Why Does Capital Flow from Equal to Unequal Countries?*

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Abstract

Capital flows from equal to unequal countries. We document this empirical regularity in a large sample of advanced economies. The capital flows are largely driven by private savings. We propose a theory that can rationalize these findings: more unequal countries endogenously develop deeper financial markets. Households in unequal counties, in turn, borrow more, driving the observed direction of capital flows.

Keywords: Inequality, Current Account, Capital Flows

JEL codes: F32, F41, E21

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1 Introduction

Understanding the determinants of the current account balance of open economies is a classic question in international macroeconomics. Interest in this question has grown in the past twenty years, alongside the large increase in the magnitude of capital flows. Traditional answers to this question range from the persistence of shocks, to the nature of the shocks (demand, supply or financial), to demographic forces, to the ability of a country to raise resources on international financial markets.

We propose a novel explanation that drives current account imbalances across countries: we document that a country's degree of income inequality is negatively correlated with its current account balance. We then propose a theory that can rationalize the observed empirical regularity among advanced countries. Recent research has emphasized the importance of inequality for macroeconomic outcomes in a closed-economy setting. Here we continue in that same spirit to understand how domestically incomplete markets can address open-economy puzzles. To address this novel question, we build on the empirical and theoretical tools developed by the literature on household heterogeneity in closed economy, bridging its insights with those of open economy macroeconomics.

Our main empirical finding is that advanced economies with greater income inequality (as measured by the net-income Gini coefficient) run larger current account deficits. Figure 1 provides a visual representation of this fact. Moreover, we find that this relationship has grown stronger in recent years. Private savings drive the relationship—being systematically lower in countries with greater inequality. Public savings are also lower in high-inequality countries, but to a lesser degree. Aggregate investment is also significantly lower in countries with high inequality. Hence, the negative relationship between inequality and aggregate savings is sufficiently strong to imply a negative relationship between inequality and the external balance, despite the counteracting effect of investment.

Next, we develop a theoretical model that can rationalize our empirical findings. Standard *exogenously* incomplete-market models used in heterogeneous-agent macroeconomics can not replicate the pattern that we observe in the data—if higher inequality is due to higher income risk. In particular, in the workhorse Bewley-Huggett-Aiyagari-İmrohoroğlu framework higher inequality yields higher aggregate saving—odds with our findings. In our model, financial

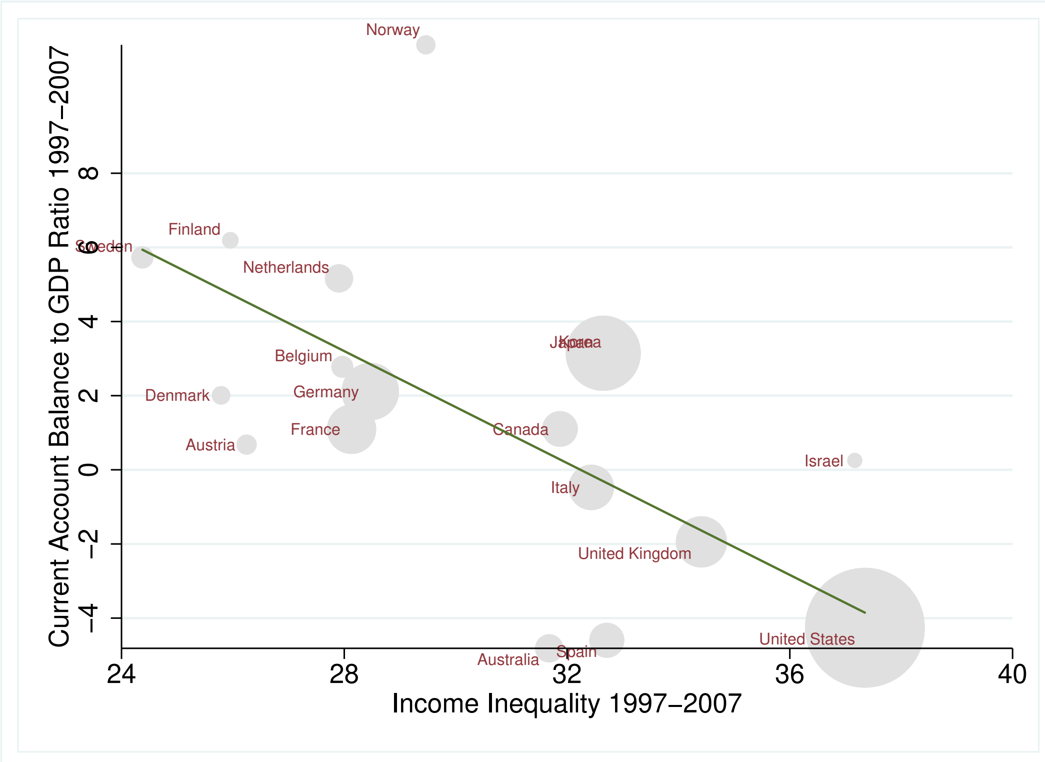

Figure 1: Relationship between income inequality and the current account balance in advanced economies.

markets are *endogenously* incomplete, due to an underlying friction on contract enforcement. As a result, higher income risk gives rise to deeper financial markets and looser borrowing constraints. Households who experience significant fluctuations in income have a strong incentive to repay their debts to retain access to borrowing. Hence, in high-inequality countries, borrowing constraints are less severe and aggregate savings are lower than in low-inequality countries. When residents of different countries can borrow and lend with each other, capital flows from equal to unequal countries, as savers in equal countries lend to borrowers in unequal countries. This prediction of the model is consistent with our empirical findings of higher current account deficits in countries with high inequality, driven by differences in private saving rates.

The paper is organized as follows. Section 1.1 discusses further our contribution relative to the existing literature. We outline our empirical analysis in section 2 and we present our theoretical model in section 3. Finally, section 4 concludes.

1.1 Related Literature

We contribute to a vast literature on international capital flows. Lucas (1990) is a seminal contribution investigating the causes of the observed direction of international capital flows. We address a related question, analyzing the systematic relationship between inequality and the direction of capital flows.

Mendoza et al. (2009) and Caballero et al. (2008) have studied how differences in financial markets development and in the ability to generate financial assets across countries may lead to current account imbalances. We build on that important contribution, showing that heterogeneity in income inequality across countries contributes to differences in financial development and thus to the observed pattern of current account deficits and surpluses.

Our model with heterogeneous households builds on the contribution by Krueger and Perri (2006) and the previous work by Alvarez and Jermann (2000) and Kehoe and Levine (2001). Our theoretical framework features multiple countries interacting in general equilibrium, and it provides a characterization of consumption inequality in each country, as well as of international capital flows and of international external positions. Our focus is on how cross-sectional differences across countries on the severity of income inequality lead to heterogeneity in their external borrowing. Krueger and Perri (2006) study instead the increase over time in income inequality observed in the United States in recent years. They show that it has not been accompanied by a corresponding rise in consumption inequality, and they provide a closed-economy model with endogenous debt constraints that can account for this finding.[1] Broer (2014) also studies an open economy version of a similar model. His focus is only on the United States, and on how trends in income risk may given rise to a sustained fall in the external asset position.

Ranciere et al. (2012) is a closely related paper analyzing the relationship between trends in income inequality and in countries' external balance. The empirical analysis in our paper differs in that we are able to identify a systematic relationship between the long-run average current account balance of a country and its average degree of income inequality in the sample period. Instead, that paper focuses on changes over time in the two variables within each country, by considering data at yearly frequency, and on a different measure of inequality,

[1]See Aguiar and Bils (2015) for an alternative interpretation.

