (2013) analyze the issue of recapitalizing a banking sector that restricts lending due to a debt overhang problem. In contrast, our motivation for such an *intervention* is to provide incentives for banks to voluntarily recapitalize when faced with unexpected losses and we show how this can be done without using government funds.

The rest of the paper is organized as follows. Section 1 presents the main model, including the mechanisms available to the regulator, the capital raising game between the bank and the market, and our equilibrium concept and refinements. Section 2 characterizes the equilibria of the capital raising game given the regulator's choice of mechanism. Section 3 characterizes the optimal SA-type (pooling) and IRB-type (separating) mechanisms. Section 4 presents our main result which characterizes when the SA-type, IRB-type, or UI mechanism is optimal given the proportion of banks with good news. Section 5 presents the policy implications of our results and Section 6 concludes. Section 7 is devoted to extending the main results beyond the two type case to a continuum of types. All proofs are relegated to the appendix in Section 8.

1 A Model of Capital Regulation Under Asymmetric Information

1.1 Baseline Model

The basic set up of the model is similar to Myers and Majluf (1984). The bank starts at time $t = 0$ with assets in place that generate a gross return captured by the random variable A. We assume for simplicity that A is a binary random variable whose return at time $t = 1$ is equal to a_h with probability p and a_ℓ with probability $(1 - p)$ where $a_h > a_\ell \geq 0$. The assets in place were purchased by the bank at time $t = -1$ and financed with 100% equity.[12] We assume that at time $t = 0$ the bank receives private information regarding the time $t = 1$ return of its assets in place. We assume that the bank's type can be represented by $\theta \in \Theta = \{h, \ell\}$ whereby a bank of type θ knows that its time $t = 1$ return will be a_θ.[13]

After learning its type at time $t = 0$ the bank, whose manager acts in the interest of the incumbent shareholders, receives an investment opportunity that costs I and generates a net return $B \sim G$ with expected value $\hat{b} := \mathbb{E}[B] > 0$. We assume that the distribution G has a bounded support over $\mathbb{R}$, has a density g that is continuous over its support, and that g is weakly increasing for returns less than the mean and weakly decreasing for returns greater than the mean. All asset returns are generated at time $t = 1$ in which case the bank is liquidated and the funds distributed to the bank's creditors and shareholders.

1.2 Capital Securities

We endow the regulator with the right to set *capital requirements* which dictate that some amount of the new investment $K = \gamma \cdot I$ must be financed through the sale of a security that the regulator qualifies as a *capital security*. We assume that the fraction of the investment not financed by the sale of a capital security is financed with insured deposits so that whenever the bank generates funds $P \geq K$ via the sale of some capital security, then the remainder $I - P$ is financed with insured deposits. Further, given that deposits are insured, we assume that they are issued at the risk free rate which we normalize to zero. In this case, deposits represent the cheapest form of financing to the banks.

We will now present our conditions for admissible capital securities. First note that a security is a mapping from the bank's return (net deposits) z to a payment $s(z)$ to the owner of the security. In what follows we will restrict attention to capital securities satisfying the following standard assumptions.

[12] Extending the results to the case where the bank finances its assets in place with less than 100% capital is straightforward.

[13] In the extensions section we show how our results can be extended to the case where Θ is a continuum.

Definition 1.1. *A capital security s is admissible if it satisfies the following conditions.*

(1) $s(z)$ is non-decreasing in the value of the bank z.

(2) $z - s(z)$ is non-decreasing in the value of the bank z.

(3) $s(z) \geq 0$ for all $z \in \mathbb{R}$.

We denote by $\mathcal{S}$ the set of admissible capital securities.

The conditions of Definition 1.1 are standard assumptions on the design of securities (see e.g. Innes (1990) and Nachman and Noe (1994)). If Condition (1) is not satisfied so that $s(z) < s(z')$ for some $z > z'$, then the bank could engage in a risk free arbitrage opportunity whereby whenever its return is z', it borrows $z - z'$ and reports z as it's earnings, gaining a profit of $s(z) - s(z')$. Similarly, if Condition (2) is not satisfied, then the bank could engage in a similar arbitrage by burning money (e.g. by liquidating assets below their market value). Finally, condition (3) represents the limited liability of the investors purchasing the security. In what follows we restrict attention to general securities in $\mathcal{S}$.

The purpose of *capital* is to absorb bank losses but can be defined differently given the regulator's objective. Namely, if the bank is large and systemic then the bank's insolvency can have spillover effects on the real economy (e.g. Lehman Brothers). In this case a capital security should be defined as a security with the ability to absorb losses before the bank becomes insolvent (e.g. equity).[14] If instead the bank is small and financed with deposits then the regulator may only care about protecting the deposit insurance fund, in which case, securities that absorb losses post insolvency may also qualify as capital (e.g. bail-inable/subordinated debt). In light of this discussion, we proceed throughout by assuming that equity qualifies as capital (this will be useful to prove some of our results) but that other securities may also qualify. The only important aspect of capital securities that we model is that they are admissible and junior to deposits.[15]

[14]In this sense, standard debt with face value D does not qualify as a capital security as if the value of the bank's assets is z, then whenever $z < D$ the bank's creditors would force liquidation of the bank leading to default. On the other hand, if s is equity, then the bank only fails whenever $z < 0$. Thus, if P is the value of the funds generated by the sale of equity then the bank can absorb additional losses (above the pre-investment equity stock) up to P before becoming insolvent.

[15]In practice, the most widely accepted *capital security* is equity. For example, the key capital requirement of

1.3 The Regulatory Environment

The regulator's capital requirement $K \geq 0$ dictates that the bank must raise an amount of funds (used to finance the new investment) greater than or equal to K by selling an admissible capital security $s \in \mathcal{S}$. Given that our distribution G is bounded there exists a level of capital $\bar{K}$ such that $K > \bar{K}$ provides no benefit to society.[16] Therefore, the first best outcome would be one whereby the regulator imposes a capital requirement $K = \bar{K}$ and both bank types invest in the new project. We will see below how high capital requirements lead to underinvestment by the h-type banks, precluding this first best outcome. We further endow the regulator with the ability to impose a lump sum ex-ante tax T on the bank (e.g. a deposit insurance premium) and to restrict the set of securities (to a subset of $\mathcal{S}$) that the bank can use to finance the capital requirement (e.g. to equity). We assume that the bank has the right to forgo the new investment (and receive a payoff of a_θ) whenever it finds it unprofitable to meet the requirements of the regulator's mechanism.[17]

Naturally, the requirements of the regulator can also depend on the bank's type θ so that when the bank *reports* that its type is θ then it must generate K_θ funds through the sale of a capital security in the restricted set $\mathcal{S}_\theta \subset \mathcal{S}$, and to pay a transfer T_θ. We assume throughout that the report of the bank's type is observed by the regulator but not by the market so that the revelation principle holds. Instead, we assume that the market observes the bank's commitment to meet the requirement K_θ and pay the transfer T_θ.[18] Note that while the mechanism can signal the bank's type through

the Basel III accords requires that at least 7% of the bank's risk weighted assets be financed with common equity (the Common Equity Tier 1 requirement (CET1) plus the Capital Conservation Buffer). Other than equity, contingent convertible capital (first motivated as a prudential regulation tool by and Squam Lake (2009)) has been gaining traction for potential use in prudential regulation. Contingent convertible debt is a debt contract that either converts to equity or is written down conditional on a market based or discretionary trigger. For example, perpetual debt that converts to equity when the bank's CET1 ratio falls below 5.125% qualifies (along with equity) to meet the Basel III Additional Tier 1 capital requirement of 1.5%. That being said, in practice, the main constraint faced by the banks are that they maintain a sufficient ratio of common equity to debt.

[16] Typically we would assume $K \leq I$ so that the banks are never required to raise more capital than the cost of their investment, but given that bad news in this model represents a devaluation of a bank's assets in place, then it also reflects a decrease in the bank's effective equity stock. Therefore, $K > I$ represents the case whereby the regulator requires the bank to recapitalize its pre-investment balance sheet before being allowed to invest in the new asset.

[17] For this reason, raising capital and making the new investment will never dilute the value of existing shareholder equity.

[18] This only matters when $K_h = K_\ell$ and $T_h = T_\ell$.

its reported commitment from (K_θ, T_θ), the bank's potential freedom to issue different securities $s \in \mathcal{S}_\theta$ may also act as an alternative signaling device in the capital raising game introduced below. We restrict attention to the following class of mechanisms.

Definition 1.2. *The regulator's mechanism $\mathcal{M}$ consists of a menu $\{(K_\theta, T_\theta, \mathcal{S}_\theta)\}_{\theta \in \{h,\ell\}}$ such that the option $\theta \in \{h, \ell\}$ requires the bank to generate funds worth at least $K_\theta \in \mathbb{R}_+$ through the sale of a capital security $s \in \mathcal{S}_\theta \subset \mathcal{S}$ and to pay an ex-ante transfer $T_\theta \in \mathbb{R}_+$ to the regulator.*

Note that our class of mechanisms could be potentially extended to the case whereby the regulator reports a noisy signal of the bank's type to the market. We do not model this signaling problem so that the only signaling of the bank's type through the mechanism comes from the (potential) difference in capital requirements and transfers. While generating a noisy signal regarding the bank's type may improve upon our class of mechanisms we note that it requires significant commitment power by the regulator.[19]

Another restriction of our mechanism is that we specify transfers as lump-sum and to be paid *ex-ante* in the spirit of a deposit insurance premium. In this case, the *ex-ante* transfer, T, will effect the pricing of a given security issued by the bank as it decreases the ex-post value of the bank from z to $z - T$. We make this restriction as an ex-ante transfer can be financed through the sale of the capital security so that there are no issues with the bank's ability to pay given its limited liability nor the regulators commitment to enforce payments in bad states of the world: prior to making the investment the bank raises funds to meet the capital requirement and to pay the transfer.[20]

[19]Namely, if the signal the regulator sends to the market is noisy, then it must be the case that it is randomly chosen, along with different capital requirements associated with the realized signal. A simple analogy is that the regulator has to flip a coin that when lands on heads yields a high capital requirement and tails a low capital requirement regardless of bank type (although the coin for different bank types has different probabilities of heads). The issue is that the regulator then has to report truthfully to the bank and the market whether the coin has landed on heads or tails and to implement the associated capital regulations. Given that the regulator will always prefer higher capital requirements (conditional on all bank types investing), not only does the regulator have to have significant commitment power, but the market has to believe that the regulator will not renege on its commitment.

[20]A more general set up would also allow for ex-post transfers dependent on the bank's realized value. We refrain from studying ex-post transfers as no such transfers currently exist in practice and the general insight can be obtained with a simpler ex-ante transfer that is inherently robust to the timing structure of the capital raising game described below.

Given a particular mechanism $\mathcal{M}$, whenever a bank of type θ chooses the menu option $\tilde{\theta} \in \{h, \ell\}$ and issues some security $s \in \mathcal{S}_{\tilde{\theta}}$ that generates funds $P \geq K_{\tilde{\theta}} + T_{\tilde{\theta}}$ (i.e. it satisfies the requirements of the mechanism) then the bank's ex-ante expected payoff is given by

$$V_\theta(s, \tilde{\theta}; P) := \mathbb{E}_\theta[max\{a_\theta + B + P - T_{\tilde{\theta}} - s, 0\}]$$

Namely, $V_\theta(s, \tilde{\theta}; P)$ represents the post investment payoff of the type θ bank who chooses menu option $\tilde{\theta}$, net deposits $I - P$, the transfer $T_{\tilde{\theta}}$, and the security payment.[21] To clarify this expression, note that the gross return of the bank's assets after making the investment is $a_\theta + I + x$ where x is the realization of B. Further, the bank raised P through the sale of s, therefore after paying the ex-ante transfer it finances the investment with $P - T_{\tilde{\theta}}$ of *new equity* and $D = I - (P - T_{\tilde{\theta}})$ of deposits. Therefore, the bank's return (accounting for limited liability) net deposits is $\max\{a_\theta + x + P - T_{\tilde{\theta}}, 0\}$. Finally, the bank must repay the security holders according to s (which we can include in the max because $s(z) = 0$ whenever $z = a_\theta + x + P - T_{\tilde{\theta}} \leq 0$). Thus we obtain our expression for $V_\theta(s, \tilde{\theta}; P)$. Note that in equilibrium the amount of funds generated, P, will be determined endogenously via the market beliefs of the bank's type given the menu option it chooses and the security it issues.

In what follows we will often differentiate between *pooling* and *separating* mechanisms which we now define.

Definition 1.3. *A pooling mechanism $\mathcal{M}$ is any mechanism satisfying $K_h = K_\ell$, $T_h = T_\ell$, and $\mathcal{S}_h = \mathcal{S}_\ell$.*

A separating mechanism $\mathcal{M}$ is any mechanism satisfying either $K_h \neq K_\ell$ or $T_h \neq T_\ell$.

Note that when the transfers T_θ are too large, then no bank type will ever find it profitable to invest. Therefore, we proceed by assuming without loss that T_θ is bounded above by the level of transfers that induce banks to forgo the investment. As we will see, denoting by $b_\theta(K_\theta)$ the intrinsic

[21] Given that B is the net return, the gross return is therefore $I + B$. Hence, the return on the new investment net deposits is $I + B - (I - P) = B + P$.

value of the new investment to the bank of type θ after raising new capital worth K_θ (formally defined in Lemma 2.2 below), this implies $T_\theta \leq b_\theta(K_\theta)$ under any separating mechanism (otherwise the type θ bank will forgo the investment) and that $T \leq \min\{b_\ell(K), b_h(K)\}$ under any pooling mechanism with transfer T and capital requirement K. The latter half of this assumption will not play a role in the analysis as we will show that transfers under the optimal pooling mechanism are always set to zero.

Given the revelation principle it is without loss to restrict attention to incentive compatible mechanisms $\mathcal{M}$ such that it is optimal for the type-θ bank to *report truthfully* (i.e. choose the menu option θ). Further, whenever $\mathcal{M}$ is pooling then $\mathcal{M}$ is trivially incentive compatible given that the bank's choice of menu does not signal any information to the market. If instead $\mathcal{M}$ is a separating mechanism, then incentive compatibility is given by the following definition.

Definition 1.4. *Let $\mathcal{M}$ be a separating mechanism. Then, $\mathcal{M}$ is incentive compatible if for each $\theta \in \{h, \ell\}$ there exists $s \in \mathcal{S}_\theta$ such that $\mathbb{E}_\theta[s] \geq K_\theta + T_\theta$ and*

$$V_\theta(s, \theta; \mathbb{E}_\theta[s]) \geq V_\theta(\tilde{s}, \tilde{\theta}; \mathbb{E}_{\tilde{\theta}}[\tilde{s}])$$

for all $\tilde{\theta} \in \{h, \ell\}$ and $\tilde{s} \in \mathcal{S}_{\tilde{\theta}}$ such that $\mathbb{E}_{\tilde{\theta}}[\tilde{s}] \geq K_{\tilde{\theta}} + T_{\tilde{\theta}}$.

Namely, $\mathcal{M}$ is incentive compatible if whenever the market belief coincides with the bank's menu choice (i.e. whenever the bank chooses menu option θ then the market believes its type is θ), then the type θ bank prefers to issue some security $s \in \mathcal{S}_\theta$ to meet the capital requirement K_θ and pay the transfer T_θ rather than issue any other security $\tilde{s} \in \mathcal{S}_{\tilde{\theta}}$ that meets the capital requirement $K_{\tilde{\theta}}$ and pay the transfer $T_{\tilde{\theta}}$. Note that this definition of incentive compatibility assumes that the market beliefs will be correct. We will show below that under a standard equilibrium refinement this will always be the case in equilibrium whenever $\mathcal{M}$ is a separating mechanism satisfying the conditions of Definition 1.4.

1.4 Welfare

We define welfare as the sum of payoffs to the bank and its creditors net the spillover costs of bank failure. Namely, given the bank's type, θ, and the capital, $K_1 = P - T_\theta$, generated from the sale of some capital security s, the bank fails when its losses from the new investment exceed its effective capital stock $a_\theta + K_1$. This is the case whenever the realization x, of B, is less than $-a_\theta - K_1$. The expected loss to the bank's creditors is therefore given by

$$L_\theta(K_1) := -\mathbb{E}[min\{a_\theta + B + K_1, 0\}].$$

$L_\theta(K_1)$ is naturally independent of the type of capital security offered and is only a function of the capital K_1 that it generates. This is due to the fact that that $s(z) = 0$ whenever $a_\theta + x + K_1 \leq 0$.

We assume that bankruptcy creates a deadweight loss to society, captured by the parameter λ, proportional to the expected loss $L_\theta(K_1)$.[22] It is important to highlight the potential interpretations for λ. One interpretation is that λ represents the dead weight loss to the bank's creditors caused by bankruptcy/liquidation proceedings. Similarly, we could also interpret λ as the deadweight loss incurred from imposing distortionary taxes on society in order to generate the funds to repay the insured deposits or the bank's creditors if the regulator cannot commit to not bailout the bank in times of distress. Finally, we can interpret λ as the spillover effects on the real economy caused by the failure of the bank caused, for example, by systemic factors.