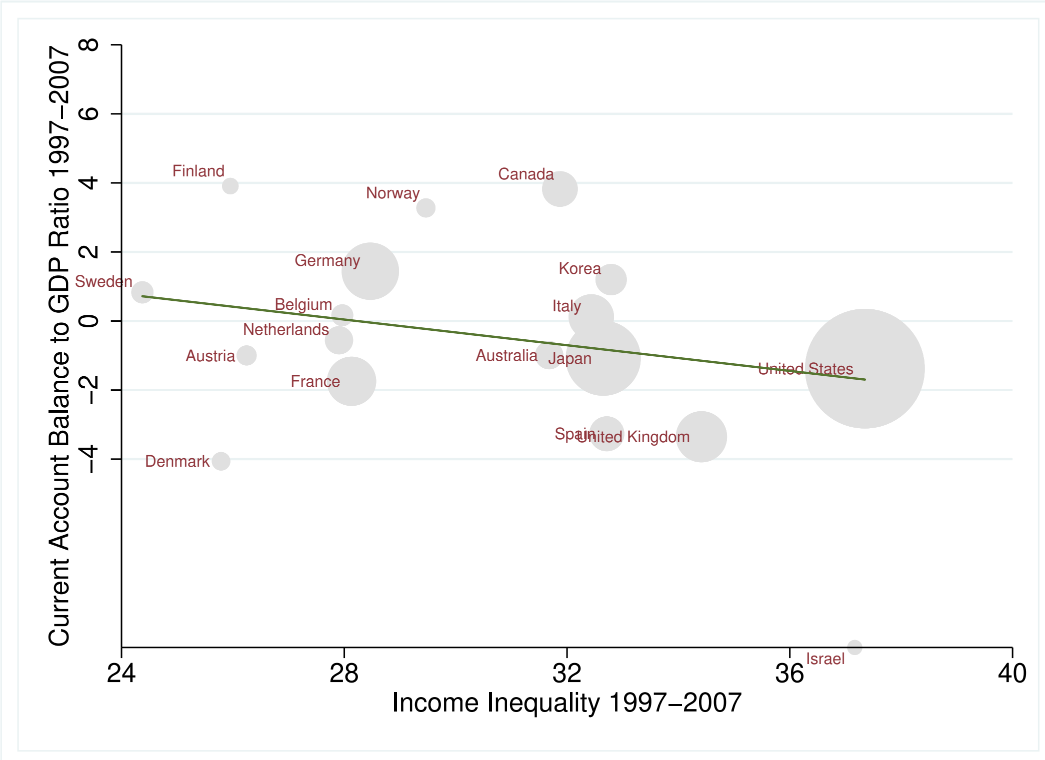

Figure 2: Relationship between income inequality and the current account balance in advanced economies, after controlling for other variables.

given by the income share of the top 5% and 1% in the income distribution. Moreover, we are able to decompose the relationship of inequality with different components of the external balance, highlighting the strong relationship with private savings and the lack of an effect on investment or on public saving.

We also build in this paper on our previous work on heterogeneous households in open economies. In de Ferra et al. (2020), we studied how changes in the distribution of foreign-currency debt holdings determined the magnitude of the effects of a contraction in capital inflows. Our focus is here instead on the relationship between income inequality and capital flows across countries, when measured over a long period of time.

2 Empirical Analysis

We study the relationship between income inequality and the current account balance in a large set of economies in the period of buoyant capital flows between 1997 and 2007.[2]

[2]We analyze a longer period of data between 1990 and 2014 in the robustness exercises we detail in Appendix A.3.

Table I: Relationship between Income Inequality and External Balance, Savings and Investment.[a]

	Emerging	Advanced	Pooled
Current Account to GDP	−0.175	−0.803***	−0.310**
	(0.132)	(0.142)	(0.097)
Trade Balance to GDP	−0.170	−1.026***	−0.455***
	(0.175)	(0.182)	(0.117)
Aggregate Savings to GDP	−0.833***	−1.296***	−0.985***
	(0.267)	(0.235)	(0.164)
Investment to GDP	−0.649***	−0.270	−0.525**
	(0.109)	(0.234)	(0.142)
Private Savings to GDP	−0.774***	−1.145***	−0.854***
	(0.133)	(0.162)	(0.169)
Public Savings to GDP	−0.044	−0.191	−0.140
	(0.149)	(0.131)	(0.095)

[a] Estimates of the parameter β_j in regression (1), with j corresponding to emerging and advanced economies in the first and second column, respectively. Standard errors in parentheses. Significantly different than zero at 99 (***), 95 (**), and 90 (*) percent confidence. The row header indicates the dependent variable Y_i in the individual regressions. Section A.1 describes the data sources and additional details.

Table II: Robustness Exercises. Relationship between Inequality, External Balance, Savings, Investment.

	Em. Baseline	Em. (A)	Ad. Baseline	Ad. (A)	EEA
Current Account to GDP	−0.175	0.127	−0.803***	−0.644***	−0.603***
	(0.132)	(0.127)	(0.142)	(0.105)	(0.101)
Trade Balance to GDP	−0.170	0.223	−1.026**	−0.779***	−0.781***
	(0.175)	(0.127)	(0.182)	(0.177)	(0.133)
Aggregate Savings to GDP	−0.833**	−0.131	−1.296***	−1.064**	−1.045****
	(0.267)	(0.184)	(0.235)	(0.390)	(0.360)
Investment to GDP	−0.649***	−0.354***	−0.270	−0.285	−0.263
	(0.109)	(0.094)	(0.234)	(0.271)	(0.270)
Private Savings to GDP	−0.774***	−0.436*	−1.145***	−1.009***	−1.033***
	(0.133)	(0.163)	(0.162)	(0.245)	(0.228)
Public Savings to GDP	−0.044	0.306***	−0.191	−0.057	−0.013
	(0.149)	(0.073)	(0.131)	(0.180)	(0.168)

[a] Estimates of the parameter β_j in regression (1), with j corresponding to alternative definitions of country groups. The first and fourth columns replicate the baseline estimates for emerging and advanced countries presented in Table I. The second and fourth column present estimates for emerging and advanced economies from an alternative specification (A) where the United States and China are excluded from the sample. The fifth column presents estimates for a sample comprising only countries belonging to the European Economic Area. Standard errors in parentheses. Significantly different than zero at 99 (***), 95 (**), and 90 (*) percent confidence. The row header indicates the dependent variable Y_i in the individual regressions. Section A.1 describes the data sources and additional details.

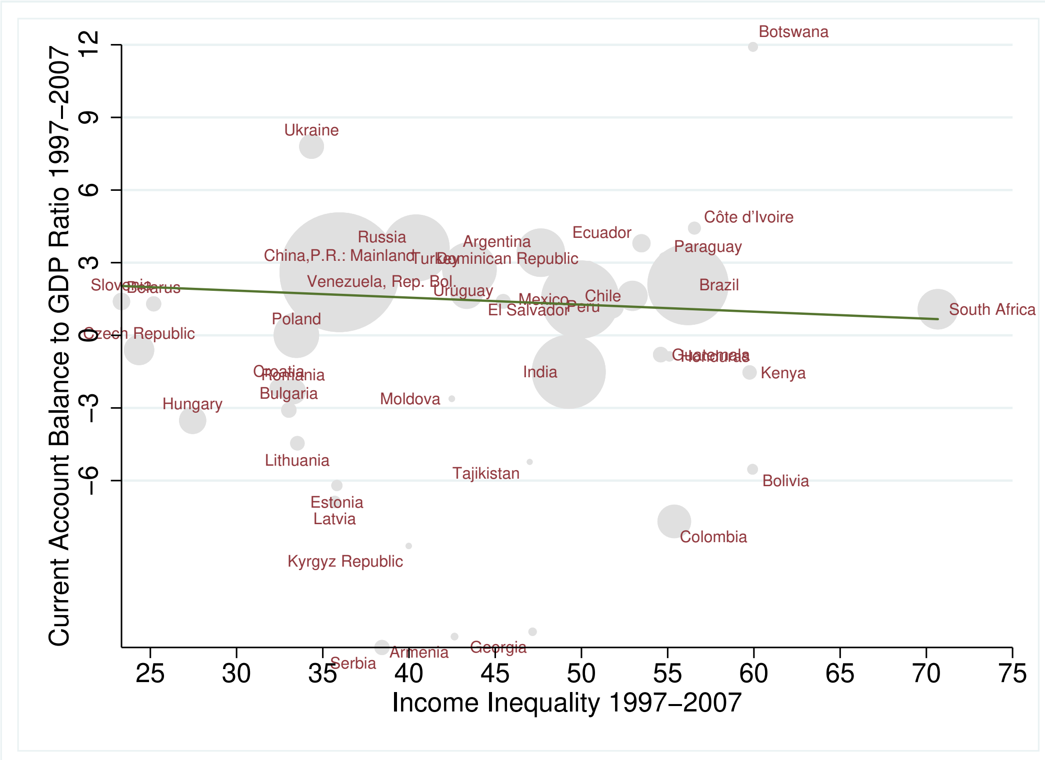

Figure 3: Relationship between income inequality and the current account balance in emerging economies, after controlling for other variables.

We estimate a weighted linear regression model of the current account balance-to-GDP ratio of individual economies against the Gini coefficient on net income and a vectors of controls that we allow to have a group specific coefficient.[3] Formally, we estimate the following regression for countries i in groups $j \in \{\text{Advanced, Emerging}\}$:

$$Y_i = \sum_j \mathbf{1}_{i \in j} \left\{ \alpha_j + \beta_j Gini_i + \gamma_j' X_i \right\} + \epsilon_i, \tag{1}$$

where X_i is the vector of controls with group-specific coefficient that includes measures of the demographic structure, the size of the government sector, GDP per capita, a continent fixed effect, and dummies for the reliance of exports on mineral resources or on fuel. For each country, all variables are included as their average in the period 1997-2007. The weight of each country in the regression is given by its GDP.

We present the results of this analysis graphically in Figures 1 and 3 and in the first row of Table I. In the figures, we plot the current account balance to GDP ratio against the Gini

[3]We present in Appendix A.1 a complete description of the data we use.

coefficient on net income in advanced and emerging economies, respectively, after controlling for all other variables in the regression (1). Advanced economies with higher inequality tend to run a larger current account deficit. This relationship seems to be absent in emerging economies, instead.The inclusion of inequality variables in the regression improves the fit substantially. Compared to a regression where the Gini coefficient is omitted from the set of independent variables, the adjusted R^2 of the main regression increases from 0.629 to 0.790 when considering this measure of income inequality.

Next, we aim to unpack the relationship between inequality and the current account balance by decomposing the drivers of the current account into savings and investment, and in particular between public and private savings. We repeat the same linear regression from (1) considering as dependent variable the ratio to GDP of the trade balance, aggregate savings and investment.

First, we verify that there is also a robust negative relationship of between income inequality and the trade balance-to-GDP ratio, of similar magnitude to that between the current account balance and income inequality. Note that the trade balance is defined as the difference between aggregate savings and investment and, in turn, aggregate savings can be decomposed in private and public savings:

$$TB = \underbrace{GDP - C - G}_{S} - I = S^{Private} + S^{Public} - I.$$

Hence, a lower trade balance may be the result of lower aggregate savings or of higher investment, or of both. We thus repeat the analysis considering as dependent variable the ratio to GDP of aggregate savings and investment. Table I presents the full set of estimates from these regressions for the parameter on income inequality, in the second, third and fourth row. In both emerging and advanced economies we find strong negative relationships between inequality and aggregate savings. We also find a negative correlation between inequality and investment, albeit of smaller magnitude and only statistically significant in the group of emerging economies. The negative effect of inequality on aggregate savings leads to a lower trade balance, whereas its the negative effect on investment would lead to a higher trade balance, implying the forces act in opposite directions. However, the negative relationship between inequality and savings is strong enough to more than entirely offset the effect on investment.

The strong negative relationship between inequality and aggregate saving is thus the key force driving the negative correlation between inequality and the current account in advanced economies.

We can further decompose the relationship between inequality and aggregate savings into public and private savings, to determine the extent to which differences in fiscal policy may be driving the results. We present these results in the two bottom rows of Table I. The relationship between inequality and aggregate savings is essentially entirely driven by private savings.

We perform a host of robustness exercises, whose main results we present in Table II. In particular, we verify that our results are not driven by the presence of the United States in the group of the advanced economies. The coefficient is smaller when excluding the U.S., but statistically the coefficients are not different. We also find that the relationship is present when restricting the analysis to countries in the European Economic Area (EEA), as we show graphically in Figure **??**. We find that China, a large, low-inequality and high-current account economy accounts for a substantial fraction of the correlation in the group of the emerging market economies. In addition, we find that our key results are robust when performing a pooled regression with all countries in the sample, rather than allowing for different coefficients in advanced and emerging economies. We also investigate the strength of the relationship between inequality and capital flows for different sample periods. We find that the magnitude of the key coefficient of interest, $\beta_{1,j}$ has grown over time, indicating that the relationship between inequality and the current account in advanced economies has grown stronger in recent years.[4]

3 Theory

We present in this section our theoretical framework, that we employ to investigate the economic forces behind the relationship between inequality and capital flows we document empirically. We present a simple model that allows for analytical results. In future research, we will develop a richer framework to conduct a quantitative investigation of the forces highlighted in the simpler model.

[4]We show in detail in Appendix A.3 our results with varying sample periods.

3.1 Model

We first present the environment of the model economy. We then illustrate a closed economy version of the model, which is helpful to clarify the building blocks of the open-economy setting. In particular, we present the closed-economy planner allocation and its decentralization. We then move on to the full open-economy equilibrium, both in the setting where the allocation is the outcome of an international planner's maximization, and in a decentralized setup.

Environment. Our model economy is closely related to the one in the simple model of Krueger and Perri (2006), building on the contributions of Kehoe and Levine (2001) and Alvarez and Jermann (2000). While the model there is one of a closed economy with a focus on planner allocations, we consider an open economy instead, and we also explicitly derive the decentralized allocation that corresponds to the planner's choice, with the corresponding borrowing constraints.

The world economy is composed of two countries, $i = 1, 2$. Time is discrete and indexed by $t = 1, 2, \ldots, \infty$. Each country is inhabited by two households of equal mass, indexed by j. There is a single, homogeneous consumption good, which can be traded across households in every country. All households receive in each period an endowment of consumption good $y_{j,i,t}$, whose realizations follow a two-point stochastic process. We denote the two endowment realizations as high (H) or low (L), $y_{H,i}$ and $y_{L,i}$, respectively and $y_{j,i,t} \in \{y_{H,i}, y_{L,i}\}$. The two possible values for the endowment are not necessarily the same in the two countries. Within each country, when one household receives high endowment, the other always receives the low endowment, and vice-versa: $y_{j,i,t} = y_{H,i} \leftrightarrow y_{j',i,t} = y_{L,i}$, with $j \neq j'$. It is thus convenient to label the two households in each country by the current level of their endowment, i.e. $j \in \{H, L\}$. We define the two endowment levels as $y_{H,i} = y(1 + \epsilon_i)$ and $y_{L,i} = y(1 - \epsilon_i)$,, with $\epsilon_1 \neq \epsilon_2$. Therefore, the value of ϵ_i captures the degree of income risk and income inequality in country i. In each country, the total endowment is equal to $y = \frac{1}{2}(y_{H,i} + y_{L,i})$. Without loss of generality, it is convenient to denote country 1 as the country with low income inequality, so that $\epsilon_1 < \epsilon_2$. The endowment follows a simple stochastic process characterized by positive persistence, and the parameter π captures the probability that a household's next-period endowment realization is equal to its current-period one.