The social welfare under the mechanism $\mathcal{M} = \{K_\theta, T_\theta, \mathcal{S}_\theta\}_{\theta \in \{h,\ell\}}$ when the type θ bank invests, reports type $\tilde{\theta}$, and the funds generated by the sale of its capital security are $P \geq K_{\tilde{\theta}} + T_{\tilde{\theta}}$ (i.e. the capital generated is $K_1 = P - T_{\tilde{\theta}}$) is given by

$$W_\theta(invest|K_1) = a_\theta + b_\theta(K_1) - (1+\lambda) \cdot L_\theta(K_1) = a_\theta + \hat{b} - \lambda \cdot L_\theta(K_1)$$

[22] The bank's losses generating a deadweight loss is a necessary ingredient to any capital regulation model as otherwise the regulator would always allow the bank to finance with 100% deposits whenever financing with capital is costly to society (e.g. capital requirements lead to underinvestment).

Namely, the bank's profit is $V_\theta(s, \tilde{\theta}; P)$ but the buyers of the bank's capital security pay P and receive $\mathbb{E}_\theta[s]$ while the bank receives P and loses $\mathbb{E}_\theta[s]$. Further, the bank pays the regulator $T_{\tilde{\theta}}$ from the funds P generated and the regulator receives the transfer $T_{\tilde{\theta}}$.[23] Hence, after canceling out these terms from the bank's profit we obtain the above expression. The second equality comes from the fact that the bank's intrinsic benefit of the new investment $b_\theta(K_1) = \hat{b} + L_\theta(K_1)$ which is the NPV of the investment plus the value of the deposit insurance to the bank (this is formally proven in Lemma 2.2 below).

If instead the bank forgoes the investment, then the social welfare is

$$W_\theta(forgo) = a_\theta.$$

Note that the welfare only depends on the decision to invest or not, regardless of the security issued. This is due to the fact that while the security may be under/over priced with respect to the bank's private information, this discrepancy acts as a direct transfer of wealth from the bank's incumbent shareholders to the owners of the security. Hence, given that the regulator does not weight the bank's shareholders any differently from external investors this transfer cancels out in the welfare function.

As we will see below, the relevant expected welfare (given that the ℓ-type will always invest) when the type θ raises $K_\theta = P_\theta - T_\theta$ from the sale of some capital security is the expected welfare when both types invest

$$W(\mathcal{M}, invest) := p \cdot W_h(invest|K_h) + (1-p) \cdot W_\ell(invest|K_\ell)$$

[23]Here we assume that transfers from the bank to the regulator are treated as taxes which are then redistributed to society via government expenditures. We do not assume that these transfers fund the deposit insurance fund for simplicity but the model could be easily extended in this direction.

and when only the ℓ-type invests

$$W(\mathcal{M}, forgo) := p \cdot a_h + (1-p) \cdot W_\ell(invest|K_\ell)$$

Therefore, the regulator's objective will be to choose a mechanism to maximize welfare conditional on the h-type's decision to invest or forgo given the mechanism and the equilibrium of the *capital raising game* which we describe in the following subsection.

1.5 The Capital Raising Game $\Gamma(\mathcal{M})$

Before introducing the capital raising game we should mention that under laissez-faire regulation ($K_h = K_\ell = 0$, $T_h = T_\ell = 0$) all bank types invest using 100% deposits as this is the cheapest form of financing. This implies that deposits serve not only as a way to prevent bank runs (see e.g. Diamond and Dybvig (1983)) but also to promote investment in the face of adverse selection.

The regulator's mechanism $\mathcal{M} = \{K_\theta, T_\theta, \mathcal{S}_\theta\}_{\theta \in \{h,\ell\}}$ induces a capital raising game $\Gamma(\mathcal{M})$ played between the bank and the market. The game $\Gamma(\mathcal{M})$, illustrated in Figure 2, proceeds as follows: at time $t = 1$ the bank of type $\theta \in \{h, \ell\}$ decides whether to forgo or invest in the new investment. If the type θ bank forgoes, the game is over and its payoff is a_θ. If instead the bank decides to invest in the new asset, it must make a report to the regulator $\theta \in \{h, \ell\}$ and issue an admissible capital security $s \in \mathcal{S}_\theta$ (e.g. equity) in order to generate funds totaling $P \geq K_\theta + T_\theta$. If the bank does not meet the specified capital requirement so that the funds generated from the sale of the security P are less than then the capital requirement K_θ and the ex-ante transfer T_θ then we assume its payoff is 0. This is consistent with the bank losing its charter and therefore being nationalized by the regulator, providing the bank's existing shareholders with a payoff of 0.[24] If the bank invests, then the market formulates a belief $\mu(s) := Pr(\theta = h|s) \in [0, 1]$ of the bank's type given the security issued, represented by the probability that the bank's type is h given the security s it issues. The market then offers a payment $P(s)$ for the security s given its beliefs $\mu(s)$.

[24] We assume that the investment decision is observable to the regulator so that such a violation will always be detected. In this case the bank will never violate the capital requirement in equilibrium as it would always prefer to forgo the investment instead.

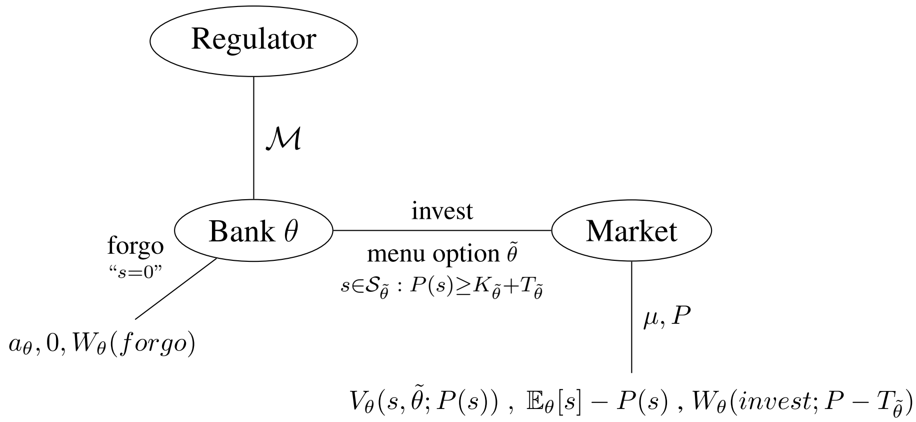


Figure 2: The capital raising game $\Gamma(\mathcal{M})$.

We denote by $\mathbb{E}_{\mu(s)}[s]$ the markets valuation of the security s given their beliefs $\mu(s)$ regarding the bank's type and $\mathbb{E}_\theta[s]$ the type θ bank's (true) valuation of the security.

Given that we assume the bank's decision to undertake the new investment is observable, we will represent the bank's decision to forgo the investment, without loss, by the issuance of the security $s = 0$ (i.e. $s(z) = 0$ for all z). In this case, whenever $s = 0$ the bank's payoff is a_θ when its true type is $\theta \in \{h, \ell\}$. If instead, the bank reports its type is $\tilde{\theta}$ and it issues some security $s \in \mathcal{S}_{\tilde{\theta}}$ that generates funds $P(s) \geq K_{\tilde{\theta}} + T_{\tilde{\theta}}$ then the bank's payoff is given by $V_\theta(s, \tilde{\theta}; P(s))$ and the markets payoff is $\mathbb{E}_\theta[s] - P(s)$.

The Underinvestment Problem: Given that the h-type security is always more valuable than the ℓ-type security (the h-type bank's distribution of returns first order stochastically dominates the ℓ-type's) we can see that for any market beliefs μ and any security $s \in \mathcal{S}$, when transfers do not depend on type (i.e. $T_h = T_\ell = T$) then we have

$$\mathbb{E}_{\mu(s)}[s] - \mathbb{E}_h[s] \leq 0 \qquad \text{and} \qquad \mathbb{E}_{\mu(s)}[s] - \mathbb{E}_\ell[s] \geq 0$$

This states that the h-type's security is always weakly underpriced while the ℓ-type's security is always weakly overpriced. In this case, the ℓ-type will always find it profitable to invest provided the transfer is not too large. The h-type on the other hand may find it optimal to forgo the investment (even with zero transfers) whenever the market puts a probability less than 1 on the bank being the h-type: $\mu(s) < 1$. The potential underinvestment created by this friction only exists when the value of the new investment is not too large. Namely, we can show that the h-type bank will never forgo the investment if the NPV of the new project $\hat{b} \geq a_h - a_\ell$ as in this case the value of the investment is so large that it is profitable for the h-type to invest even if the market holds the worst beliefs: $\mu(s) = 0$ for all $s \in \mathcal{S}$ (and optimally transfers will be zero). We therefore assume without loss that $\hat{b} < a_h - a_\ell$ throughout, noting that whenever this assumption does not hold then the regulator can achieve the first best outcome inducing all banks to invest and setting capital requirements $K_h = K_\ell = \bar{K}$ through the use of transfers $T_h = T_\ell = 0$ and securities $\mathcal{S}_h = \mathcal{S}_\ell = \mathcal{S}_{eq}$ where $\mathcal{S}_{eq}$ is the set of equity securities.

1.6 Equilibrium Concept and Refinements

In this subsection we will define our equilibrium concept for the game $\Gamma(\mathcal{M})$ and two refinements that we will be interested in. A strategy profile of the capital raising game $\Gamma(\mathcal{M})$ consists of a tuple (s_h, s_ℓ, μ, P) with $s_\theta \in \mathcal{S}_\theta \cup \{0\}$ the security issued by each type $\theta \in \{h, \ell\}$, $\mu : \mathcal{S} \to [0, 1]$ such that $\mu(s)$ is the market belief of the bank's type when it issues security s, and $P : \mathcal{S} \to \mathbb{R}$ such that $P(s)$ is the price offered by the market for a given security s. We will utilize the perfect Bayesian equilibrium solution concept which, in the context of $\Gamma(\mathcal{M})$, is defined as follows.

Definition 1.5. *Let $\mathcal{M}$ be an incentive compatible mechanism. The strategy profile $e^\star = (s_h^\star, s_\ell^\star, \mu^\star, P^\star)$ is a perfect Bayesian equilibrium if it satisfies the following conditions:*

(1) If $s_\theta^\star \neq 0$, then $P(s_\theta^\star) \geq K_\theta + T_\theta$ for each $\theta \in \{h, \ell\}$ and

$$s_\theta^\star \in \underset{s \in \mathcal{S}_\theta : P(s) \geq K_\theta + T_\theta}{\operatorname{argmax}} V_\theta(s, \theta; P(s))$$

(2) The beliefs $\mu^\star$ are consistent with the bank's type specific strategy $(s_h^\star, s_\ell^\star)$ so that $\mu^\star(s_\theta^\star)$ is computed using Bayes rule for each $\theta \in \{h, \ell\}$.

(3) The market price is competitive given the market beliefs: $P^\star(s) = \mathbb{E}_{\mu^\star(s)}[s]$ for all $s \in \mathcal{S}$.

The first two conditions represent the standard definition of perfect Bayesian equilibrium which requires that (1) if the bank invests, then the security $s^\star_\theta$ meets the capital requirement (i.e. sequential rationality of the investment decision) and the choice of security is sequentially rational with respect to the market beliefs $\mu^\star$, (2) the market beliefs are consistent with respect to the type specific strategy of the bank. Finally, condition (3) assumes that the market prices securities competitively so that the price the market offers for a security is exactly equal to the markets value of that security given its beliefs about the bank's type: $P^\star(s) = \mathbb{E}_{\mu^\star(s)}[s] = \mu^\star(s)\mathbb{E}_h[s] + (1-\mu^\star(s))\mathbb{E}_\ell[s]$.

Now, as is usual for signaling games, $\Gamma(\mathcal{M})$ has socially undesirable equilibria whereby, regardless of the mechanism $\mathcal{M}$, the h-type never invests. Namely, such an equilibrium outcome is supported by the beliefs $\mu(s) = 0$ for all $s \in \mathcal{S}$. We say that these equilibria are undesirable when there exists another equilibrium whereby the h-type invests, in which case $\mu(s) \neq 0$ for at least one security s. In this sense these equilibria are undesirable given that the h-type is unjustifiably excluded from the market even though if the market had beliefs the h-type might invest ($\mu(s) > 0$) then it would be optimal for the h-type to invest. Our first refinement will allow us to rule out the undesirable equilibria that require the market to ignore informative signals produced by the mechanism and the bank's choice of menu option. To this end we will use the intuitive criterion of Cho and Kreps (1987).

Definition 1.6 (Intuitive Criterion Cho and Kreps (1987))**.** *Let $e^\star = (s^\star_h, s^\star_\ell, \mu^\star, P^\star)$ be an equilibrium of the game $\Gamma(\mathcal{M})$ and let $u_\theta(s,\mu)$ be the payoff of the type-θ bank when issuing security s under beliefs μ. The equilibrium $e^\star$ satisfies the intuitive criterion if for any security $s \in \mathcal{S}$ such that for some $\theta, \theta' \in \{h, \ell\}$*

$$u_\theta(s^\star_\theta, \mu^\star) < \max_\mu u_\theta(s,\mu)$$

and

$$u_{\theta'}(s,\mu)|_{\mu(s):Pr_\mu(\theta|s)=0} \geq u_{\theta'}(s^\star_{\theta'}, m^\star)$$

then $\mu^\star(s)$ is such that $Pr_\mu(\theta'|s) = 1$.

Note that this definition is simplified from the original definition of Cho and Kreps due to the fact that we are dealing only with two possible types. Namely, in the language of the general definition, whenever s is equilibrium dominated for type-θ (i.e. issuing s yields a lower off-path payoff for the type-θ than the equilibrium strategy no matter the off path beliefs) but not equilibrium dominated for type θ' then the market should not believe that the bank is type-θ when it observes security s being issued. This implies the market believes the bank is type θ' whenever there are only two types. The intuition here is that when such a condition is satisfied, then when seeing the out of equilibrium security s issued, the market should believe that the bank's type is θ' if there are no out of equilibrium beliefs that would make issuing s more profitable than $e^{\star}$ for type θ while the type θ' bank could profit by issuing s whenever the market believes the bank's type is θ' after s is issued.

One remaining issue is that there still exist equilibria of pooling mechanisms that satisfy the intuitive criterion but still arbitrarily deter investment by the h-type. Namely, in such an equilibrium, markets believe that only the ℓ-type will invest so that $\mu(s) = 0$ for all $s \in \mathcal{S}$. Therefore, it may be the case that the h-type will find it optimal to forgo the new investment even though it is optimal for the h-type to invest when the market believes both types invest (i.e. $\mu(s) = p$ for some $s \in \mathcal{S}$). When the optimal mechanism is pooling then such bad equilibria are always dominated by any equilibrium where both types invest (provided such an equilibrium exists, which is the only case where the equilibria we attempt to rule out are in fact undesirable). In that case, we would like to think that the regulator's choice of a pooling mechanism should signal that both types will invest as optimality of the pooling mechanism is publicly observable. We therefore introduce the following assumption.

Assumption 1.1. *The regulator's choice of mechanism acts as a credible signal to the market of the h-type's investment decision. Namely, if the welfare of some equilibrium of a pooling mechanism where both types invest generates higher welfare than any equilibrium of any separating mechanism then the choice of the pooling mechanism credibly signals to the market that the h-type* *will invest.*

Both Assumption 1.1 and the Intuitive Criterion refinement are not necessary if the regulator has the possibility to purchase the bank's security through a government recapitalization program. Namely, if the Intuitive Criterion or Assumption 1.1 do not hold then it may be the case that bad equilibria are coordinated on, but given the dynamic nature of security issuance this opens up a possibility for the banks to report when markets are undervaluing their securities. As proven in Lemma 2.1 below, by purchasing the bank's security in this situation the regulator can achieve a strict welfare improvement over the inefficient equilibrium outcome.

2 Preliminary Results and Equilibria of the Capital Raising Game

Before proceeding to characterize the equilibria of the capital raising game we will first present a few preliminary results.

2.1 Preliminary Results

First, we will show that the intuitive criterion and Assumption 1.1 are not necessary if the regulator has access to government recapitalizations.

Lemma 2.1. *Let $\mathcal{M}$ be a socially optimal mechanism. If there exists an equilibrium of $\Gamma(\mathcal{M})$ whereby the h-type invests and the regulator has the ability to purchase the banks' securities at their equilibrium prices, then doing so yields a strict expected welfare improvement over any equilibrium of $\Gamma(\mathcal{M})$ whereby the h-type forgoes the investment.*

Proof. See appendix Section 8.2.1. □

The idea behind this lemma is that if a socially optimal mechanism permits an equilibrium whereby the h-type invests, then inducing investment by the h-type must be socially optimal (otherwise the regulator could increase capital requirements or transfers to induce the bank to forgo this investment). Whenever this is the case, then it is easy to show that by agreeing to purchase the security of the bank at the price specified in the equilibrium that induces investment, the regulator strictly increases expected welfare as he successfully induces the h-type to invest and breaks even in expectation on the purchase of the security. This is a subtle argument but it is relevant as when the pooling mechanism is optimal it would always benefit society to allow state sponsored recapitalizations in the case that the market and the bank coordinate on a bad equilibrium.

The next lemma provides us with an easier expression for $V_\theta(s, \tilde{\theta}; P)$.

Lemma 2.2. *Let s be an admissible security that generates funds P and denote by $K_1 = P - T_{\tilde{\theta}}$ the capital generated from the sale of s. Then,*

$$V_\theta(s, \tilde{\theta}; P) := \mathbb{E}_\theta[max\{a_\theta + B + P - T_{\tilde{\theta}} - s, 0\}] = a_\theta + b_\theta(K_1) + K_1 - \mathbb{E}_\theta[s]$$

where

$$b_\theta(K_1) := \int_{-a_\theta - K_1}^{\infty} x dG(x) - G(-a_\theta - K_1) \cdot (a_\theta + K_1) = \hat{b} + L_\theta(K_1)$$

Proof. See appendix Section 8.2.2. □

Note here that $b_\theta(K_1)$ represents the net present value of the new investment to the bank given the newly raised capital $K_1 = P - T_{\tilde{\theta}}$ net the *contamination cost* of the risk that the new investment imposes on the bank's post investment capital stock $a_\theta + K_1$. Namely, once the bank has made the new investment, it losses its existing capital $a_\theta + K_1$ whenever the loss incurred by the new investment exceeds this value, which happens with probability $G(-a_\theta - K_1)$. It is easy to check that $b_\theta(K_1) = \hat{b} + L_\theta(K_1)$ so that the added value to the firm from the new investment is equal to its full liability expected value $\hat{b}$ plus the expected liability that the new investment imposes on the deposit insurance fund. This is equivalent to saying that the added value of the new investment to the firm is exactly equal to the investment's value under full liability plus the value of the deposit insurance (i.e. the value of the put option on the bank's assets with strike price $I - K_1$).

Finally, the next result will be useful for characterizing the equilibria of the capital raising game.

Lemma 2.3. *If $s \neq 0$ is an admissible security that generates funds P when the market believes* *the bank's type is $\theta \in \{h, \ell\}$ then*

$$\mathbb{E}_\theta[s] = \int_{-a_\theta - P}^{\infty} s(x + a_\theta + P)dG(x).$$

Further, whenever $T_h = T_\ell = T$ then for all $s \in \mathcal{S}$ and all values of P:

(1) $\mathbb{E}_h[s] > \mathbb{E}_\ell[s]$

(2) $\mathbb{E}_h[s] - \mathbb{E}_\ell[s]$ is increasing in a_h.

Proof. See appendix Section 8.3.1. □

This result states two important conditions that our admissible securities satisfy. The first is that the value of any security is always higher when the bank is the h-type (excluding the effect of transfers). Intuitively this due to the fact that the h-type's existing assets are more valuable than the ℓ-type's and they face the same new investment. The second result states that the difference in this value $\mathbb{E}_h[s] - \mathbb{E}_\ell[s]$, which we call the *information sensitivity* of s, is strictly increasing a_h (keeping a_ℓ fixed).

2.2 Equilibria of Pooling Mechanisms

First, we characterize the properties of perfect Bayesian equilibria of $\Gamma(\mathcal{M})$ for all pooling mechanisms $\mathcal{M}$. We show that there are effectively three types of equilibria.

Proposition 2.1. *Let $\mathcal{M}$ be a pooling mechanism with capital requirement K and transfer $T \leq \min\{b_h(K_1), b_\ell(K_1)\}$ where $K_1 \geq K$ is the capital raised, net the ex-ante transfer. Then, any equilibrium $e = (s_h, s_\ell, \mu, P)$ of $\Gamma(\mathcal{M})$ that satisfies the intuitive criterion satisfies one (and only one) of the following three properties:*

(i) $s_h = 0$, $\mathbb{E}_\ell[s_\ell] = K + T$.

(ii) $s_\ell = s_h = s$, $\mathbb{E}_p[s] \geq K + T$.

(iii) $s_\ell \neq s_h$, $\mathbb{E}_\ell[s_\ell] = K + T$, $\mathbb{E}_h[s_h] = K' + T$ *where* $K' > K$ *and* s_h *satisfy*

$$s_h \in \underset{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s_h] = K' + T}}{\operatorname{argmin}} \ \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']$$

$$b_\ell(K) = b_\ell(K') + K' - \mathbb{E}_\ell[s_h]$$

Proof. See appendix Section 8.3.2. □

The first type-(i) equilibrium is the *undesirable* equilibrium discussed in Section 1.6. Important to note is that even under Assumption 1.1 this equilibrium will still be relevant when we discuss the *underinvestment* (UI) mechanism. Namely, it may be the case that the underlying parameters of the model are such that the rents paid by the regulator to the bank (in the form of lower capital requirements) do not outweigh the benefit of inducing the h-type bank to invest, regardless of whether a separating or pooling mechanism is utilized. In this case, the optimal mechanism sets a pooling requirement $K = \bar{K}$ and the h-type optimally forgoes the investment yielding the type-(i) equilibrium.

The second type-(ii) equilibrium will be the relevant *pooling equilibrium* whereby both types issue the same security $s \in \mathcal{S}$ and the market prices that security at its average price ($\mu(s) = p$) so that $\mathbb{E}_p[s] = p\mathbb{E}_h[s] + (1-p)\mathbb{E}_\ell[s] = K + T$. Important to note is that the security issued in any pooling equilibrium is the one that minimizes the information rents paid by the h-type to the market: $\mathbb{E}_h[s] - \mathbb{E}_\ell[s]$. In this case, this is equivalent to the banks issuing securities that minimize the *information sensitivity* of the security.

Finally, we show that there may exist a type-(iii) *separating equilibrium* whereby the h-type issues a security that generates more than the capital requirement so that the ℓ-type prefers to just meet the capital requirement and signal its type to the market than to raise the additional capital to mimic the h-type. As explained below we can effectively ignore this equilibrium as whenever it exists as it can be implemented by a separating mechanism. Further, the fact that the type-(iii) equilibrium dominates the type-(ii) equilibrium implies that whenever it exists for the optimal pooling capital requirement K, then the best pooling mechanism is (weakly) dominated by the best separating mechanism.

2.3 Equilibria of Separating Mechanisms

The following result characterizes the properties of equilibria of $\Gamma(\mathcal{M})$ whenever $\mathcal{M}$ is an incentive compatible separating mechanism.

Proposition 2.2. *Let $\mathcal{M}$ be an incentive compatible separating mechanism with capital requirements K_ℓ and K_h. Then, any equilibrium (s_h, s_ℓ, μ, P) of $\Gamma(\mathcal{M})$ that satisfies the intuitive criterion with $s_h \neq 0$ and $s_\ell \neq 0$ is such that*
(i) $\mu(s_\ell) = 0$ and $\mu(s_h) = 1$.
(ii) $\mathbb{E}_\ell[s_\ell] = K_\ell + T_\ell$ and $\mathbb{E}_h[s_h] = K_h + T_h$.

Proof. See appendix Section 8.3.3. □

What this proposition states is that incentive compatibility guarantees that when the intuitive criterion is satisfied then the market beliefs always coincide with the bank's menu choice (as signaled through their choice of capital requirement and transfers). Furthermore, we show that the capital requirements will be optimally binding for both types. This is again due to the fact that the banks prefer to be as highly leveraged as possible (deposits are subsidized). Hence, the only thing that can prevent the banks from having binding capital requirements is if the market has strange beliefs that the bank that issues security s_θ but exactly meets the capital requirement (i.e. $\mathbb{E}_\theta[s_\theta] = K_\theta + T_\theta$) is not type θ. Such a belief is ruled out by the intuitive criterion given that the mechanism is incentive compatible whether the capital requirement is binding or not.

3 Optimal Mechanisms

As mentioned above, for some parameters of the model (conditions will be given below) having the h-type forgo investment in exchange for setting a high capital requirement for the ℓ-type will be socially optimal. In this case we assume, without loss, that the regulator utilizes the *underinvestment* pooling mechanism $\mathcal{M}^{\star}_{und}$ that sets $T_h = T_\ell = 0$ and $K^\star = \bar{K}$.[25]

In this section we will characterize the optimal mechanism when the regulator is restricted to the class of pooling mechanisms and then proceed to characterize the optimal mechanism when the regulator is restricted to separating mechanisms that dominate the optimal pooling mechanism. The reader can feel free to skip to the main results in Section 4 which is a characterization of the optimal mechanism, stating when the optimal pooling mechanism, optimal separating mechanism, or optimal underinvestment mechanism is preferred by the regulator with respect to the underlying parameters.

3.1 Optimal Pooling Mechanisms

Here we first note that we can focus without loss on type-(ii) equilibria of pooling mechanisms. Namely, given that any type-(iii) equilibrium of a pooling mechanism is payoff equivalent to an equilibrium of the separating mechanism $\mathcal{M}$ with $K_\ell = K^\star$, $K_h = K'$, $T_h = T_\ell = 0$ implies that whenever the pooling mechanism optimally sets a capital requirement $K^\star$ and permits a type-(iii) equilibrium for some $K' > K^\star$, then it is weakly dominated by the optimal separating mechanism. Therefore, in what follows we will only consider type-(ii) equilibria of pooling mechanisms as these are the relevant equilibria (under Assumption 1.1) when the pooling mechanism is not dominated.

The next proposition characterizes the optimal pooling mechanism.

Proposition 3.1. *Let $\mathcal{M}^{\star}_{pool}$ with $K_\ell = K_h = K^\star$, $T_\ell = T_h$, be the optimal pooling mechanism. Then, $T_\ell = T_h = 0$,*

$$\mathcal{S}_\ell = \mathcal{S}_h = \{s \in \mathcal{S} : s \in \operatorname*{argmin}_{\substack{s\in\mathcal{S} \\ \mathbb{E}_p[s]=K^\star}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]\}$$

and $K^\star$ is the unique value that solves

$$b_h(K^\star) = (1-p) \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_p[s]=K^\star}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

[25]Our assumption that $\hat{b} < a_h - a_\ell$ guarantees that the h-type forgoes the investment under $\mathcal{M}^{\star}_{und}$.

Proof. See appendix Section 8.4.2. □

Proposition 3.1 states that the optimal pooling mechansim sets transfers to zero and restricts banks to issue securities that minimize the information sensitivity. The latter point is optimal as the security that minimizes the information sensitivity allows the regulator to set the highest possible capital requirement. This is due to the fact that under a pooling mechanism capital requirements are set to induce investment by the h-type and the cost the h-type pays when to investing is proportional to the information sensitivity of the security issued. Namely, when the capital requirement is K the bank invests and issues security s such that $\mathbb{E}_p[s] \geq K$ if and only if

$$b_h(K) \geq \mathbb{E}_h[s] - \mathbb{E}_p[s] = (1-p)(\mathbb{E}_h[s] - \mathbb{E}_\ell[s])$$

Hence, the regulator would like to minimize the information sensitivity of the security utilized as it allows him to weakly increase capital requirements. Then, the capital requirement $K^\star$ of the optimal pooling mechanism is set as high as possible to make the h-type bank indifferent between investing or not. We then show that this equation always yields an interior solution given that $b_h(K)$ is decreasing in K (banks have a preference for leverage) and the information sensitivity is increasing in the capital requirement:

$$\min_{s\in\mathcal{S}:\mathbb{E}_p[s]=K} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] > \min_{s\in\mathcal{S}:\mathbb{E}_p[s]=K'} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']$$

whenever $K > K'$. Finally, whenever the bank is indifferent between investing and not investing under the capital requirement $K^\star$ then it is easy to see that there is a unique type-(ii) pooling equilibrium that induces investment whereby capital requirements bind so that $\mathbb{E}_p[s^\star] = K^\star$.

Next we characterize when it is optimal for the regulator to want to induce the h-type to invest through the optimal pooling mechanism $\mathcal{M}^\star_{pool}$ rather than setting the maximal capital requirement $\bar{K}$ and only having the ℓ-types invest through the optimal underinvestment mechansim $\mathcal{M}^\star_{und}$.

Lemma 3.1. *Let $K^\star$ be the capital requirement of the optimal pooling mechanism $\mathcal{M}^\star_{pool}$. $\mathcal{M}^\star_{pool}$*

dominates the optimal underinvestment mechanism $\mathcal{M}^\star_{und}$ if and only if

$$\hat{b} \geq \frac{\lambda}{p}(pL_h(K^\star) - (1-p)(L_\ell(K^\star) - L_\ell(\bar{K})).$$

Proof. See appendix Section 8.4.1. □

3.2 Optimal Separating Mechanisms

In this section we will proceed to characterize the optimal separating mechanisms. We will first characterize when inducing investment by the h-type in a separating mechanism is preferred to the optimal underinvestment mechanism $\mathcal{M}^\star_{und}$.

Lemma 3.2. *Let $\mathcal{M}^\star_{sep} = \{(K^\star_h, T^\star_h, \mathcal{S}^\star_h), (K^\star_\ell, T^\star_\ell, \mathcal{S}^\star_\ell)\}$ be the optimal separating mechanism. $\mathcal{M}^\star_{sep}$ dominates the optimal underinvestment mechanism $\mathcal{M}^\star_{und}$ if and only if*

$$\hat{b} \geq \frac{\lambda}{p}(p \cdot L_h(K^\star_h) + (1-p) \cdot (L_\ell(K^\star_\ell) - L_\ell(\bar{K}))).$$

Proof. See appendix Section 8.4.3. □

Before proceeding to characterize the optimal separating mechanism, we will first note that the incentive compatibility conditions can be written as

$$(IC_\ell) \qquad T_h - T_\ell \geq b_\ell(K_h) - b_\ell(K_\ell) + \mathbb{E}_h[s_h] - \mathbb{E}_\ell[s_h]$$

and

$$(IC_h) \qquad T_h - T_\ell \leq b_h(K_h) - b_h(K_\ell) + \mathbb{E}_h[s_\ell] - \mathbb{E}_\ell[s_\ell]$$

where s_h and s_ℓ are such that $\mathbb{E}_h[s_h] = K_h + T_h$ and $\mathbb{E}_\ell[s_\ell] = K_\ell + T_\ell$ (conditions satisfied in equilibrium). Further, under any incentive compatible separating mechanism, both bank types are indifferent between which security they issue when investing is optimal (i.e. the transfer is not too large). This is due to the fact that under any incentive compatible separating mechanism the bank's choice of capital requirement credibly reveals to the market its true type. Therefore, once the bank's type is revealed, what ever security it issues is correctly priced and thus pays in expectation exactly the funding that it generates.

Proposition 3.2. *Let $\mathcal{M}^\star$ be an optimal separating mechanism with*

$$\mathcal{S}_h^\star = \{s \in \mathcal{S} : s \in \underset{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s'] = K_h + T_h}}{\operatorname{argmin}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']\}$$

and

$$\mathcal{S}_\ell^\star = \{s \in \mathcal{S} : s \in \underset{\substack{s' \in \mathcal{S} \\ \mathbb{E}_\ell[s'] = K_\ell + T_\ell}}{\operatorname{argmax}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']\}$$

then this mechanism weakly dominates all other separating mechanisms and strictly dominates any mechanism that sets $S_h \neq S_h^\star$ or $S_\ell \neq S_\ell^\star$ for some underlying parameters $(a_h, a_\ell, p, \hat{b})$.

Proof. See appendix Section 8.4.4. □

Proposition 3.2 states that any optimal separating mechanism is weakly dominated by the separating mechanism that restricts the h-type to issue the least information sensitive security and the ℓ-type to issue the most information sensitive security subject to binding capital requirements (dictated by the equilibrium conditions). This comes from the fact that restricting securities to these sets can only relax the incentive constraints (allowing for the possibility of improving welfare).

Lemma 3.3. *Let $\mathcal{M}_{sep}^\star$ be the optimal separating mechanism. If*

$$\max_{\substack{s \in \mathcal{S} \\ \mathbb{E}_\ell[s] = \bar{K} + T_\ell}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] > \min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_h[s] = \bar{K} + T_h}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

and

$$\min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_h[s] = \bar{K} + T_h}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] \leq \hat{b}$$

then $\mathcal{M}^{\star}_{sep}$ achieves the first best: $K_\ell = K_h = \bar{K}$. There exists $\hat{p}$ such that whenever $p > \hat{p}$, if $\mathcal{M}^{\star}_{sep}$ achieves the first best, then so does $\mathcal{M}^{\star}_{pool}$.