Household preferences are given by the following lifetime utility function:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_{j,i,t}\right) \tag{2}$$

where β captures the subjective discount factor, $c_{j,i,t}$ is the level of consumption of household j in country i at time t and $u(\cdot)$ is the period utility function which takes the standard CRRA form.

Closed-Economy Planner Allocation. We consider the closed-economy allocation that results from a planner's maximization problem. In closed economy, no transfers take place across countries. We present this simpler allocation for explanatory purposes, and later move on to the full open-economy allocation.

The objective of the social planner is to maximize the sum of welfare of the two households in a given country, subject to two incentive compatibility constraints. Each household has the option to refuse to make transfers to the other household, and to revert to autarky. In a decentralized allocation, this refusal corresponds to a default on promises made in the past. When a household is relegated into autarky, it remains there indefinitely. Therefore, the social planner must offer households the same level of welfare that they would each attain in autarky, to prevent their default. Formally, the planner faces the incentive compatibility constraint that households' value must at least equal their value in autarky. As in Krueger and Perri (2006), we focus on symmetric allocations. In a symmetric allocation, consumption of the households with currently low and high income realizations can be expressed as:[5]

$$c_j = y(1 + \tilde{\epsilon}_j), \tag{3}$$

where $\tilde{\epsilon}_H = -\tilde{\epsilon}_L = \tilde{\epsilon}$ and $\tilde{\epsilon}$, captures the degree of consumption inequality or risk-sharing among households in the country.[6] The planner chooses $\tilde{\epsilon}$ and her problem is formally defined

[5]To simplify the notation, we ignore the country index i.

[6]This is the case because in closed economy the resource constraint $\sum_j c_j = y$ holds.

as follows:

$$
\begin{aligned}
&\max_{\tilde{\epsilon},\tilde{\epsilon}_H,\tilde{\epsilon}_L} \sum_{j\in\{H,L\}} V\left(\tilde{\epsilon}_j\right), \\
&\text{s.t. } V\left(\tilde{\epsilon}_j\right) \geq V_{j,AUT} \quad \forall j \in \{H,L\} \\
&\tilde{\epsilon}_H = -\tilde{\epsilon}_L = \tilde{\epsilon}.
\end{aligned}
\tag{4}
$$

$V\left(\tilde{\epsilon}_j\right)$ denotes the level of welfare attained by the household j which, in a symmetric allocation, is characterized as follows:

$$
V(\tilde{\epsilon}_j) = u\left(c_j\right) + \beta\left[\pi V(\tilde{\epsilon}_j) + (1-\pi)V(\tilde{\epsilon}_{-j})\right]. \tag{5}
$$

Imposing that $\tilde{\epsilon}_{-j} = -\tilde{\epsilon}_j$, the value function simplifies to:

$$
V(\tilde{\epsilon}_j) = \frac{1}{(1-\beta)(1+\beta-2\pi\beta)}\left[(1-\beta\pi)u\left(y(1+\tilde{\epsilon}_j)\right) + \beta(1-\pi)u\left(y(1-\tilde{\epsilon}_j)\right)\right]. \tag{6}
$$

Finally, the incentive-compatibility constraints impose that the planner must grant each household at least the level of welfare they would attain in autarky, captured by $V_{j,AUT}$.

Figure 4 graphically represents the household value function, for positive and negative values of its argument. The value function has a global maximum at $\tilde{\epsilon}_j = \epsilon^* > 0$. The value function is increasing for negative values of its argument. For the household with currently low income, the incentive-compatibility constraint does not bind if risk sharing is better than in autarky , i.e. for $\tilde{\epsilon}_L \in (\epsilon_L, 0]$. For the household with currently high income, it is convenient to define two specific levels of $\tilde{\epsilon}_j$. First, we define $\bar{\epsilon} > 0$ as the degree of risk-sharing that equates this household's value to the one it would attain under full risk-sharing: $V\left(\bar{\epsilon}\right) = V\left(0\right)$. Second, we define $\tilde{\epsilon}_{H,AUT} < \epsilon^*$ as the lowest degree of risk-sharing that equates this household's value to the one it would attain under autarky: $V\left(\tilde{\epsilon}_{H,AUT}\right) = V_{H,AUT}$.

The possible values of $\tilde{\epsilon}$ that the planner can choose while satisfying the incentive-compatibility constraints depend on whether ϵ_i falls in one of three regions. First, suppose the degree of income inequality is high, $\epsilon \geq \bar{\epsilon}$. If this is the case, the incentive compatibility constraints never bind for $\tilde{\epsilon} \in [0, \bar{\epsilon}]$. Hence the planner can deliver households the allocation with full risk-sharing, which is given by $\tilde{\epsilon} = 0$. This allocation corresponds to the unconstrained maximum of the planner's objective. Second, consider the case where income inequality falls

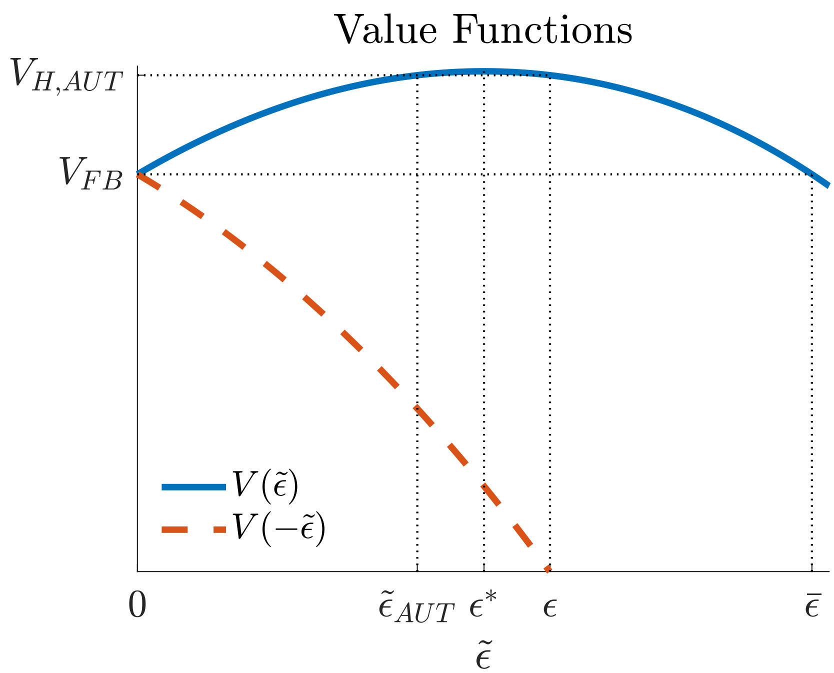

Figure 4: Household value functions defined in (5). The blue, solid line and the red, dashed line display the value function for positive and negative values of the argument $\tilde{\epsilon}$, respectively, imposing that $\tilde{\epsilon}_{-j} = -\tilde{\epsilon}_j$.