Proof. See appendix Section 8.4.5. □

Lemma 3.3 states conditions under which the optimal separating equilibrium leads to the first best outcome. We do not expect the conditions of Lemma 3.3 to hold in practice for sensible distributions G and we can show that they do not hold numerically (e.g. under a normal distribution). Additionally, we can show that if $\bar{K}$ is arbitrarily large, then the second condition will fail under the assumption that $\hat{b} < a_h - a_\ell$. We proceed assuming that these conditions do not hold in order to characterize the second best separating mechanism.

We proceed with the following lemma which states that whenever the optimal separating mechanism $\mathcal{M}^{\star}_{sep}$ attains a higher level of welfare than the optimal pooling mechanism then the constraint IC_ℓ is always binding.

Lemma 3.4. *Let $\mathcal{M}^{\star}_{sep} = \{(K^{\star}_h, T^{\star}_h, \mathcal{S}^{\star}_h), (K^{\star}_\ell, T^{\star}_\ell, \mathcal{S}^{\star}_\ell)\}$ be the optimal separating mechanism. If $\mathcal{M}^{\star}_{sep}$ dominates $\mathcal{M}^{\star}_{pool}$ and does not achieve the first best outcome then,*

(i) IC_ℓ is always binding.

(ii) If $K^{\star}_\ell > K^{\star}_h$ then IC_h is not binding.

Proof. See appendix Section 8.4.6. □

Finally, the following proposition summarizes the optimal separating mechansim.

Proposition 3.3. *Let $\mathcal{M}^{\star}_{sep} = \{(K^{\star}_h, T^{\star}_h), (K^{\star}_\ell, T^{\star}_\ell)\}$ be the optimal separating mechanism. If $\mathcal{M}^{\star}_{sep}$ dominates the optimal pooling mechanism $\mathcal{M}^{\star}_{pool}$ then,*

(i) if $K^{\star}_\ell > K^{\star}_h$ then $T^{\star}_h = b_h(K^{\star}_h)$ and $T^{\star}_\ell = 0$.

(ii) if $K^{\star}_h \geq K^{\star}_\ell$ then $T^{\star}_h$ and $T^{\star}_\ell$ are chosen to solve the program

$$\min_{K_\ell, K_h, T_\ell, T_h} p \cdot L_h(K_h) + (1-p) \cdot L_\ell(K_\ell)$$

$$b_\ell(K_h) - b_\ell(K_\ell) + \min_{\substack{s\in\mathcal{S}\\ \mathbb{E}_h[s]=K_h+T_h}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] = T_h - T_\ell$$

$$b_h(K_h) - b_h(K_\ell) + \max_{\substack{s\in\mathcal{S}\\ \mathbb{E}_\ell[s]=K_\ell+T_\ell}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] \geq T_h - T_\ell$$

$$T_h \in [0, b_h(K_h)] \quad \textit{and} \quad T_\ell \in [0, b_\ell(K_\ell)]$$

Proof. See appendix Section 8.4.7. □

We note that whenever the optimal separating mechanism sets $K_\ell^\star > K_h^\star$, then we can determine the optimal transfers. Otherwise, it the optimal transfers in general will depend on the distribution of returns. Hence, we obtain a partial characterization in this latter case. The following lemma will prove useful when characterizing when separating is preferred to pooling and vice-versa

Lemma 3.5. *Let $\mathcal{M}_{sep}^\star$ be the optimal separating mechanism. There exists $\bar{p} \in [0,1]$ such that whenever $p < \bar{p}$ then $K_\ell^\star > K_h^\star$ and when $p > \bar{p}$ then $K_h^\star > K_\ell^\star$. Further, $\bar{p}$ is strictly increasing in $a_h - a_\ell$.*

Proof. See appendix Section 8.4.8. □

Example 3.1. *Suppose that $B \sim \mathcal{N}(\hat{b}, \sigma^2)$, $I = 10$, $a_\ell = 0$, and $a_h = 2 \cdot I$ and let $\bar{p}_{\sigma^2}$ be the threshold of Lemma 3.5. Then,*

$$\bar{p}_1 > .9999999999$$

$$\bar{p}_{10} > .9999998377$$

$$\bar{p}_{50} > .9711209822$$

$$\bar{p}_{100} > .8745895451$$

This example will prove to be relevant in the context of our main results below. Namely, it shows that practically we expect $\bar{p}$ to be arbitrarily close to 1 when returns are normally distributed with reasonable variance.

4 Comparison of Optimal Mechanisms

We will now proceed to characterize under what conditions each of the mechanisms $\mathcal{M}^{\star}_{sep}$, $\mathcal{M}^{\star}_{pool}$, and $\mathcal{M}^{\star}_{und}$ are optimal.

Proposition 4.1. *Let $\mathcal{M}^{\star}$ be the optimal regulatory mechanism. There exists $p_{pool}, p_{sep}, p_{und} \in [0, 1)$ such that $p_{pool} \geq p_{sep} \geq p_{und}$ and*

(i) Whenever $p \geq p_{pool}$ then $\mathcal{M}^{\star} = \mathcal{M}^{\star}_{pool}$.

(ii) Whenever $p \in (p_{und}, p_{sep})$ then $\mathcal{M}^{\star} = \mathcal{M}^{\star}_{sep}$.

(iii) Whenever $p \leq p_{und}$ then $\mathcal{M}^{\star} = \mathcal{M}^{\star}_{und}$.

If $p_{sep} \neq p_{pool}$ then either $\mathcal{M}^{\star} = \mathcal{M}^{\star}_{pool}$ or $\mathcal{M}^{\star} = \mathcal{M}^{\star}_{sep}$ when $p \in (p_{sep}, p_{pool})$ depending on the underlying parameters.

Proof. See appendix Section 8.5.1. □

Proposition 4.1 states that the optimal mechanism is pooling for large values of p, separating for intermediate values, and underinvestment for small values of p. This idea is conveyed in Figure 1 in the introduction. Namely, as the proportion of good banks goes to 1 then the market price of the equilibrium security converges to the good type bank's true valuation of the security. In that case the cost of raising capital goes to zero and therefore the regulator can set higher and higher capital requirements while still inducing investment. On the other hand, as the proportion of good banks goes to zero then the cost of underinvestment goes to zero as good banks are the only type who forgo investment when the capital requirement is too high. In that case, the benefit of setting higher capital requirements for the low type banks eventually becomes larger than the cost of underinvestment as p goes to zero. Finally, we note that there are strictly positive values of K_ℓ and K_h that satisfy the incentive compatibility constraints. In this case, separation will be optimal over pooling whenever p is small and separation will be optimal over underinvestment whenever p is large.

One issue here is that it may be the case that $p_{sep} \neq p_{pool}$ in which case there may be values of $p_1, p_2 \in (p_{sep}, p_{pool})$ such that $p_1 < p_2$ and pooling is optimal when $p = p_1$ yet separation is optimal when $p = p_2$. This is due to the fact that while $K^\star$ is increasing as p increases, so does $K_h^\star$ (because the regulator puts a higher weight on the h-type) so it is not clear whether $K_h^\star$ increases faster or slower than $K^\star$ for intermediate values of p. The next proposition allows us to state when we have a full characterization.

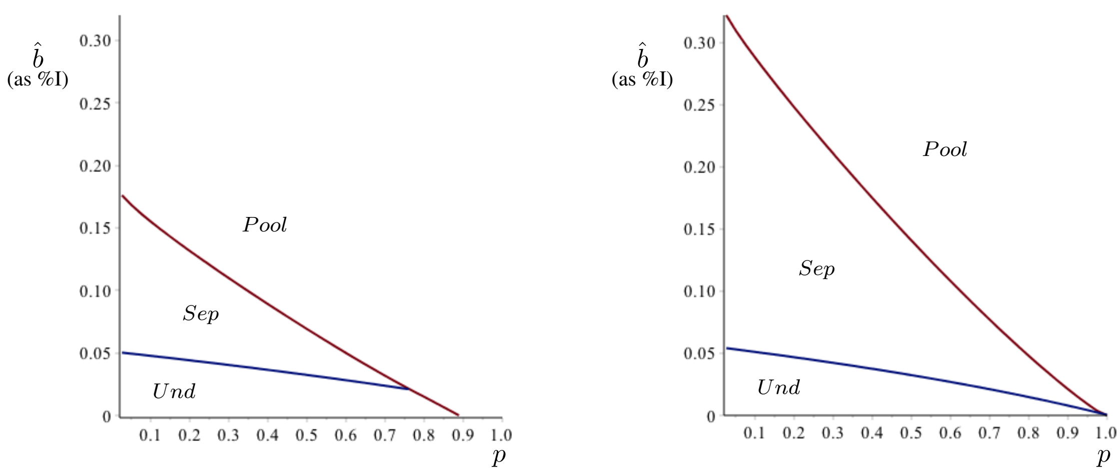
Corollary 4.1. *Let p_{pool}, p_{sep}, and p_{und} be the values of Proposition 4.1. Then,*

(i) If $p_{pool} < \bar{p}$ then $p_{pool} = p_{sep}$.

(ii) There exists $\bar{a}$ such that whenever $a_h > \bar{a}$, then $p_{pool} < \bar{p}$ and therefore $p_{pool} = p_{sep}$.

Proof. See appendix Section 8.5.2. □

Corollary 4.1 gives us a full characterization of the optimal mechanism for all $p \in [0, 1]$. Namely, it states that (i) whenever $p_{pool} < \bar{p}$ of Lemma 3.5 then we have a full characterization and that (ii) there always exists $\bar{a}$ such that $p_{pool} > \bar{p}$ whenever $a_h > \bar{a}$. It is worth noting that although we do not have a full characterization whenever $p_{pool} < \bar{p}$, it is straightforward to extend our results to a full characterization as soon as the distribution of returns G is specified.



(a) $a_h - a_\ell = 25\%$ of I (b) $a_h - a_\ell = 50\%$ of I

Figure 3: Optimal mechanism given NPV $\hat{b}$ and proportion of good banks p when $B \sim \mathcal{N}(\hat{b}, 4)$, $\lambda = \frac{1}{3}$, $I = 10$.

Our main result is represented for a fixed NPV of the investment $\hat{b}$ in Figure 1 in the introduction. Figure 3 shows the optimal mechanism by region as both a function of the proportion of banks with good news p and the NPV of the investment. As we can see, regardless of the proportion p, when the NPV of the new project is large then the pooling mechanism is optimal. In fact, as mentioned earlier, whenever $\hat{b} \geq a_h - a_\ell$ then the optimal pooling mechanism achieves the first best outcome whereby all bank types invest and the regulator can set the maximal capital requirement $K^\star = \bar{K}$. Next, we note that whenever the value of the investment is low, then the optimal mechanism is the underinvestment mechanism. The difference between the two plots of Figure 3 is with respect to the difference in the value of the bank's existing assets with respect to their private information. We can think of this difference as a measure of the opacity of the bank's assets (e.g. the maximal bid-ask spread). In this case, we can see that as $a_h - a_\ell$ increases the value of separation increases represented by an outward shift in the line where separating and pooling generate the same welfare.

Finally, we note that Proposition 4.1 does not necessarily imply that $p_{sep} > p_{und}$. Namely, we do not rule out the case where $p_{sep} = p_{und}$, in which case separation is never optimal. The next proposition states that whenever $\hat{b}$ is large enough, then it must be the case that $p_{sep} > p_{und}$.

Proposition 4.2. *Let p_{pool}, p_{sep}, and p_{und} be the values of Proposition 4.1. Then,*

(i) If $b_\ell(\bar{K}) \geq b_\ell(0) - b_h(0)$ then $p_{und} = 0$.

(ii) There exists $\bar{b}$ such that whenever $\hat{b} > \bar{b}$ then $b_\ell(\bar{K}) < b_\ell(0) - b_h(0)$ and therefore $p_{und} = 0$.

Proof. See appendix Section 8.5.3. □

5 Policy Implications

In this section we will present the main policy implications of our results.

1. Internal Ratings Based v.s. Standardized Approach Regulations. From a cross sectional perspective, our results would suggest that the regulator should impose the IRB approach regulations on large and opaque banks, in line with its current use, while the SA approach should be utilized for more transparent banks. The key insight here is that transparency is an important parameter to determine the optimal regulation and therefore regulators should work to develop accurate measures of bank transparency to utilize for regulatory purposes. This observation can potentially lend insight into why the spill over effects of the financial crisis were so large, given that banks' balance sheets had become increasingly opaque prior to the housing market crash through the widespread use of off balance sheet activities and the origination and trading of opaque assets such as mortgage backed securities.

Another point to note is that under current regulations the largest banks have discretion over which approach (IRB or SA) they use to determine their capital requirements. We note that, in our model, if the bank were to have the ability to choose the separating (IRB-type) or pooling (SA-type) mechanism before learning their type, then it is easy to show that whenever the IRB-type mechanism is socially optimal, the bank would prefer to utilize the SA-type mechanism. Similarly, whenever the SA-type mechanism is socially optimal the bank would prefer to utilize the IRB-type mechanism in most cases (whenever $p < \bar{p}$). Therefore, our results suggest that the regulator should remove the discretion of the banks to choose which approach they utilize in determining their capital requirements. Basel III has introduced a revised capital requirement *output floor* that limits the benefit banks can receive from utilizing the IRB approach which limits their capital requirement to be at least 72.5% of the SA requirement. This backstop can help to limit inefficiencies due to banks choosing the suboptimal framework but in our model would still lead to a suboptimal outcome.

2. Counter Cyclical Capital Buffer (CCyB). Basel III has introduced a counter cyclical capital buffer requiring an additional capital surcharge of 0-2.5% of core tier 1 capital to risk weighted assets. The purpose of this buffer is to allow local regulators to increase capital requirements during booms in order to prevent the excessive build up of aggregate credit and to be able to relax capital requirements during recessions in order to reduce credit rationing. This is an idea that is at the heart of this paper. Most importantly, we provide a foundation for how capital requirements lead to credit growth and rationing. The implications of our results to the CCyB are that the regulator should only expect changes in the credit supply to come from opaque banks with good news. Therefore, the regulator can increase capital requirements on banks with transparent balance sheets, or banks that have recently been stress tested by the regulator (provided that the results of the stress test are public). Similarly, whenever the regulator utilizes the IRB-type mechanism that we propose in this paper then there will be no credit rationing so that capital requirements can be set as high as possible subject to meeting incentive compatibility of truthful reporting. Finally, we note that while local jurisdictions are encouraged to utilize the credit-to-GDP ratio in determining their CCyB, a key implication of our model is that the profitability of new investments should also influence capital requirement buffers as the more profitable the investment is, the higher capital requirements the regulator can optimally set.

3. Government Interventions During Crisis Periods. During the financial crisis, government interventions were crucial to restore the faith in the banking system. While these interventions, such as TARP, served as a way to recapitalize banks, they also served as a way to signal information about the bank's quality to the market given that banks were only accepted to the programs after being heavily screened by the regulator. Our IRB-type mechanism is in effect a private solution to this problem. Namely, once the regulator designs capital requirements and transfers correctly, the banks will be screened into different classes (without imposing monitoring costs on the regulator), providing an informative signal to the market regarding their quality. We further note, as explained in Section 2.1, that government intervention may be necessary due to mis-coordination of the bank and market on inefficient signaling equilibria. Namely, we show that there can exist inefficient signaling equilibria of the capital raising game whereby at certain times the market forms an extraneous belief that only the banks with bad news will invest. In such a situation, this can cause the banks with good news to forgo the investment given the markets under pricing of their securities, thereby enforcing the market's belief. We show in Lemma 2.1 that the regulator can resolve this issue through a government recapitalization program such as TARP by agreeing to purchase the bank's security at the efficient equilibrium price and that this is strictly welfare improving with respect to the inefficient equilibrium outcome.

4. Stress Testing. We have yet to discuss stress testing of banks, a highly utilized regulatory practice since the crisis. Stress testing would complement the mechanisms in our model provided that the results of the stress test are made public and reveal credible information about the bank's asset quality. This lends to the debate regarding whether the results of regulatory supervision should be disclosed to the market, highlighting how doing so will help to resolve the adverse selection problem that raising capital presents.[26] In this sense, it would be most appropriate to utilize stress tests when the level of bank opacity is large. Namely, while the regulator can utilize our IRB-type mechanism to reveal the bank's private information, such an approach requires paying information rents in the form of lower capital requirements in order to credibly induce this information revelation. Further, these information rents are strictly increasing in the opacity of the bank's assets. Hence, when the level of bank opacity is large, the benefit of information revelation through stress testing will outweigh the cost of performing the test. This comes from the fact that once the regulator reveals the information gathered during the bank's stress test, then that bank's capital security will be more accurately priced allowing the regulator to set a higher capital requirements (through either mechanism) without inducing underinvestment. These insights complement the current literature on stress testing and information disclosure (e.g. Leitner and Williams (2017) and Goldstein and Leitner (2018)).