in the region of moderately high values $\epsilon \in (\epsilon^*, \bar{\epsilon})$. If this is the case, the planner cannot deliver the full risk-sharing allocation, since doing so would violate the incentive-compatibility constraint of the currently high-income household. However, in this setting, allocations with $\tilde{\epsilon} \in (\tilde{\epsilon}_{H,AUT}, \epsilon)$ are feasible and Pareto-dominate the autarkic allocation. The low-income household benefits from the greater degree of risk-sharing, and the high-income household also benefits from a reduction in the volatility of consumption. The allocation that maximizes the planner's objective is the one associated with the lowest $\tilde{\epsilon}$ that satisfies the two constraints, i.e. $\tilde{\epsilon} = \tilde{\epsilon}_{AUT}$. Finally, consider the case where income inequality is low, $\epsilon \in (0, \epsilon^*)$. In this region, the value function for the households with high and low income realizations are increasing and decreasing in $\tilde{\epsilon}$, respectively. Here, the planner cannot deliver Pareto improvements over the autarkic allocation: increasing welfare of one household would imply a reduction in welfare of the other below its autarky level, thereby violating one incentive-compatibility constraint. In this region, the planner must thus choose a degree of consumption inequality that equals that of income inequality $\tilde{\epsilon} = \epsilon$. Figure 5 graphically displays the planner's choice for

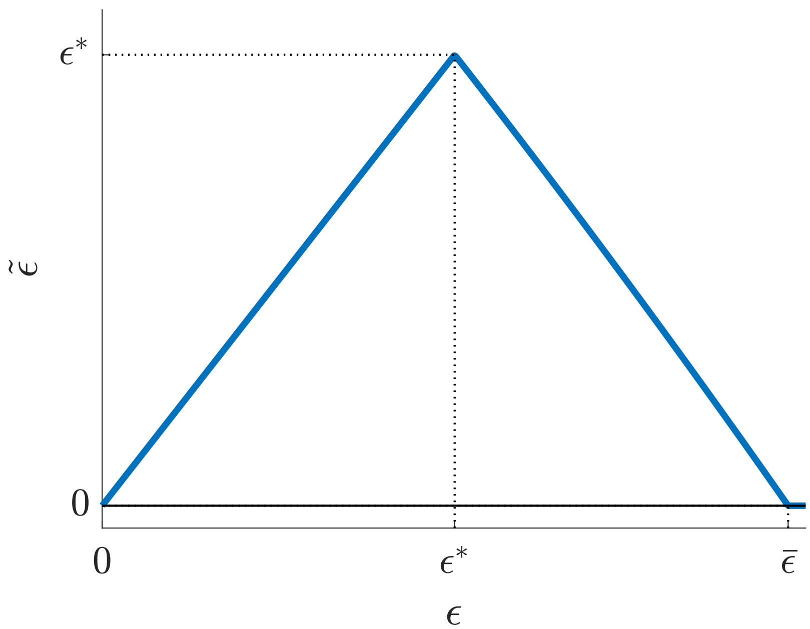

Figure 5: Solution of the closed-economy planner problem (4), formally given by (7). The blue, solid line represents the degree of consumption inequality $\tilde{\epsilon}$ that the planner chooses, as a function of the degree of income inequality ϵ. The planner cannot improve upon autarky for low degrees of inequality, $\epsilon \in [0, \epsilon^*)$, and it delivers a Pareto improvement when inequality is relatively high, $\epsilon \in (\epsilon^*, \bar{\epsilon})$. The planner implements the full risk-sharing allocation when inequality is sufficiently high: $\epsilon \geq \bar{\epsilon}$.

$\tilde{\epsilon}$ which we also formally summarize below:

$$\epsilon \begin{cases} \in [0, \epsilon^*) & \tilde{\epsilon} = \epsilon, \\ \in [\epsilon^*, \bar{\epsilon}) & \tilde{\epsilon} = \tilde{\epsilon}_{H,AUT}, \\ \geq \bar{\epsilon} & \tilde{\epsilon} = 0. \end{cases} \tag{7}$$

Closed-Economy Decentralized Allocation. We describe here a setting with decentralized trade in goods and assets. The equilibrium allocation in this decentralized setting is identical, in terms of consumption and welfare, to the planner allocation above described. In each period, households can issue state-contingent securities. These securities pay off one unit of consumption good in the following period, conditional on the realization of the household's own endowment shock. The budget constraint of a generic household j writes as:

$$c_j = y_j - b_j + q_{j,H} b'_{j,H} + q_{j,L} b'_{j,L}, \tag{8}$$

where we again omit the country index i for brevity and primes denote next period variables. b_j denotes the amount of claims issued in the previous period that pay off in the current one, $b'_{j,H}$ and $b'_{j,L}$ are the claims issued in the current period that pay off in the next one, conditional on the household's endowment being high or low, respectively, and $q_{j,H}$ and $q_{j,L}$ are the unit prices of these securities. In addition, households are subject to constraints, similar to borrowing constraints, that limit the amount of state contingent securities they can issue:

$$b'_{j,H} \leq \bar{b}_H \text{ and } b'_{j,L} \leq \bar{b}_L. \tag{9}$$

The borrowing constraints ensure that households never find it optimal, ex-post, to renege on promises made in the past.

The optimality conditions associated with the problem of a household with current endowment j are as follows:

$$\begin{aligned} u'(c_j)\, q_{j,j} &= \beta \pi u'(c_j) + \psi_{j,j}, \\ u'(c_j)\, q_{j,-j} &= \beta (1-\pi)\, u'(c_{-j}) + \psi_{j,-j}, \end{aligned} \tag{10}$$

where $\psi_{j,j}$ and $\psi_{j,-j}$ denote the Lagrange multipliers associated with the two constraints in (9).

The two households' endowment processes are perfectly negatively correlated. Hence, either both households' future endowments will be identical to the current ones, or neither will. Thus, securities issued by two households conditional on their endowment realization remaining unchanged (or changing) in the next period pay off in the same state of the world. In equilibrium, securities markets clear. It must then be the case that the amounts of securities issued by the two household conditional on their endowment realization remaining unchanged (or changing) must be the opposite of each other:

$$b'_{H,H} = -b'_{L,L} \text{ and } b'_{H,L} = -b'_{L,H}. \tag{11}$$

It must then also be the case in equilibrium that the prices of these securities are pair-wise equal:

$$q_{H,H} = q_{L,L} \text{ and } q_{H,L} = q_{L,H}. \tag{12}$$

Again, we focus on symmetric and stationary allocations. In a symmetric allocation, all households issue the same portfolio of state contingent securities, and thus in each period all households with a given endowment realization have the same level of wealth: $b'_{H,H} = b'_{L,H} = b_H$ and $b'_{H,L} = b'_{L,L} = b_L$. Hence, in equilibrium all households with the same endowment in the current period enjoy the same level of consumption: $y_j = y_H \leftrightarrow c_j = c_H$ and $y_j = y_L \leftrightarrow c_j = c_L$.

We solve for the equilibrium allocation through a guess-and-verify method. We guess that constraints on securities' issuance never bind for the high-endowment household: $\psi_{H,H} = \psi_{H,L} = 0$. Given this guess, and imposing the levels of consumption from the planner allocation, securities' prices must satisfy:

$$q_{H,H} = q_{L,L} = \beta\pi \text{ and } q_{H,L} = q_{L,H} = \beta(1-\pi)\frac{u'(y(1-\tilde{\epsilon}))}{u'(y(1+\tilde{\epsilon}))}. \tag{13}$$

Given these prices, the low-endowment household would like to borrow against her future high endowment realizations. However, she is prevented from doing so by the presence of the securities issuance constraint, (9). The constraint is binding in equilibrium if the allocation does not feature full risk-sharing: $\tilde{\epsilon} > 0 \rightarrow \psi_{L,H} > 0$.