5. Capital Security Design. Finally, we would like to mention our results on security design. In current regulations equity is considered the *highest quality* capital instrument. This is due to the fact that equity allows the bank to absorb maximal losses before becoming insolvent in comparison to other securities such as subordinated debt that only absorb losses after the bank fails. Yet, this begs the question of whether the regulator should be concerned with absorbing losses pre-insolvency or post-insolvency. While for large and systemic banks it is clear that pre-insolvency loss absorption provides a much larger benefit to society, this may not be the case for smaller,

[26] Note that this relates potentially more to regulatory supervision of bank solvency rather than stress testing.

less-systemic banks. What we show in this paper is that in the latter case the regulator may want to consider the use of less *informationally sensitive* securities for capital regulation (e.g. subordinated debt). Similarly, given the current interest in hybrid debt securities such as contingent convertible bonds (see e.g. Squam Lake (2010)), our paper states that, barring any potential pricing or other issues that these new securities may impose, the use of these instruments can allow the regulator to set higher capital requirements without inducing underinvestment and yet still maintaining the same level of pre-insolvency loss absorption. Finally, we show how under the IRB-type mechanism the optimal capital security that the regulator restricts the banks with bad news to issue is the one that maximizes the informational sensitivity. Namely, the regulator should force the banks with bad news sell their existing assets in order to finance the new investment, something that we saw done in practice through the use of the TARP program during the crisis.

6 Conclusion

In this paper we have analyzed how capital requirements should optimally be set when banks must issue new securities to meet regulatory capital requirements. We show that when banks have private information about their assets in place, then under a pooled capital requirement there may be underinvestment. We then proceed to characterize the problem of designing the optimal mechanism in this environment and show that three regulatory frameworks may be optimal over the underlying parameter space.

The first type of mechanism bypasses the investment incentives of the firms by inducing them to truthfully reveal their private information to the market. Namely, we show that under such a mechanism the bank's securities are correctly priced by the market and therefore all banks optimally invest regardless of the capital requirement. That being said, the regulator is restricted to set capital requirements to ensure that it is incentive compatible for the banks to truthfully reveal their private information, thereby paying information rents to induce truthful revelation. The second type my mechanism instead pools the information of the banks by setting a single capital requirement. In this case the capital requirement is set as high as possible subject to inducing investment by the banks with good news. Finally, we show that it also may be optimal for the regulator to set capital requirements very high, purposefully inducing the banks with good news to forgo the new investment. We characterize under what conditions each of these three mechanisms is optimal given the underlying parameters of the model and the resulting policy implications. Given the lack of micro-foundations for the cost of capital in the existing literature, we hope that this model and its insights will prove to be useful for studying more complex issues of banking regulation in future research.

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7 Extensions

7.1 Continuum of Types

In this section we will show that our main results extend to the case where the bank's private information is the updated value of its assets in place a which falls in some interval $[\underline{a}, \bar{a}]$. In this case, we assume the market and the regulator have a prior belief $p \in \Delta([\underline{a}, \bar{a}])$ over $[\underline{a}, \bar{a}]$. In this case, we will parameterize the asymmetric information problem by $\mathbb{E}_p[a] := \hat{a} \in [\underline{a}, \bar{a}]$, the market expectation of the bank's assets in place with respect to the prior p. In this sense, as $\hat{a}$ increases this is equivalent to saying that p puts a higher probability on good news types.

7.1.1 Pooling

In a pooling mechanism, the regulator sets a single capital requirement K and the bank's type specific decision is given by $d_a(K) \in \{0, 1\}$ where $d_a(K) = 1$ implies that the bank issues a security $s \in \mathcal{S}$ and makes the investment when its type is a while $d_a(K) = 0$ implies the bank forgoes the investment. Note that without loss we can focus on pooling equilibria of the pooling mechanism as if there exist some semi-separating equilibria that dominate the pooling mechanism then the regulator can implement these equilibria using a separating mechanism and therefore the pooling mechanism is dominated. Furthermore, the regulator can rule out any other semi separating equilibria from being coordinated on by restricting $\mathcal{S}_a = \mathcal{S}_{a'} = \{s\}$ to be a single security s for all $a, a' \in [\underline{a}, \bar{a}]$ thereby removing the possibility of the bank's security signaling its type.

Now, given the nature of the problem we know that for any K, if all bank types invest, then

$$b_{\bar{a}}(K) \geq \min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_{pool}[s]=K}} \mathbb{E}_{\bar{a}}[s] - \mathbb{E}_{pool}[s]$$

where

$$\mathbb{E}_{pool}[s] := \int_{\underline{a}}^{\bar{a}} \mathbb{E}_a[s] p(a) da.$$

Otherwise, for each K, there exists a unique threshold $\tau(K)$ such that for all $a > \tau(K)$ the bank forgoes the project ($d_a(K) = 0$) and for all $a < \tau(K)$ the bank undergoes the investment $d_a(K) = 1$. Of course, in this case $\hat{a}$ is determined by this threshold. Therefore we denote by

$$\hat{a}(K) = \int_{\underline{a}}^{\tau(K)} ap(a)da \tag{1}$$

the market expectation when all banks $a > \tau(K)$ forgo the investment. In that case, for any K, $\tau(K)$ is determined by 1 and

$$b_{\tau(K)}(K) = \min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_{p_\tau}[s]=K}} \mathbb{E}_{\bar{a}}[s] - \mathbb{E}_{p_\tau}[s] \tag{2}$$

where

$$\mathbb{E}_{p_\tau}[s] := \int_{\underline{a}}^{\tau(K)} \mathbb{E}_a[s] p_\tau(a) da.$$

and $p_\tau(a) := p(a|a \geq \tau(K))$.

Therefore, denoting by $L_a(K)$ the liability of the a-type bank, the regulator chooses the optimal pooling mechanism to solve the program

$$\max_K \int_{\underline{a}}^{\tau(K)} (\hat{b} - \lambda \cdot L_a(K)) p(a) da$$

where $\tau(K)$ solves 1 and 2. It should be straightforward to see that as $\hat{a} \to \bar{a}$ then $\tau(K) \to \bar{a}$ and $K \to +\infty$.

7.1.2 Separating

Now, we have characterized the optimal pooling mechanism we can proceed to characterize the optimal separating mechanism. Note that we will assume here that full separation is optimal which may not always be the case. It should be straightforward to extend our characterization to the case of semi-separation. We further assume that p is single peaked so that p is weakly increasing for all $a \in [\underline{a}, \hat{a})$ and weakly decreasing for all $a \in (\hat{a}, \bar{a}]$.

Now, denoting by K_a the capital requirement of type a and T_a the transfer to be paid by type a, incentive compatibility requires that for any $a, a' \in [\underline{a}, \bar{a}]$ we have

$$T_{a'} - T_a \geq b_a(K_{a'}) - b_a(K_a) + \mathbb{E}_{a'}[s_{a'}] - \mathbb{E}_a[s_{a'}]$$

and

$$T_{a'} - T_a \leq b_{a'}(K_{a'}) - b_{a'}(K_a) + \mathbb{E}_{a'}[s_a] - \mathbb{E}_a[s_a]$$

While we can say more about the optimal design of the separating mechanism it is not necessary to present the extension of our results to a continuum of types.

7.1.3 Extension of Proposition 4.1 to the Continuum Case

Now, before presenting the analog to Proposition 4.1 we first note that the no investment mechanisms are more complicated in the continuum setting as the regulator can set capital requirements to induce investment from all types $a < \tilde{a}$ for any threshold $\tilde{a}$. Denote by $\mathcal{M}_{no(\tilde{a})}$ the optimal pooling mechanism with capital requirement $K^\star$ such that $\tau(K^\star) = \tilde{K}$. In this case, the optimal *full investment* pooling mechanism is simply $\mathcal{M}^\star_{pool} = \mathcal{M}^\star_{no(\bar{a})}$. In this case we can see that there is some added insight as $\hat{a} \to 0$ then it must be the case that $\mathcal{M}^\star_{pool}$ is dominated by all mechanisms $\mathcal{M}^\star_{no(\tilde{a})}$ with $\tilde{a} < \bar{a}$.

Proposition 7.1. *Let $\mathcal{M}^\star$ be the optimal regulatory mechanism. There exists a_{pool}, a_{sep}, a_{no} $\in$*

$[\underline{a}, \bar{a})$ such that $a_{pool} \geq a_{sep}$ and

(i) Whenever $\hat{a} > a_{pool}$ then $\mathcal{M}^\star = \mathcal{M}^\star_{pool}$.

(ii) Whenever $\hat{a} \in (a_{no}, a_{sep})$ then $\mathcal{M}^\star = \mathcal{M}^\star_{sep}$.

(iii) Whenever $\hat{a} < a_{no}$ then $\mathcal{M}^\star = \mathcal{M}^\star_{no(\underline{a})}$.

Proof. First, we note that as $\hat{a} \to \bar{a}$ then it must be the case that $\mathcal{M}^\star_{pool}$ dominates both $\mathcal{M}^\star_{sep}$ and $\mathcal{M}^\star_{no(\tilde{a})}$ for all $\tilde{a} < \bar{a}$. The latter case is trivial given that as $\hat{a} \to \bar{a}$ the regulator puts approximately probability 1 on the bank's type being $\bar{a}$. In that case it cannot be that $\mathcal{M}^\star_{no(\tilde{a})}$ dominates $\mathcal{M}^\star_{pool}$ for some $\tilde{a} < \bar{a}$ as the capital requirement of $\mathcal{M}^\star_{pool}$ is such that $K^\star \to +\infty$ as $\hat{a} \to \bar{a}$.

To prove that there exists a_{pool} such that whenever $\hat{a} > a_{pool}$ then $\mathcal{M}^\star_{pool}$ dominates $\mathcal{M}^\star_{sep}$ we simply note that this is the case whenever there are only two types $\underline{a}$ and $\bar{a}$ as shown in the proof of Proposition 4.1. Given that satisfying the incentive compatibility conditions for types $a \in (\underline{a}, \bar{a})$ must weakly decrease the optimal capital requirements $K_{\underline{a}}$ and $K_{\bar{a}}$ then it must be the case that if $\mathcal{M}^\star_{pool}$ dominates $\mathcal{M}^\star_{sep}$ assuming only two types the it must further dominate $\mathcal{M}^\star_{sep}$ when there are more types as $\mathcal{M}^\star_{pool}$ sets the same capital requirement when there are two types $\underline{a}$ and $\bar{a}$ as well as when there is a continuum of types $[\underline{a}, \bar{a}]$.

Now to prove that there exists a_{no} such that $\mathcal{M}^\star_{no(\underline{a})}$ dominates both $\mathcal{M}^\star_{sep}$ and $\mathcal{M}^\star_{pool}$ whenever $\hat{a} < a_{no}$ we note that as $\hat{a} \to \underline{a}$ then K is strictly decreasing (assuming that $K < \bar{K}$ for all $\hat{a}$ for some arbitrarily large $\bar{K}$) as

$$\min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_{pool}[s] = K}} \quad \mathbb{E}_{\bar{a}}[s] - \mathbb{E}_{pool}[s]$$

is strictly decreasing in $\hat{a}$. Hence, at some point it must be the case that in the limit $K < \bar{K}$ and therefore $\mathcal{M}^\star_{no(\underline{a})}$, which sets $K^\star_{no} = \bar{K}$, dominates $\mathcal{M}^\star_{no(\underline{a})}$.

Finally given the statement of this proposition, we note that if for all $\hat{a} < a_{pool}$ it is the case that $\mathcal{M}^\star_{sep}$ dominates $\mathcal{M}^\star_{no(\underline{a})}$ then $p_{no} = 0$. Similarly, if $\mathcal{M}^\star_{no(\underline{a})}$ dominates $\mathcal{M}^\star_{sep}$ for all $a < a_{pool}$, then $a_{sep} = a_{no}$. Otherwise, there exists $a_{sep} > a_{no}$ such that $\mathcal{M}^\star_{sep}$ is optimal whenever $\hat{a} \in (a_{sep}, a_{no})$ and $\mathcal{M}^\star_{no(\underline{a})}$ is optimal whenever $\hat{a} < a_{no}$. $\square$

8 Appendix

8.2 Proofs of Section 2

8.2.1 Proof of Lemma 2.1

Proof. First note that it is without loss to assume that inducing investment by the h-type is socially optimal if $\mathcal{M}$ is socially optimal and separating. Namely, if a separating mechanism induces the h-type to forgo the investment then the equivalent outcome can be implemented by a pooling mechanism that sets capital requirements, security restrictions, and transfers equal to the ℓ-type's in the separating mechanism. Therefore, assume without loss that if an optimal separating mechanism induces investment by the h-type then doing so is socially desirable. In this case, if the market coordinates on an equilibrium whereby the h-type does not invest because the market believes the h-type will never invest then the regulator can purchase the h-types security s_h at a price $\mathbb{E}_h[s_h]$. The expected gain from purchasing this security is 0 as given that the mechanism is incentive compatible only the h-type will ask the regulator to purchase its security (while meeting the terms of the h-type menu option). In that case the regulator breaks even on the h-type's security but induces the h-type to invest over the alternative equilibrium yielding a strict expected welfare improvement of $p \cdot \hat{b}$.

If instead the optimal mechanism is pooling, then if there exists an equilibrium whereby the h-type invest it must be the case that investment is optimal for society. Namely, under our assumption $\hat{b} < a_h - a_\ell$ implies that if the market prices the bank's security at the pooling average so that $\mu(s) = p$, then the h-type will forgo the new investment if the pooling capital requirement K is too large. Therefore, if inducing investment by the h-type is not socially optimal then the regulator should increase the capital requirement, contradicting the fact that an equilibrium exists whereby the h-type invests.

Now suppose $\mathcal{M}$ is a socially optimal pooling mechanism and that there exists an equilibrium of $\mathcal{M}$ that induces the h-type to invest whereby both banks issue the same security s. In that case, by purchasing the bank's security at the price $\mathbb{E}_p[s]$ the regulator breaks even on the security. This is due to the fact that when the bank is the ℓ-type the regulator loses $\mathbb{E}_p[s]-\mathbb{E}_\ell[s]$ but when the bank is the h-type the regulator gains $\mathbb{E}_h[s]-\mathbb{E}_p[s]$. Therefore, given that $\mathbb{E}_p[s]=p\mathbb{E}_h[s]+(1-p)\mathbb{E}_\ell[s]$ the regulator breaks even in expectation on the purchase of s. Hence, the regulator obtains a strict expected welfare gain over the equilibrium whereby the h-type doesn't invest equal to $p\hat{b}$.

Finally, if $\mathcal{M}$ is a socially optimal pooling mechanism with an equilibrium whereby the two bank types issue different securities, then this equilibrium could be implemented through the use of a separating mechanism, in which case we know that agreeing to purchase the bank's security at the appropriate price is strictly welfare improving over any equilibrium whereby the h-type forgoes investment. □

8.2.2 Proof of Lemma 2.2

Proof. To prove this we simply use the definitions to obtain

$$\begin{aligned}
V_\theta(s,\tilde{\theta};P) = &\ \mathbb{E}_\theta[max\{a_\theta+B+P-T_{\tilde{\theta}}-s,0\}] \\
= &\ \textstyle\int_{-a_\theta-P+T_{\tilde{\theta}}}^{\infty}(x+a_\theta+P-T_{\tilde{\theta}}-s)dG(x) \\
= &\ \textstyle\int_{-a_\theta-P+T_{\tilde{\theta}}}^{\infty}(x-s)dG(x)+(1-G(-a_\theta-P+T_{\tilde{\theta}}))(a_\theta+P-T_{\tilde{\theta}}) \\
= &\ \textstyle\int_{-a_\theta-P+T_{\tilde{\theta}}}^{\infty}xdG(x)-\mathbb{E}_\theta[s]+(1-G(-a_\theta-P+T_{\tilde{\theta}}))(a_\theta+P-T_{\tilde{\theta}}) \\
= &\ a_\theta+b_\theta(P-T_{\tilde{\theta}})+P-\mathbb{E}_\theta[s]-T_{\tilde{\theta}}
\end{aligned}$$

Finally, we substitute $K_1=P-T_{\tilde{\theta}}$ to obtain the result. Note that the fourth equality is valid due to the fact that the bank's limited liability implies that $z-s(z)\geq 0$ for all $z\in\mathbb{R}$ so that $x+a_\theta+P-T_{\tilde{\theta}}-s\geq 0$ if and only if $x+a_\theta+P-T_{\tilde{\theta}}\geq 0$ which is the case whenever $x\geq -a_\theta-P+T_{\tilde{\theta}}$.

To show that $b_\theta(K_1)=\hat{b}+L_\theta(K_1)$ we note that

$$\hat{b}+L_\theta(K_1)=\int_{-\infty}^{\infty}xdG(x)-\int_{-\infty}^{-a_\theta-K_1}(x+a_\theta+K_1)dG(x)=b_\theta(K_1)$$

□

8.3 Proofs of Section 3

8.3.1 Proof of Lemma 2.3

Proof. The first expression for $\mathbb{E}_\theta[s]$ comes from the fact that if s is admissible, then $s(z) = 0$ whenever $z \leq 0$ and therefore given $z = x + a_\theta + P - T_\theta$ then $s(x + a_\theta + P - T_\theta) = 0$ whenever $x < -a_\theta - P + T_\theta$.

To prove that $\mathbb{E}_h[s] > \mathbb{E}_\ell[s]$ when $T_h = T_\ell = T$ for all admissible securities we note that in this case denoting $K_1 = P - T$ then

$$\mathbb{E}_h[s] = \int_{-a_h-K_1}^{\infty} s(x+a_h+K_1)dG(x) = \int_{-a_\ell-K_1}^{\infty} s(x+a_h+K_1)dG(x)+\int_{-a_h-K_1}^{a_\ell-K_1} s(x+a_h+K_1)dG(x)$$

Now, given that $s(z)$ is monotone in z we know that $s(x + a_h + K_1) \geq s(x + a_\ell + K_1)$ for all $x \geq -a_\ell - K_1$. Now, if there exists some measurable set $C \subset [-a_\ell - K_1, +\infty)$ such that $s(x + a_h + K_1) > s(x + a_\ell + K_1)$ for all $x \in C$ then

$$\int_{-a_\ell-K_1}^{\infty} s(x + a_h + K_1)dG(x) > \int_{-a_\ell-K_1}^{\infty} s(x + a_\ell + K_1)dG(x) = \mathbb{E}_\ell[s]$$

and therefore $\mathbb{E}_h[s] > \mathbb{E}_\ell[s]$.

Otherwise, $s(x + a_h + K_1) = s(x + a_\ell + K_1)$ for all measurable sets $C \subset [-a_\ell - K_1, +\infty)$. In this case, suppose that $s(z) = s(z') = d > 0$ for all measurable sets $C \subset [z_0, +\infty)$ with $z, z' > z_0 \geq d$, then

$$\mathbb{E}_h[s] = (1 - G(z_0 - a_h - K_1)) \cdot d + \int_{-a_h-K_1}^{z_0} s(x + a_h + K_1)dG(x)$$

while

$$\mathbb{E}_\ell[s] = (1 - G(z_0 - a_\ell - K_1)) \cdot d + \int_{-a_\ell-K_1}^{z_0} s(x + a_\ell + K_1)dG(x)$$

Now given that

$$\int_{-a_h-K_1}^{z_0} s(x+a_h+K_1)dG(x) = \int_{-a_\ell-K_1}^{z_0} s(x+a_h+K_1)dG(x) + \int_{-a_h-K_1}^{-a_\ell-K_1} s(x+a_h+K_1)dG(x)$$

we can see, again by monotonicity, of s that

$$\int_{-a_\ell-K_1}^{z_0} s(x+a_h+K_1)dG(x) \geq \int_{-a_\ell-K_1}^{z_0} s(x+a_\ell+K_1)dG(x)$$

and given that $G(z_0-a_\ell-K_1) > G(z_0-a_h-K_1)$ for all z_0 implies again that $\mathbb{E}_h[s] > \mathbb{E}_\ell[s]$.

Now, based on the above proof the only difference between the type ℓ and type h banks is that $a_h > a_\ell$. In this sense, we could always introduce a third type h' such that $a_{h'} > a_h$ and reproduce the same proof to obtain that $\mathbb{E}_{h'}[s] > \mathbb{E}_h[s] > \mathbb{E}_\ell[s]$. Therefore, it must be the case that $\mathbb{E}_h[s] - \mathbb{E}_\ell[s]$ is strictly increasing in a_h. □

8.3.2 Proof of Proposition 2.1

Proof. We will prove the Proposition in the following steps.

Claim (1): There exist no equilibria of $\Gamma(\mathcal{M})$ *with* $s_\ell = 0$. Denoting by $P_{\mu(s)}$ the market price of security s under beliefs $\mu(s)$ we can see

$$\min_{\mu}\ a_\ell + b_\ell(P_{\mu(s)}) + P_{\mu(s)} - \mathbb{E}_\ell[s] - T > a_\ell$$

coming from the fact that $P_{\mu(s)} - \mathbb{E}_\ell[s] = \mathbb{E}_{\mu(s)}[s] - \mathbb{E}_\ell[s] \geq 0$ for all beliefs μ and $b_\ell(K_1) - T > 0$ for all equilibrium prices $P_{\mu(s)}$ given that $T \leq \min\{b_h(K_1), b_\ell(K_1)\}$ where $K_1 = P_{\mu(s)} - T$. Hence, the ℓ-type always finds it profitable to invest. □

Claim (1) states that all equilibria of $\Gamma(\mathcal{M})$ are such that $s_\ell \neq 0$. Therefore the condition that either $s_h = 0$, $s_h = s_\ell$, or $s_h \neq s_\ell$ is trivial. What is left to prove are the remaining conditions on points (i) and (iii) (the condition on (ii) that $\mathbb{E}_p[s] \geq K+T$ comes from the sequential rationality of the investment decision).

Claim (2): Any equilibrium of $\Gamma(\mathcal{M})$ *satisfying the intuitive criterion and* $s_h = 0$ *is such that* $\mathbb{E}_\ell[s_\ell] = K + T$. In order to prove this, we first note that in any equilibrium with $s_h = 0$, it must be the case that $\mu(s_\ell) = 0$ and therefore the payoff to the ℓ-type is $a_\ell + b_\ell(\mathbb{E}_\ell[s_\ell] - T)$. Now suppose that in some equilibrium with $s_h = 0$ it is the case that $\mathbb{E}_\ell[s_\ell] > K + T$. Then, given that $b_\ell(K_1)$ is strictly decreasing in K_1 as $\frac{\partial}{\partial K_1} b_\ell(K_1) = -G(-a_\ell - K_1) < 0$, we know that if the ℓ-type issues a security s such that $\mathbb{E}_\ell[s] = K + T$, then it achieves a strictly higher payoff, regardless of the market beliefs (beliefs can only improve the ℓ-types payoff when switching securities). Hence, any sequentially rational strategy s_ℓ must satisfy $\mathbb{E}_\ell[s_\ell] = K + T$.

Claim (3): Any equilibrium with $s_\ell \neq s_h$ *must satisfy* $\mathbb{E}_h[s_h] = K' + T > K + T = \mathbb{E}_\ell[s_\ell]$

$$s_h \in \underset{s' \in \mathcal{S}: \mathbb{E}_h[s'] = K' + T}{\operatorname{argmin}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s'] \quad \text{with } K' \text{ satisfying} \quad b_\ell(K) = b_\ell(K') + \mathbb{E}_h[s_h] - \mathbb{E}_\ell[s_h]$$

In order to prove this, we first note that $s_\ell \neq s_h$ implies $\mu(s_\ell) = 0$ and $\mu(s_h) = 1$. Therefore, as concluded from the proof of the previous claim it must be the case that $\mathbb{E}_\ell[s_\ell] = K + T$. Furthermore, if there is no profitable deviation for the ℓ-type to mimic the h type, then it must be the case that

$$a_\ell + b_\ell(K) \geq a_\ell + b_\ell(K') + \mathbb{E}_h[s_h] - \mathbb{E}_\ell[s_h]$$

which implies

$$b_\ell(K) - b_\ell(K') \geq \mathbb{E}_h[s_h] - \mathbb{E}_\ell[s_h] \tag{3}$$

and given that $\mathbb{E}_h[s_h] - \mathbb{E}_\ell[s_h] > 0$ and $b_\ell(K)$ is decreasing in K implies that it must be the case that $K' > K$.

Now, the h-type should always issue a security that minimizes $\mathbb{E}_h[s_h]$ while still satisfying (3) given that the intuitive criterion states that any security s_h that satisfies (3) must have equilibrium beliefs $\mu(s_h) = 1$. In this case the security that satisfies this condition is the security s_h that minimizes $\mathbb{E}_h[s_h] - \mathbb{E}_\ell[s_h]$ and sets (3) to equality. Hence, in any equilibrium with $s_h \neq s_\ell$, we have $\mathbb{E}_\ell[s_\ell] = K + T < K' + T = \mathbb{E}_h[s_h]$,

$$s_h \in \underset{s' \in \mathcal{S}: \mathbb{E}_h[s'] = K' + T}{\operatorname{argmin}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']$$

and K' such that

$$b_\ell(K) = b_\ell(K') + \mathbb{E}_h[s_h] - \mathbb{E}_\ell[s_h]$$

□

8.3.3 Proof of Proposition 2.2

Proof. If $\mathcal{M}$ is incentive compatible then it must be the case that the h-type is weakly better off choosing the h-option of the menu and the ℓ-type weakly better off choosing the ℓ-option of the menu. Therefore, given that we always assume without loss that the banks choose their own menu when they are indifferent and that the market correctly believes this, then it must be the case that $\mu(s_h) = 1$ and $\mu(s_\ell) = 0$ for any equilibrium satisfying the intuitive criterion.

To show that capital requirements are binding we note that under an incentive compatible mechanism the bank of type $\theta \in \{h, \ell\}$ receives a payoff of $a_\theta + b_\theta(K_1)$ where $K_1 = P - T \geq K_\theta$ is the capital generated by the sale of the security s_θ. In that case, given that the mechanism is incentive compatible when the capital requirements are binding and the bank's equilibrium payoff is strictly decreasing in the capital generated K_1 (coming from $\frac{\partial}{\partial P} b_\theta(K_1) = -G(-a_\theta - K_1) < 0$). Therefore, sequential rationality of the bank's strategy implies that it must be the case that both types generate exactly the capital required so that $\mathbb{E}_\theta[s_\theta] = K_\theta + T_\theta$ for each $\theta \in \{h, \ell\}$. □

8.4 Proofs of Section 4

8.4.1 Proof of Lemma 3.1

Proof. If investment is not socially desirable, then the regulator will optimally set $K = \bar{K}$ and only the ℓ-type bank will invest. Therefore, investment is socially desirable under the pooling equilibrium whenever the welfare of both banks investing with pooling requirement $K^\star$ is greater than the welfare of just the ℓ-type investing with capital requirement $\bar{K}$: $W(K^\star|invest) \geq W(I|forgo)$. Further, using the fact that $\hat{b} = \hat{b}(K^\star)+\hat{L}(K^\star)$ we can see that, after rearranging, $W(K^\star|invest) \geq W(\bar{K}|forgo)$ if and only if $\hat{b} \geq \frac{\lambda}{p}(\hat{L}(K^\star) - (1-p)\cdot L_\ell(\bar{K}))$. □

8.4.2 Proof of Proposition 3.1

Proof. First note that $T_h = T_\ell = 0$ is optimal under any mechanism with $K_\ell = K_h$ given that transfers cancel out in the welfare function and $T_h = T_\ell > 0$ will only lead to lower capital requirements to induce the h-type to invest. Hence, optimally $T_h = T_\ell = 0$. Further, we know that under any type-(ii) equilibrium both types issue a security s such that $\mathbb{E}_p[s] \geq K$. Further, this equilibrium exists only if the h-type bank prefers investment and selling an underpriced security as opposed to forgoing the investment. The first step is to show that the regulator should optimally restrict securities to the set

$$\mathcal{S}' := \{s \in \mathcal{S} : s \in \operatorname*{argmin}_{\mathbb{E}_p[s] \geq K} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]\}$$

In order to prove this, we first note that investment by the h-type is optimal only if

$$a_h + b_h(K') + K' - \mathbb{E}_h[s] \geq a_h$$

where $K' = \mathbb{E}_p[s] \geq K$. This can equivalently be expressed as

$$b_h(K') \geq \mathbb{E}_h[s] - K' = (1-p)(\mathbb{E}_h[s] - \mathbb{E}_\ell[s])$$

Now, given that $b_h(K)$ is decreasing in K, then this expression tells us that it is weakly optimal for the regulator to restrict securities to the set $\mathcal{S}'$ as if $s \notin \mathcal{S}'$ then it must be the case that the regulator can increase the capital requirement which weakly improves welfare (strictly if the capital requirement is binding in the pooling equilibrium).

Next we will show that

$$\min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_{pool}[s]=K}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] \tag{4}$$

is increasing in K. In order to do so, consider $K' > K$ and denote by $s, s' \in \mathcal{S}$ the two securities such that such that

$$s \in \operatorname*{argmin}_{\substack{s \in \mathcal{S} \\ \mathbb{E}_{pool}[s]=K}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] \quad \text{and} \quad s' \in \operatorname*{argmin}_{\substack{s \in \mathcal{S} \\ \mathbb{E}_{pool}[s]=K'}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s].$$

We claim that no matter the values of K' and K, as long as $K' > K$, then $\mathbb{E}_h[s'] - \mathbb{E}_\ell[s'] > \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$. To prove this, we simply note that there exists $\phi \in (0, 1)$ such that if we let $\tilde{s} = \phi s'$, then $\mathbb{E}_{pool}[\tilde{s}] = K$ and

$$\mathbb{E}_h[s'] - \mathbb{E}_\ell[s'] > \phi(\mathbb{E}_h[s'] - \mathbb{E}_\ell[s']) = \mathbb{E}_h[\tilde{s}] - \mathbb{E}_\ell[\tilde{s}] \geq \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

where the last inequality comes from the fact that s minimizes $\mathbb{E}_h[s] - \mathbb{E}_\ell[s]$ among all securities such that $\mathbb{E}_{pool}[s] = K$. Hence, we have proven our claim that (4) is strictly increasing in K.

Now, the regulator would like to increase K as large as possible just until the h-type bank is indifferent between investing or not. If there exists a pooling equilibrium under the capital requirement K such that $\mathbb{E}_p[s] > K$ then this cannot be optimal as it implies that

$$b_h(K) > (1-p) \min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_p[s]=K}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

which implies that there exist equilibria where the banks raise $K < K'$ which is strictly worse than the equilibrium whereby the banks raise exactly K'. If instead the capital requirement K is set so that

$$b_h(K) = (1-p) \min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_p[s]=K}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

then the bank and the market are always guaranteed to coordinate on the (unique) equilibrium that generates the highest possible level of capital. $\square$

8.4.3 Proof of Lemma 3.2

Proof. If investment by the h-type is not socially desirable, then the regulator will optimally set $K = \bar{K}$ and only the ℓ-type bank will invest. Therefore, investment is socially desirable under the pooling equilibrium whenever the welfare of both banks investing with separating requirements $K_h^\star$ and $K_\ell^\star$ is greater than the welfare of just the ℓ-type investing with capital requirement $\bar{K}$: $W(K_\ell^\star, K_h^\star | invest) \geq W(I|forgo)$. This is the case whenever

$$\hat{b} - \lambda(p \cdot L_h(K_h^\star) + (1-p) \cdot L_\ell(K_\ell^\star)) \geq (1-p)(\hat{b} - \lambda \cdot L_\ell(\bar{K}))$$

and after rearranging we obtain our result. $\square$

8.4.4 Proof of Proposition 3.2

Proof. First assume that the first best $K_\ell = K_h = \bar{K}$ is not possible under any separating mechanism. This implies that for any separating mechanism, one of the incentive compatibility constraints is binding. Now, let $\mathcal{M} = \{K_\theta, T_\theta, S_\theta\}_{\theta \in \{h,\ell\}}$ be a mechanism with $\tilde{s} \in S_h$ such that

$$\mathbb{E}_h[\tilde{s}] - \mathbb{E}_\ell[\tilde{s}] > \min_{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s'] \geq K_h + T_h}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']$$

We claim that $\mathcal{M}$ is weakly dominated by a mechanism $\mathcal{M}' = \{K'_\theta, T'_\theta, S'_\theta\}_{\theta \in \{h,\ell\}}$ that sets

$$\mathcal{S}'_h = \{s \in \mathcal{S} : s \in \operatorname*{argmin}_{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s'] \geq K_h + T_h}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']\}.$$

In order to prove this claim, suppose the IC_ℓ constraint is binding under $\mathcal{M}$ and assume $\tilde{s} \notin \mathcal{S}'_h$. Then, given that IC_ℓ must hold for all $s \in S_h$ implies that

$$T_h - T_\ell = b_\ell(K_h) - b_\ell(K_\ell) + \mathbb{E}_h[\tilde{s}] - \mathbb{E}_\ell[\tilde{s}] > b_\ell(K_h) - b_\ell(K_\ell) + \min_{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s'] \geq K_h}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s'] \quad (5)$$

Now, if IC_h is not binding, then there exists $\hat{K} > K_\ell$ such that $\mathcal{M}'$ is incentive compatible with $K'_\ell = \hat{K}$ and $K'_h = K_h$ implying that $\mathcal{M}'$ strictly dominates $\mathcal{M}$. To show that this is the case, we simply note that the inequality of (5) implies that one can increase K_ℓ by a small amount ϵ without violating the incentive compatibility constraint IC_ℓ whenever

$$\mathcal{S}'_h = \{s \in \mathcal{S} : s \in \underset{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s'] \geq K_h + T_h}}{\operatorname{argmin}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']\}$$

Further, given that IC_h is not binding, we can always find an $\bar{\epsilon}$ such that for all $\epsilon < \bar{\epsilon}$ setting $K'_\ell = K_\ell + \epsilon$ produces a mechanism that satisfies both incentive compatibility constraints. If instead, IC_h is also binding so that we cannot make such a welfare improvement then $\mathcal{M}'$ can achieve the same welfare as $\mathcal{M}$ even when restricting securities to the set $\mathcal{S}'_h$ by setting $K'_\theta = K_\theta$ and $T'_\theta = T_\theta$ as this restriction only relaxes IC_ℓ.