We can explicitly solve for the asset positions chosen by the two households as a function of the exogenous income inequality ϵ and the endogenous consumption inequality $\tilde{\epsilon}$ that characterize the allocation:[7]

$$b_H = -b_L = \frac{\epsilon - \tilde{\epsilon}}{1 - q_{H,H} + q_{H,L}} = \frac{\epsilon - \tilde{\epsilon}}{1 - \beta\left[\pi - (1-\pi)\frac{u'(y(1-\tilde{\epsilon}))}{u'(y(1+\tilde{\epsilon}))}\right]}. \tag{14}$$

The expression above also characterizes the borrowing constraint $\bar{b}_H$, which only binds for the low-endowment household in allocations without full risk-sharing. The borrowing constraint $\bar{b}_L = 0$ never binds. Finally, the above prices and quantities verify the guess that the borrowing constraints do not bind for the high-endowment household. The above allocation is thus the equilibrium one, satisfying households' optimality conditions, budget constraints and market clearing conditions.

[7]These expressions follow from plugging prices into the budget constraints for the two households (8) and imposing market clearing conditions (11) and securities' prices (13).

Open-Economy Planner Allocation. We now consider an open economy setting where the planner can make transfers across the two countries that compose the world economy. The objective of the planner is now to maximize the sum of welfare of the four households that live in the two countries. The planner is again subject to incentive compatibility constraints, ensuring that all households prefer the allocation the planner chooses to simply consuming their stochastic income in autarky. The planner sets the amount of goods that one country transfers to the other in each period, and the degree of consumption risk. Importantly, we restrict the degree of relative consumption risk, $\tilde{\epsilon}$, to be the same across countries. This restriction allows the open-economy planner allocation to emerge as the outcome of a decentralized competitive equilibrium.[8] We denote by τ the fraction of income that households in the more unequal country 2 must transfer to households in the more equal country 1, as set by the planner. Hence, in a symmetric allocation, consumption of the households in the two countries is given by:

$$c_{j,i} = y(1 + \tilde{\epsilon}_j)(1 + \tau_i) \tag{15}$$

where, as before, $\tilde{\epsilon}_H = -\tilde{\epsilon}_L = \tilde{\epsilon}$ is the degree of consumption risk and the transfer across countries is given by $\tau_1 = -\tau_2 = \tau$.

The open-economy planner problem is defined as follows:

$$\begin{aligned} \max_{\tilde{\epsilon},\tilde{\epsilon}_H,\tilde{\epsilon}_L,\tau,\tau_1,\tau_2} \; & \sum_{i=1}^{2} \sum_{j\in\{H,L\}} V\left(\tilde{\epsilon}_j, \tau_i\right), \\ \text{s.t. } & V\left(\tilde{\epsilon}_j, \tau_i\right) \geq V_{j,AUT,i} \quad \forall j \in \{H, L\} \\ & \tilde{\epsilon}_H = -\tilde{\epsilon}_L = \tilde{\epsilon}, \\ & \tau_1 = -\tau_2 = \tau. \end{aligned} \tag{16}$$

With slight abuse of notation, the households' value functions are defined in a similar way to the closed economy setup:

$$V(\tilde{\epsilon}_j, \tau_i) = u\left(c_{j,i}\right) + \beta\left[\pi V(\tilde{\epsilon}_j, \tau_i) + (1-\pi)V(\tilde{\epsilon}_{-j}, \tau_i)\right]. \tag{17}$$

We restrict attention to the subset of the parameter space $\epsilon_1 > \tilde{\epsilon}_2$, where $\tilde{\epsilon}_2$ is defined as the degree of consumption risk that arises in the closed-economy planner allocation in country

[8] We characterize the decentralized competitive equilibrium in the following paragraph.

2.[9] In addition, we can ignore the incentive compatibility constraints of the low-endowment households in both countries, which never bind at an optimum.[10]

It is convenient to break down the solution to the open-economy planner problem into three regions. First, the unconstrained optimum for the open-economy planner is given by $\tilde{\epsilon} = \tau = 0$. This solution also amounts to the full problem's solution when the incentive compatibility constraints do not bind, as inequality is sufficiently high in both countries: $\epsilon_2 > \epsilon_1 > \bar{\epsilon}$. Second, for lower degrees of inequality, it may either be the case that the incentive compatibility constraint of the high-income household binds in both countries, or only in country 1.[11] In the former case, the solution to the planner problem is given by the unique couple of values $(\hat{\epsilon}, \hat{\tau})$ with $\hat{\epsilon}\, \epsilon^{\star}$ that simultaneously satisfies the incentive compatibility constraints of both countries' high-endowment households:[12]

$$V(\hat{\epsilon}, \hat{\tau}) = V_{H,AUT,1} \quad \text{and} \quad V(\hat{\epsilon}, -\hat{\tau}) = V_{H,AUT,2}. \tag{18}$$

Figure 6 graphically presents the intersection of the two incentive compatibility constraints that determines the solution for $(\hat{\epsilon}, \hat{\tau})$.

In the latter case, the solution to the planner problem is equivalent to the solution to a simplified problem.[13] In this simplified problem the incentive compatibility constraint in country 2 is omitted and the incentive constraint in country 1 holds with equality. We denote the solution to this simplified problem by the couple (ϵ_1^*, τ_1^*).

We now characterize the solution of the full problem. If the transfer $\tau_1^* > 0$ that solves the simplified problem is smaller than the transfer $\hat{\tau}$ for which both constraints bind, $\hat{\tau} > \tau_1^* > 0$, only the incentive compatibility constraint in country 1 binds and the planner sets $\tau = \tau_1^*$.

Intuitively, if the solution to the simplified problem features a relatively low transfer, $\tau_1^* < \hat{\tau}$, it must also feature a relatively high degree of consumption risk, $\epsilon_1^* > \hat{\epsilon}$ for the the incentive compatibility constraint of the high-endowment household in country 1 to hold.

[9] We show in Appendix B.1 that no solution to the planner problem exists outside of this parameter space. This condition implicitly limits our attention to cases where $\epsilon_2 > \epsilon^*$, as otherwise $\epsilon_1 < \epsilon_2$ and $\epsilon_1 > \tilde{\epsilon}_2$ could not both hold at the same time.

[10] We prove this result in Appendix B.5.

[11] We show in Appendix B.3 there that it is never the case that the high-income household incentive compatibility constraint binds in country 2, but not in country 1.

[12] We prove the existence and uniqueness of $(\hat{\epsilon}, \hat{\tau})$ in Appendix B.2.

[13] We formally present this simplified problem and its solution in Appendix B.4.3.