Next consider the case where IC_ℓ is not binding. In this case IC_h should be binding otherwise $\mathcal{M}$ is not optimal. Further, given that IC_h is binding and is independent of the security s_h, then $\mathcal{M}'$ generates the same welfare as $\mathcal{M}$ whenever the capital requirements and transfers are set equal. Therefore, we have shown that restricting securities to $\mathcal{S}'_h$ weakly improves welfare. Finally, to conclude the first part of the proof we will show that restricting to $\mathcal{S}^\star_h$ over $\mathcal{S}'_h$ is also without loss. Namely, we will show that

$$\min_{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s'] \geq K_h + T_h}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s'] = \min_{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s'] = K_h + T_h}} \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']$$

To do so we will show that for any s such that $\mathbb{E}_h[s] > K + T$ there exists s' such that $\mathbb{E}_h[s'] = K + T$ and $\mathbb{E}_h[s'] - \mathbb{E}_h[s'] < \mathbb{E}_h[s] - \mathbb{E}_h[s]$. Namely, consider $s'(z) = \phi \cdot s(z)$ for all $z \in \mathbb{R}$. Then, letting $\phi = \frac{K+T}{\mathbb{E}_h[s]} < 1$ we can see that $\mathbb{E}_h[s'] = \mathbb{E}_h[\phi \cdot s] = \phi \cdot \mathbb{E}_h[s] = K + T$. Further, this implies that

$$\mathbb{E}_h[s'] - \mathbb{E}_h[s'] = \phi \cdot (\mathbb{E}_h[s] - \mathbb{E}_h[s]) < \mathbb{E}_h[s] - \mathbb{E}_h[s]$$

We will now prove that it is weakly optimal for the regulator to restrict the ℓ-type to issue securities in the set

$$S'_\ell = \{s \in \mathcal{S} : s \in \underset{\substack{s' \in \mathcal{S} \\ \mathbb{E}_\ell[s'] \geq K_\ell + T_\ell}}{\text{argmax}} \quad \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']\}$$

suppose in a similar vein that $\mathcal{M}$ is a mechanism such that there exists $\tilde{s} \in S_\ell$ with

$$\mathbb{E}_h[\tilde{s}] - \mathbb{E}_\ell[\tilde{s}] < \max_{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s'] \geq K_\ell + T_\ell}} \quad \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']$$

Now, if IC_h is binding then

$$T_h - T_\ell = b_h(K_h) - b_h(K_\ell) + \mathbb{E}_h[\tilde{s}] - \mathbb{E}_\ell[\tilde{s}] < b_h(K_h) - b_h(K_\ell) + \max_{\substack{s' \in \mathcal{S} \\ \mathbb{E}_h[s'] \geq K_\ell + T_\ell}} \quad \mathbb{E}_h[s'] - \mathbb{E}_\ell[s'].$$

Therefore, if IC_ℓ is not binding then the regulator can strictly increase K_h by a positive amount when restricting

$$S_\ell = \{s \in \mathcal{S} : s \in \underset{\substack{s' \in \mathcal{S} \\ \mathbb{E}_\ell[s'] \geq K_\ell + T_\ell}}{\text{argmax}} \quad \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']\}$$

if otherwise IC_ℓ is binding, making such a restriction yields the same welfare when setting the same capital requirements and transfers as $\mathcal{M}$. Finally, we note that when choosing from a security $s \in \mathcal{S}_\ell$, the ℓ-type bank will optimally choose a security such that $\mathbb{E}_\ell[s_\ell] = K_\ell + T_\ell$ in any equilibrium and therefore it is without loss to restrict

$$S_\ell = \{s \in \mathcal{S} : s \in \underset{\substack{s' \in \mathcal{S} \\ \mathbb{E}_\ell[s'] = K_\ell + T_\ell}}{\text{argmax}} \quad \mathbb{E}_h[s'] - \mathbb{E}_\ell[s']\}$$

In order to conclude the proof we note that we have just shown that restricting securities to $\mathcal{S}^\star_\theta$ can only improve welfare under the optimal capital requirements and transfers. Therefore, if there exists a separating mechanism that achieves the first best (so that no incentive constraints are binding) then it is without loss to restrict the securities of that mechanism to $\mathcal{S}^\star_\theta$. $\square$

8.4.5 Proof of Lemma 3.3

Proof. The conditions given in the lemma are precisely the conditions necessary to achieve incentive compatibility when $K_\ell = K_h = \bar{K}$. Namely, incentive compatibility in this case becomes

$$\max_{\substack{s\in\mathcal{S} \\ \mathbb{E}_\ell[s]=\bar{K}+T_\ell}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] > T_h - T_\ell > \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_h[s]=\bar{K}+T_h}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

but inducing investment requires $T_h < b_h(\bar{K}) = \hat{b}$. Therefore, whenever

$$\hat{b} \geq \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_h[s]=\bar{K}+T_h}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

the regulator can achieve the first best by setting $T_\ell = 0$ and $T_h = T \leq \hat{b}$ achieving incentive compatibility of the first best level of capital requirements $K_\ell = K_h = \bar{K}$.

For the second part of the proof we note that there always exists $\hat{p}$ such that

$$(1-\hat{p}) \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_{pool}[s]=\bar{K}}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] = \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_h[s]=\bar{K}+T_h}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] \leq \hat{b}$$

which implies that the regulator can implement the first best through the optimal pooling mechanism whenever $p > \hat{p}$. □

8.4.6 Proof of Lemma 3.4

Proof. First, suppose that $K_\ell > K_h$. We will show that when $\mathcal{M}^\star_{sep}$ dominates $\mathcal{M}^\star_{pool}$ then no matter the choice of $K_\ell > K_h$ the constraint IC_h is never binding and therefore it must be the case that IC_ℓ is binding. To do so, note that based on the characterization of the optimal pooling capital requirement $K^\star$, we note that if $\mathcal{M}_{sep}$ dominates $\mathcal{M}_{pool}$ then it must be the case that $K_\ell > K^\star$. This implies that

$$b_h(K_\ell) < (1-p) \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_{pool}[s]=K_\ell}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] < \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_{pool}[s]=K_\ell+T_\ell}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

where the second inequality comes from the fact that securities that generate more funds always increase the minimum information sensitivity (see the proof of Proposition 3.2) and we can drop the $(1-p)$ as these values are strictly positive. This implies that

$$b_h(K_h) - b_h(K_\ell) + \max_{\substack{s_\ell \in \mathcal{S} \\ \mathbb{E}_\ell[s_\ell] = K_\ell + T_\ell}} \left(\mathbb{E}_h[s_\ell] - \mathbb{E}_\ell[s_\ell]\right) >$$

$$b_h(K_h) - \min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_{pool}[s] = K_\ell + T_\ell}} \left(\mathbb{E}_h[s] - \mathbb{E}_\ell[s]\right) + \max_{\substack{s_\ell \in \mathcal{S} \\ \mathbb{E}_\ell[s_\ell] = K_\ell}} \left(\mathbb{E}_h[s_\ell] - \mathbb{E}_\ell[s_\ell]\right)$$

Further, given that $T_h - T_\ell < b_h(K_h)$ (coming from the fact that the bank will optimally forgo investment if $T_h > b_h(K_h)$) implies that IC_h is never binding whenever

$$\min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_{pool}[s] = K_\ell + T_\ell}} \left(\mathbb{E}_h[s] - \mathbb{E}_\ell[s]\right) < \max_{\substack{s_\ell \in \mathcal{S} \\ \mathbb{E}_\ell[s_\ell] = K_\ell + T_\ell}} \left(\mathbb{E}_h[s_\ell] - \mathbb{E}_\ell[s_\ell]\right)$$

We will show that this inequality holds when s_ℓ is equity. Namely denoting by s^1_{eq} the equity security satisfying $\mathbb{E}_{pool}[s^1_{eq}] = K_\ell + T_\ell$ then denoting by $V_\theta(K_\ell)$ the value of the firm type θ after issuing equity worth $K_\ell + T_\ell$ and making the investment we obtain

$$\min_{\substack{s \in \mathcal{S} \\ \mathbb{E}_{pool}[s] = K_\ell + T_\ell}} \left(\mathbb{E}_h[s] - \mathbb{E}_\ell[s]\right) \leq \mathbb{E}_h[s^1_{eq}] - \mathbb{E}_\ell[s^1_{eq}] = \frac{V_h(K_\ell) - V_\ell(K_\ell)}{\hat{V}(K_\ell)} K_\ell$$

where $\hat{V}(K_\ell) = pV_h(K_\ell) + (1-p)V_\ell(K_\ell)$. Now if s^2_{eq} is the equity security satisfying $\mathbb{E}_\ell[s^2_{eq}] = K_\ell + T_\ell$ then

$$\max_{\substack{s_\ell \in \mathcal{S} \\ \mathbb{E}_\ell[s_\ell] = K_\ell + T_\ell}} \left(\mathbb{E}_h[s_\ell] - \mathbb{E}_\ell[s_\ell]\right) \geq \mathbb{E}_h[s^2_{eq}] - \mathbb{E}_\ell[s^2_{eq}] = \frac{V_h(K_\ell) - V_\ell(K_\ell)}{V_\ell(K_\ell)} K_\ell$$

and therefore, given that $\hat{V}(K_\ell) > V_\ell(K_\ell)$ implies

$$\frac{V_h(K_\ell) - V_\ell(K_\ell)}{\hat{V}(K_\ell)} K_\ell < \frac{V_h(K_\ell) - V_\ell(K_\ell)}{V_\ell(K_\ell)} K_\ell$$

and we obtain our result.

Now we turn to the case where $\mathcal{M}^{\star}_{sep}$ sets $K_h > K_\ell$. In this case, suppose by contraposition that IC_ℓ is not binding. Then, it must be the case that IC_h is binding so that

$$T_h - T_\ell = b_h(K_h) - b_h(K_\ell) + \max_{\substack{s\in\mathcal{S} \\ \mathbb{E}_\ell[s]=K_\ell+T_\ell}} \mathbb{E}_h[s] - \mathbb{E}_h[s].$$

Now let

$$s_\ell \in \operatorname*{argmax}_{\substack{s\in\mathcal{S} \\ \mathbb{E}_\ell[s]=K_\ell+T_\ell}} \mathbb{E}_h[s] - \mathbb{E}_h[s]$$

be the chosen security of the ℓ-type. Given that

$$\frac{\partial}{\partial K_\ell}[b_h(K_h) - b_h(K_\ell) + \mathbb{E}_h[s_\ell] - K_\ell] = G(-a_h - K_\ell) + \frac{\partial}{\partial K_\ell}(\mathbb{E}_h[s_\ell] - K_\ell))$$

implies that if $\mathbb{E}_h[s_\ell] - \mathbb{E}_\ell[s_\ell]$ is strictly increasing in K_ℓ then the regulator could do strictly better by increasing K_ℓ by some positive amount without violating IC_h and therefore IC_ℓ which is assumed to be non-binding. In order to prove this, we will show that for any value of $K'_\ell > K_\ell$, if

$$s_\ell \in \operatorname*{argmax}_{\substack{s\in\mathcal{S} \\ \mathbb{E}_\ell[s]=K_\ell+T_\ell}} \mathbb{E}_h[s] - \mathbb{E}_h[s] \qquad \text{and} \qquad s'_\ell \in \operatorname*{argmax}_{\substack{s\in\mathcal{S} \\ \mathbb{E}_\ell[s]=K'_\ell+T_\ell}} \mathbb{E}_h[s] - \mathbb{E}_h[s]$$

then $\mathbb{E}_h[s_\ell] - \mathbb{E}_\ell[s_\ell] < \mathbb{E}_h[s'_\ell] - \mathbb{E}_\ell[s'_\ell]$.

In order to prove this, simply note that s'_ℓ can always be constructed as $s'_\ell(z) = s_\ell(z) + s_0(z)$ for some appropriately constructed $s_0(z)$ such that $s_0(z) \in [0, \max\{z - s_\ell(z), 0\}]$ and $s_0(z) \in \mathcal{S}$. Namely, s'_ℓ pays the same as s_ℓ plus an additional residual s_0 which in expectation is worth $K'_\ell - K_\ell$. In that case given that $\mathbb{E}_h[s] - \mathbb{E}_\ell[s] > 0$ for all $s \in \mathcal{S}$ implies

$$\max_{\substack{s\in\mathcal{S} \\ \mathbb{E}_\ell[s]=K'_\ell+T_\ell}} \mathbb{E}_h[s] - \mathbb{E}_h[s] \geq \mathbb{E}_h[s'_\ell] - \mathbb{E}_\ell[s'_\ell] = \mathbb{E}_h[s_\ell] - \mathbb{E}_\ell[s_\ell] + \mathbb{E}_h[s_0] - \mathbb{E}_\ell[s_0] > \mathbb{E}_h[s_\ell] - \mathbb{E}_\ell[s_\ell]$$

and we have proven our claim.

Therefore, we have just shown that if IC_h is binding and IC_ℓ is not binding, then the regulator can increase $K_\ell^\star$ by a small amount increasing the RHS of IC_h. If this increase in $K_\ell^\star$ increases the RHS of IC_h by more than it increases the RHS of IC_ℓ then the regulator would increase $K_\ell^\star$ until $K_\ell^\star \geq K_h^\star$ in which case we are no longer in this case. Otherwise, the regulator will increase $K_\ell^\star$ until IC_ℓ is binding. □

8.4.7 Proof of Proposition 3.3

Proof. By Lemma 3.4 we know that the constraint IC_h is never binding whenever the separating mechanism is optimal and chooses $K_\ell^\star > K_h^\star$. Hence, the relevant binding constraint is IC_ℓ

$$T_h - T_\ell = b_\ell(K_h) - b_\ell(K_\ell) + \min_{\substack{s_h \in \mathcal{S} \\ \mathbb{E}_h[s_h] = K_h + T_h}} \mathbb{E}_h[s_h] - \mathbb{E}_\ell[s_h]$$

Now, given that no term on the RHS of IC_ℓ depends on T_ℓ implies that optimally $T_\ell = 0$ whenever the optimal separating equilibrium dominates the optimal pooling mechanism. Namely, if $T_\ell > 0$ then the regulator can decrease T_ℓ which relaxes the IC_ℓ constraint allowing for an increase in capital requirements. If IC_h binds in this case then either we contradict the fact that the optimal separating equilibrium dominates the optimal pooling mechanism or the fact that $K_\ell^\star > K_h^\star$.

In order to prove that optimally $T_h = b_h(K_h^\star)$ we note that if IC_ℓ is binding with $T_h < b_h(K_h^\star)$, then it is not binding when $T_h = b_h(K_h^\star)$. In order to prove this we note that as T_h increases, $\mathbb{E}_h[s_h]$ increases identically no matter the security chosen as $\mathbb{E}_h[s_h] = K_h + T_h$. This implies that by increasing T_h the security s_h must yield a higher expected payment and therefore $\mathbb{E}_\ell[s_h]$ must weakly increase in value (e.g. if s_h is equity or standard debt then $\mathbb{E}_\ell[s_h]$ will strictly increase in value as T_h increases). If $\mathbb{E}_\ell[s_h]$ strictly increases in value then the increase in T_h is larger than the increase in the RHS of IC_ℓ and therefore setting $T_h = b_h(K_h^\star)$ is strictly optimal as it relaxes the IC_ℓ constraint. Otherwise setting $T_h = b_h(K_h^\star)$ it is weakly optimal. While it can be shown that the security that minimizes the information sensitivity will always yield a strict increase in $\mathbb{E}_\ell[s_h]$ we exclude the proof as this shorter proof suffices.

Now, if instead the optimal mechanism sets $K_h^\star > K_\ell^\star$, then we know that IC_ℓ is still binding and equal to $T_h - T_\ell$. Therefore, $T_h - T_\ell$ should be chosen so that $K_\ell^\star$ and $K_h^\star$ maximize the welfare given investment which is equivalent to minimizing the expected liability subject to incentive compatibility and $T_\ell \in [0, b_h(K_h^\star)]$ and $T_h \in [0, b_\ell^\star]$. $\square$

8.4.8 Proof of Lemma 3.5

Proof. Note that the regulator's objective is to maximize welfare which, when the mechanism is incentive compatible, is equivalent to minimizing the expected liability given by $p \cdot L_h(K_h) + (1-p) \cdot L_\ell(K_\ell)$. Therefore, there always exists a value of p_1 large such that when $p > p_1$ the increase in welfare from increasing K_h by any amount Δ is larger than the decrease in welfare from decreasing K_ℓ to zero. Similarly, there exists p_2 such that when $p < p_2$ then the benefit of increasing K_ℓ by any amount Δ is larger than the decrease in welfare from decreasing K_h to zero. Further, optimally, K_ℓ is weakly decreasing in p while K_h is weakly increasing. Therefore, there are three cases: (i) $K_\ell^\star > K_h^\star$ for all $p > p_2$, in which case $\bar{p} = 1$, (ii) $K_h^\star > K_\ell^\star$ for all $p < p_1$ in which case $\bar{p} = 0$, and (iii) there exists $p_0 \in (p_2, p_1)$ such that $K_\ell^\star > K_h^\star$ whenever $p < p_0$ and $K_h^\star > K_\ell^\star$ when $p > p_0$ in which case $\bar{p} = p_0$.

To show that $\bar{p}$ is strictly increasing in a_h we simply note that the marginal benefit of increasing K_h is given by $p \cdot G(-a_h - K_h)$ which goes to zero as $a_h \to +\infty$. Therefore, as the marginal benefit of increasing K_h decreases, p must be larger to induce the regulator to set $K_h > K_\ell$. $\square$

8.5 Proofs of Section 5

8.5.1 Proof of Proposition 4.1

Proof. We will proceed to prove this proposition in steps.

Claim 8.1. *There exists p_{sep} such that whenever $p < p_{sep}$ then $\mathcal{M}_{sep}^\star$ dominates $\mathcal{M}_{pool}^\star$.*

Proof. Denote by s_{pool} and s_{sep} the securities such that

$$s_{pool}(K) \in \underset{\substack{s \in \mathcal{S} \\ \mathbb{E}_{pool}[s]=K}}{\operatorname{argmin}} \ \mathbb{E}_h[s] - \mathbb{E}_\ell[s] \quad \text{and} \quad s_{sep}(K) \in \underset{\substack{s \in \mathcal{S} \\ \mathbb{E}_h[s]=K+T_h}}{\operatorname{argmin}} \ \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

then, we know that the capital requirement $K^\star$ of $\mathcal{M}^\star_{pool}$ is chosen to solve

$$b_h(K^\star) = (1-p)\cdot(\mathbb{E}_h[s_{pool}(K^\star)] - \mathbb{E}_\ell[s_{pool}(K^\star)])$$

but from the definition of s_{pool} we know that $p\cdot\mathbb{E}_h[s_{pool}(K^\star)] + (1-p)\cdot\mathbb{E}_\ell[s_{pool}(K^\star)] = K^\star$ which implies that

$$\mathbb{E}_h[s_{pool}(K^\star)] - \mathbb{E}_\ell[s_{pool}(K^\star)] = \frac{1}{p}(K^\star - \mathbb{E}_\ell[s_{pool}(K^\star)])$$

and therefore, $K^\star$ is chosen to solve

$$b_h(K^\star) = \frac{1-p}{p}(K^\star - \mathbb{E}_\ell[s_{pool}(K^\star)])$$

which shows that $K^\star$ is increasing in p. Further, as $p \to 0$ it must be the case that $K^\star - \mathbb{E}_\ell[s_{pool}(K^\star)] \to 0$ given that $b_h(K^\star)$ is positive and strictly greater than 0 for all $K^\star$. Further, $K^\star - \mathbb{E}_\ell[s_{pool}(K^\star)] \to 0$ only if $K^\star \to 0$ coming from the fact that $\mathbb{E}_h[s_{pool}(K)] - \mathbb{E}_\ell[s_{pool}(K)] > 0$ and strictly increasing in $K^\star$ for all $s \in \mathcal{S}$ (see the proof of Proposition 3.1).

Now, we know that there exists $\bar{p}$ such that whenever $p < \bar{p}$ then optimally $K^\star_\ell > K^\star_h$ and therefore $K^\star_\ell$ and $K^\star_h$ must satisfy

$$b_h(K^\star_h) = b_\ell(K^\star_h) - b_\ell(K^\star_\ell) + \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_h[s]=K^\star_h+T^\star_h}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

Hence, we simply note that if we set $K^\star_\ell = K^\star_h = 0$ then we obtain

$$b_h(0) > \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_h[s]=b_h(0)}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

where the inequality comes from the fact that the security that minimizes the information sensitivity satisfies $\mathbb{E}_h[s_h] = b_h(0)$ and therefore as long as $\mathbb{E}_\ell[s_h] > 0$ then we obtain our result. Further, once this result holds we know that $K_\ell > 0$ and $K_h = 0$ is incentive compatible and therefore there exists p_{sep} such that whenever $p < p_{sep}$, $\mathcal{M}^\star_{sep}$ dominates $\mathcal{M}^\star_{pool}$.

To prove that $\mathbb{E}_\ell[s_h] > 0$ whenever $\mathbb{E}_h[s_h] = b_h(0)$, we note that the only case where $\mathbb{E}_h[s_h] = b_h(0)$ and $\mathbb{E}_\ell[s_h] = 0$ is if s_h is such that $s_h(z) > 0$ if and only if $z \leq (a_h - a_\ell)$: the ℓ-type pays 0 under s_h whenever the h-type pays a positive amount under s_h. But in that case, this implies that $s_h(z) = 0$ for large values of z and $s_h(z) > 0$ for small values of z (this must be the case as $\mathbb{E}_h[s_h] > 0$) contradicting the fact that $s(z)$ is non-decreasing in z.

□

Claim 8.2. *There exists p_{pool} such that whenever $p > p_{pool}$ then $\mathcal{M}^\star_{pool}$ dominates $\mathcal{M}^\star_{sep}$.*

Proof. In order to prove this claim, we note that as $p \to 1$ then $K^\star \to \bar{K}$. Namely, denoting by $s_e(K^\star)$ the equity security such that $\mathbb{E}_{pool}[s_e(K^\star)] = K^\star$, then we know

$$\frac{1-p}{p}(K^\star - \mathbb{E}_\ell[s_{pool}(K)]) \leq \frac{1-p}{p}(K^\star - \mathbb{E}_\ell[s_e(K)]) =$$

$$\frac{1-p}{p} \cdot K^\star(1 - \frac{a_\ell + b_\ell(K^\star) + K^\star}{p \cdot (a_h + b_h(K^\star)) + (1-p)(a_\ell + b_\ell(K^\star)) + K}) =$$

$$\frac{1-p}{p} \cdot K^\star \cdot \frac{p \cdot (a_h + b_h(K^\star) - a_\ell - b_\ell(K^\star))}{p \cdot (a_h + b_h(K^\star)) + (1-p)(a_\ell + b_\ell(K^\star)) + K} \leq (1-p) \cdot (a_h - a_\ell)$$

where the last inequality comes from the fact that the LHS is strictly increasing in $K^\star$ and the RHS is obtained by taking the limit as $K^\star \to +\infty$. Therefore, as $p \to 1$ we know that

$$\frac{1-p}{p}(K^\star - \mathbb{E}_\ell[s_{pool}(K)]) \to 0$$

and therefore it must be the case that $K^\star \to \bar{K}$.

Given this, the only way $\mathcal{M}^\star_{pool}$ does not dominate $\mathcal{M}^\star_{sep}$ as $p \to 1$ is if both $K^\star_\ell \to \bar{K}$ and $K^\star_h \to \bar{K}$ as $p \to 1$. Now, we always know that for any values of $K^\star_h$ and $K^\star_\ell$ it must be the case that IC_ℓ is binding. Therefore, using the fact that $b_h(K_h) \geq T_h - T_\ell$ we can see that under the optimal separating mechanism

$$b_h(K_h^\star) \geq T_h - T_\ell = b_\ell(K_h^\star) - b_\ell(K_\ell^\star) + \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_h[s]=K_h^\star+T_h^\star}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s]$$

Further, as $p \to 1$ we know that $K_h^\star > K_\ell^\star$. Therefore, if $\mathcal{M}_{sep}^\star$ dominates $\mathcal{M}_{pool}^\star$ then it must be the case that $K_h^\star > K_{pool}^\star$ which implies

$$b_h(K_h^\star) < (1-p) \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_{pool}[s]=K_h^\star}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s] \leq (1-p)(a_h - a_\ell)$$

which implies that if $\mathcal{M}_{sep}^\star$ dominates $\mathcal{M}_{pool}^\star$ for all $p > \bar{p}$ then

$$(1-p)(a_h - a_\ell) > b_h(\bar{K}) \geq b_\ell(\bar{K}) - b_\ell(K_\ell^\star) + \min_{\substack{s\in\mathcal{S} \\ \mathbb{E}_h[s]=\bar{K}+T_h^\star}} \mathbb{E}_h[s] - \mathbb{E}_\ell[s].$$

Which can only be the case if $K_\ell^\star < \bar{K}$ as the information sensitivity is always positive and therefore as p approaches 1 this inequality can only be satisfied if $b_\ell(K_\ell^\star) - b_\ell(\bar{K}) > 0$ which implies $K_\ell^\star < \bar{K}$. Hence, there must exist a level p_{pool} such that whenever $p > p_{pool}$, $\mathcal{M}_{pool}^\star$ dominates $\mathcal{M}_{sep}^\star$. □

Claim 8.3. *There exists $p_{und} \in [0, 1)$ such that whenever $p < p_{und}$ then $\mathcal{M}_{und}^\star$ dominates both $\mathcal{M}_{pool}^\star$ and $\mathcal{M}_{sep}^\star$.*

Proof. By lemma's 3.1 and 3.2 we know that $\mathcal{M}_{und}^\star$ dominates both $\mathcal{M}_{pool}^\star$ and $\mathcal{M}_{sep}^\star$ whenever

$$b < \min\{\frac{\lambda}{p}(p \cdot L_h(K^\star) + (1-p)(L_\ell(K^\star) - L_\ell(\bar{K})), \frac{\lambda}{p}(p \cdot L_h(K_h) + (1-p)(L_\ell(K_\ell) + L_\ell(\bar{K}))\}$$

further, we know that $K^\star \to 0$ as $p \to 0$ and therefore

$$\frac{\lambda}{p}(p \cdot L_h(K^\star) + (1-p)(L_\ell(K^\star) - L_\ell(\bar{K})) \to +\infty$$

Thus, there exists $p_{und} > 0$ such that $\mathcal{M}^\star_{und}$ dominates $\mathcal{M}^\star_{pool}$ whenever $p < p_{und}$.

Finally, the only way that $\mathcal{M}^\star_{und}$ dominates $\mathcal{M}^\star_{sep}$ is if $K^\star_\ell < \bar{K}$ for all p close to zero. This is the case if and only if

$$b_h(0) < b_\ell(0) - b_\ell(\bar{K})$$

or equivalently

$$b_\ell(\bar{K}) > b_\ell(0) - b_h(0)$$

Namely, this condition states that $K^\star_\ell = \bar{K}$ and $K^\star_h = 0$ is not incentive compatible. When it is satisfied then there exists $p_{und} > 0$ such that $p < p_{und}$ implies $\mathcal{M}^\star_{und}$ dominates $\mathcal{M}^\star_{sep}$. Whenever this condition is not satisfied then it implies that setting $K^\star_\ell = \bar{K}$ and $K^\star_h = 0$ is incentive compatible for all $p > 0$ and therefore $\mathcal{M}^\star_{sep}$ weakly dominates $\mathcal{M}^\star_{und}$ for all $p > 0$ in which case $p_{und} = 0$. □

Now, note that Claim 8.2 and Claim 8.3 imply together that there exists p_{und} and p_{sep} such that $p < p_{und}$ implies $\mathcal{M}^\star = \mathcal{M}^\star_{und}$ and $p \in (p_{und}, p_{sep})$ implies $\mathcal{M}^\star = \mathcal{M}^\star_{sep}$ whenever $p_{sep} > p_{und}$. This comes from the fact that we can always take p_{und} to be the largest value of p such that whenever $p < p_{und}$ then $\mathcal{M}^\star = \mathcal{M}^\star_{und}$.

Claim 8.4. *There exists p_{pool} such that whenever $p > p_{pool}$, then $\mathcal{M}^\star = \mathcal{M}_{pool}$*

Proof. First note that if $p_{sep} > p_{und}$ then $p_{pool} \geq p_{sep} \geq p_{und}$ and therefore by Claim 8.1 and the definition of p_{und} we know that $\mathcal{M}^\star_{pool}$ dominates $\mathcal{M}^\star_{sep}$ which in turn dominates $\mathcal{M}^\star_{und}$.

Now, if instead $p_{sep} < p_{und}$ then $\mathcal{M}^\star_{sep}$ is never optimal. Further, in that case we know that as $p \to 1$ then $K^\star \to \bar{K}$, in which case there exists p_{pool} such that $\mathcal{M}^\star = \mathcal{M}_{pool}$ whenever $p > p_{pool}$. □

□

8.5.2 Proof of Proposition 4.1

Proof. (i) First, note that it must be the case that $\mathcal{M}_{pool}$ dominates $\mathcal{M}_{sep}$ for all $p \in (p_{sep}, \bar{p})$. Namely, as p increases above p_{sep}, then $K_\ell^\star$ is weakly decreasing as the marginal benefit of higher capital for the ℓ-type decreases given that the probability of the ℓ-type decreases. Further, we have shown that $K^\star$ is strictly increasing in p. Therefore, as long as $K_\ell^\star > K_h^\star$ then $K^\star > K_\ell^\star > K_h^\star$ and therefore $\mathcal{M}_{pool}$ strictly dominates $\mathcal{M}_{sep}$. Hence, the only way that $\mathcal{M}_{sep}$ can dominate $\mathcal{M}_{pool}$ when $p \in (p_{sep}, p_{pool})$ is if $p > \bar{p}$ and therefore $K_h^\star > K_\ell^\star$.

(ii) We know that for all p' there exists $\bar{a}$ such that $\bar{p} > p'$ whenever $a_h > \bar{a}$. This comes from the fact that the marginal benefit of increasing K_h is $p \cdot G(-a_h - K_h)$ which goes to zero as $a_h \to +\infty$. What is left to prove is that p_{pool} is bounded away from 1 as $a_h \to +\infty$. In order to show this, we note that if s_{pool} were equity, then denoting K_e the optimal capital requirement when the banks are restricted to issuing equity then we know that $K^\star \geq K_e$. Further, K_e is determined by

$$b_h(K_e) = \frac{1-p}{p} \frac{a_h - a_\ell + b_h(K_e) - b_\ell(K_e)}{a_h + b_h(K_e) + K_e} \cdot K_e$$

or equivalently

$$K_e = \frac{p}{1-p} \cdot \frac{a_h + b_h(K_e) + K_e}{a_h - a_\ell + b_h(K^\star) - b_\ell(K_e)} b_h(K_e)$$

Next, note that $b_h(K_e)$ is strictly decreasing in a_h and as $a_h \to +\infty$ it is the case that $b_h(K_e) \to \hat{b}$ for all values of K_e. Further,

$$\lim_{a_h \to +\infty} \frac{a_h + b_h(K_e) + K_e}{a_h - a_\ell + b_h(K_e) - b_\ell(K_e)} = 1$$

and this expression is strictly decreasing in a_h. Therefore,

$$K_e \to \frac{p}{1-p} \cdot \hat{b}$$

as $a_h \to +\infty$. Furthermore, we know that K_e is decreasing in a_h and therefore

$$K^\star > \frac{p}{1-p} \cdot \hat{b}$$

for all $a_h > a_\ell$.

Hence, we have just shown that $K^\star \to +\infty$ as $p \to 1$ for all $a_h > a_\ell$ and therefore there exists $\tilde{p} < 1$ such that $p_{pool} < \tilde{p}$ for all a_h. Finally, given that we know there exists $\bar{a}$ such that whenever $a_h > \bar{a}$ then $\bar{p} > \tilde{p}$ for any $\tilde{p} < 1$, we have proven our claim. □

8.5.3 Proof of Proposition 4.2

Proof. Part (i) simply states the condition for $K^\star_\ell = \bar{K}$ and $K^\star_h = 0$ to be incentive compatible under the optimal separating mechanism. Therefore, if $b_\ell(\bar{K}) \geq b_\ell(0) - b_h(0)$ then the regulator could always implement $K^\star_\ell = \bar{K}$ and $K^\star_h = 0$ regardless of the value of p and therefore $\mathcal{M}^\star_{und}$ never dominates $\mathcal{M}^\star_{sep}$ so that $p_{und} = 0$.

(ii) Suppose that $b_\ell(\bar{K}) < b_\ell(0) - b_h(0)$ so that $p_{und} > 0$. Then, given that $b_\theta(K) = \hat{b} + L_\theta(K)$ we note that $p_{und} > 0$ only if

$$\hat{b} < L_\ell(0) - L_h(0) - L_\ell(\bar{K}) \tag{6}$$

and noting that as $\hat{b} \to +\infty$ then $L_\theta(K) \to 0$ as $Pr(x < -a_\theta - K) \to 0$. Hence, RHS of 6 goes to 0 as $\hat{b} \to +\infty$ and therefore there exists $\bar{b}$ such that $\hat{b} > \bar{b}$ implies that $b_\ell(\bar{K}) \geq b_\ell(0) - b_h(0)$. □