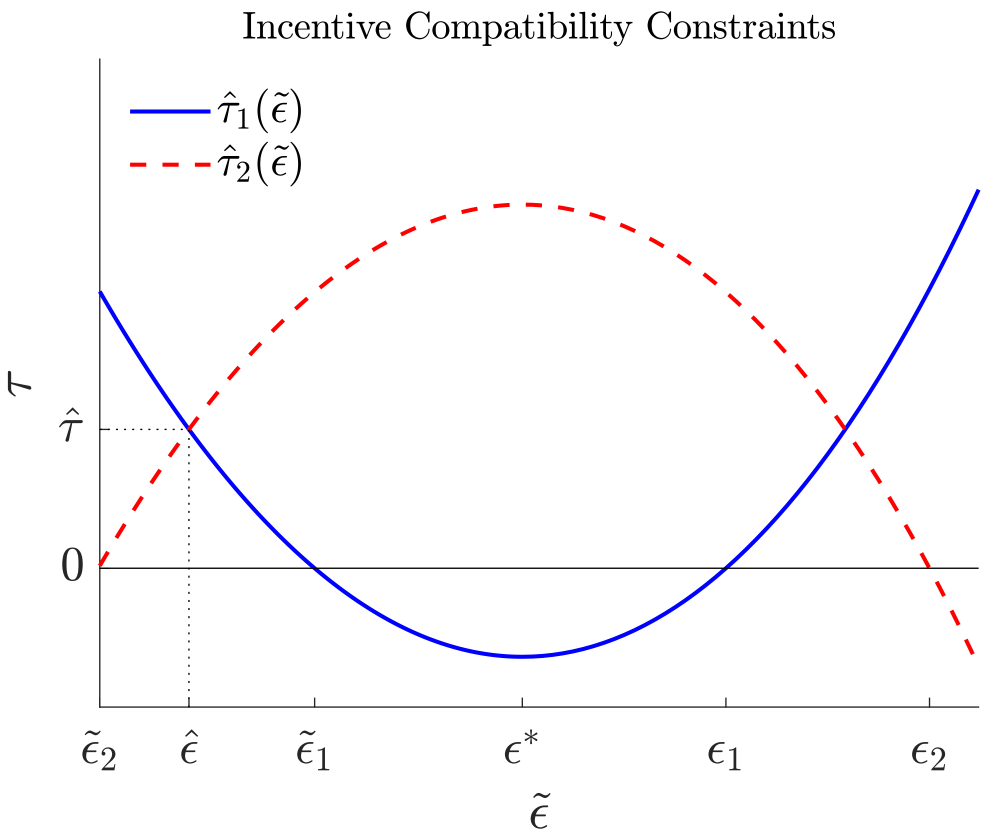

Figure 6: The blue, solid and red, dashed lines represent all allocations that give the high endowment households in countries 1 and 2,respectively, the same welfare as in autarky in the $(\tilde{\epsilon}, \tau)$ space. The two curves thus give a graphical representation of the incentive compatibility constraints in the open economy social planner problem (16). The constraint is satisfied for allocations North of the curve for the household in country 1 and South of the curve for the household in 2. The intersection $(\hat{\epsilon}, \hat{\tau})$ of the two curves represents the allocation with $\hat{\epsilon} < \epsilon^\star$ for which both constraints bind with equality.

Both the low transfer and the high degree of consumption inequality are beneficial to the high-endowment household in country 2, implying that she is better off than in autarky and that her incentive compatibility constraint does not bind.

If, instead, the solution to the simplified problem features a high transfer $\tau_1^* > \hat{\tau} > 0$ and a low degree of consumption risk $\epsilon_1^* < \hat{\epsilon}$, it would violate the incentive compatibility constraint in country 2. Thus, the simplified problem solution cannot solve the full planner problem. The open economy planner solution hence amounts to $\tau = \hat{\tau}$. If the minimum of τ_1^* and $\hat{\tau}$ is negative, no incentive compatibility constraint binds and the planner can set $\tau = 0$ and obtain the unconstrained optimum. Formally, the solution for τ to the full planner problem writes as:

$$\tau = \max\left\{\min\left\{\tau_1^*, \hat{\tau}\right\}, 0\right\}. \tag{19}$$

The planner solution for $\tilde{\epsilon}$ is given by ϵ_1^* if $\tau = \tau_1^*$, by $\hat{\epsilon}$ if $\tau = \hat{\tau}$, and by 0 if $\tau = 0$. Note that the open-economy planner solution for the degree of consumption risk falls in between the closed-economy solutions in the two countries: $\tilde{\epsilon} \in [\tilde{\epsilon}_1, \tilde{\epsilon}_2]$.

Figure 7 graphically presents the solution to the open economy planner allocation. The figure displays the incentive compatibility constraint for the high endowment households in country 1 and in country 2, for two different autarky values of the latter. Hence, the figure characterizes two different pairs of values $(\hat{\epsilon}, \hat{\tau})$. In addition, the figure also displays the set of allocations that satisfy the first-order optimality conditions of the planner's simplified problem, characterizing its solution (ϵ_1^*, τ_1^*) as the intersection of this curve with the incentive compatibility constraint of country 1. The solution to the full planner problem is thus given by the intersection of the two constraints if the constraint in country 2 is tight and it is instead given by the solution to the simplified planner problem if the constraint in country 2 is slack.

The above results show that the solution to the open economy planner allocation is characterized by a positive transfer from the unequal to the equal country and by a degree of consumption risk in between the one that emerges in closed economy in the two countries. The transfer between the two countries can be interpreted, in a stationary allocation, as interest payments on previously incurred debt made by the unequal country to the equal country. The stationary open economy allocation thus describes the outcome of the opening of international financial markets, starting from a world composed of closed economies that differ in terms of income inequality. When financial markets open, capital flows from the equal to the unequal country, as the return from saving is higher in the latter country than in the former. Eventually, the unequal country accumulates enough liabilities that it transfer a positive net amount of resources to the equal country. The next paragraph characterizes the international asset and liability position that emerge in a decentralized allocation, corresponding to the open economy planner allocation described here.

Open-Economy Decentralized Allocation We describe here the open-economy version of the allocation with decentralized trade in goods and assets. The discussion is closely related to the one for the closed-economy decentralized allocation. Again, the open-economy decentralized allocation replicates the planner one in terms of consumption and welfare, as well as capital flows. Households in all countries face a budget constraint as in (8) and borrowing constraints as in (9) so that the optimality conditions associated with their problem are again given by (10).

The key difference of the open-economy decentralized allocation with respect to the closed-economy one, is that markets clear at the world level, rather than country by country. Hence, market clearing conditions are now given by:

$$\sum_{i=1,2} \left(b'_{H,H,i} + b'_{L,L,i}\right) = 0 \text{ and } \sum_{i=1,2} \left(b'_{H,L,i} + b'_{L,H,i}\right) = 0. \tag{20}$$

As in the closed economy, (13), the price of securities issued by households conditional on their endowment realization changing (or remaining unchanged) must be equal across households and also across countries:

$$q_{H,H,1} = q_{L,L,1} = q_{H,H,2} = q_{L,L,2} \text{ and } q_{H,L,1} = q_{L,H,1} = q_{H,L,2} = q_{L,H,2}. \tag{21}$$

We solve again for the equilibrium symmetric and stationary allocation with a guess-and-verify method. In each country, all households with the same endowment realization have the same level of wealth in a symmetric allocation: $b'_{H,H,i} = b'_{L,H,i} = b_{H,i}$. Hence, in each country, all households with the same endowment have the same consumption level: $y_{j,i} = y_{H,i} \leftrightarrow c_{j,i} = c_{H,i}$ and $y_{j,i} = y_{L,i} \leftrightarrow c_{j,i} = c_{L,i}$.

We again impose the guess that constraints on securities issuance only bind, if at all, for

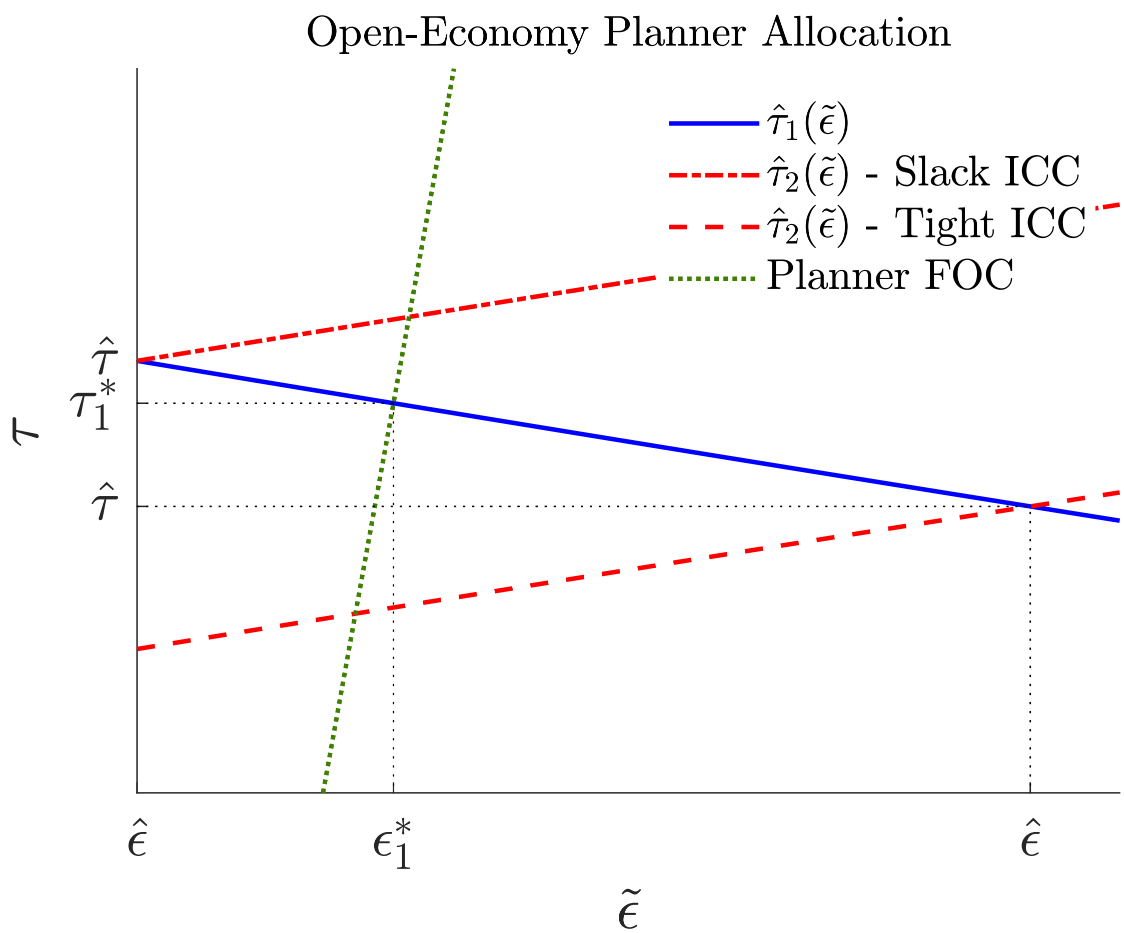

Figure 7: The blue, solid line represents the incentive compatibility constraint of the high endowment household in country 1. The red dashed and dashed-dotted lines represent the same constraint in country 2. The two curves are drawn for two different values of $V_{H,AUT,2}$ implying a tight and a slack constraint, respectively. The two allocations $(\hat{\epsilon}, \hat{\tau})$ denote the two intersections of the incentive compatibility constraint in country 1 with the two possible values of the constraint in country 2. The green, dotted line represents the set of allocations that satisfy the first-order necessary conditions of the modified planner problem (24) imposing only the incentive compatibility constraint of country 1. The intersection (ϵ_1^*, τ_1^*) is the solution to the modified planner problem. The solution to the full planner problem for τ is given by the minimum between τ_1^* and $\hat{\tau}$. The solution for $\tilde{\epsilon}$ is accordingly given by either ϵ_1^* or by $\hat{\epsilon}$.

low-endowment households. Given this guess and imposing again that consumption levels are the same as in the open-economy planner allocation we obtain once more that securities' prices are given by (13). This condition is crucial to highlight that the degree of consumption risk-sharing $\tilde{\epsilon}$ must be the same across countries in the open economy for the price of securities $q_{H,L,i}$ and $q_{L,H,i}$ to also be the same across countries.

Given securities' prices, it is now possible to solve for the vector of households' asset positions that corresponds to the planner allocation. Countries' net external debt positions satisfy:

$$b_{H,1} + b_{L,1} = -2y\frac{\tau}{1-\tilde{q}} = -b_{H,2} - b_{L,2}, \tag{22}$$

where $\tilde{q} = q_{H,H} + q_{H,L}$ denotes the price of a risk-free security. Country 1, which receives a positive transfer $2y\tau$ in the social planner allocation, is a net holder of external assets in the decentralized allocation. The international transfer corresponds to the interest payments on the country's asset held abroad. Finally, the asset positions of the individual households are as follows:

$$\begin{aligned} b_{H,1} &= \frac{y}{1 - q_{H,H} + q_{H,L}}\left[\epsilon - \tilde{\epsilon} - \tau\left(1 + \tilde{\epsilon} + 2\frac{q_{H,L}}{1-\tilde{q}}\right)\right], \\ b_{L,1} &= \frac{y}{1 - q_{H,H} + q_{H,L}}\left[\tilde{\epsilon} - \epsilon - \tau\left(1 - \tilde{\epsilon} + 2\frac{q_{H,L}}{1-\tilde{q}}\right)\right]. \end{aligned} \tag{23}$$

The asset positions closely resemble those in the closed economy, with the addition of a term that reflects the dispersion in wealth positions across countries. Specifically, households in the low-inequality country 1 are systematically richer than households in the high-inequality country 2, reflecting the positive external asset position of the former country.

Intuitively, the fact that the low-inequality country has positive external wealth vis-à-vis the high inequality country follows directly from the results in the closed economy model on borrowing constraints and interest rates. In countries with low inequality, borrowing constraints are tight and, in closed economy, interest rates are low. The reverse is true in the high-inequality country, where the sharp fluctuations in income ensure that the value of autarky is low and thus larger debt positions are sustainable, leading in turn to high interest rates in closed economy. When financial markets of the two countries integrate, savers in the low inequality country buy some of the debt issued by borrowers in the high-inequality country, until the two countries' interest rates (or securities' prices) converge to a unique world interest rate. Hence, capital flows from equal to unequal countries, and residents of the former accumulate claims against residents of the latter. Eventually, the world economy reaches a steady state (the stationary allocation just described) where residents of the low-inequality country hold wealth against residents of the high-inequality country. Hence, in each period residents of the unequal country make a transfer of resources, equivalent to interest payments on external wealth, to residents of the equal country.

4 Conclusions

We document novel empirical relationships: in a cross-section of advanced economies, the ratio of the current account balance to GDP and inequality are negatively correlated. The negative relationship between savings and inequality is the main driver of this correlation. We demonstrate that low private savings in countries with high inequality lie at the heart of this result.

We show that a model with endogenously incomplete domestic markets can rationalize these findings. In countries with high income inequality higher levels of debt can be sustained, since the value of autarky is lower. The opposite is true in countries with low income inequality. Consequently, the level of income inequality in a country is positively correlated with the supply of assets and the interest rate in that country. When international financial markets open, to clear the world financial market at a comment interest rate, capital flows from (previously) low-interest rate countries to high-interest rate ones. Equivalently, capital flow from equal to unequal countries.

The model we presented in this paper is an analytical one. The next logical step is to write a quantitative model that can be rigorously disciplined by micro data to assess the strength of the effect of inequality on capital flows. The quantitative model would also help us to understand spillover effects of redistributive policy across countries.

References

Aguiar, M. and M. Bils (2015): "Has Consumption Inequality Mirrored Income Inequality?" *American Economic Review*, 105, 2725–56.

Alvarez, F. and U. J. Jermann (2000): "Efficiency, equilibrium, and asset pricing with risk of default," *Econometrica*, 68, 775–797.

Broer, T. (2014): "Domestic or global imbalances? Rising income risk and the fall in the US current account," *Journal of Monetary Economics*, 64, 47 – 67.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas (2008): "An equilibrium model of" global imbalances" and low interest rates," *American economic review*, 98, 358–93.

de Ferra, S., K. Mitman, and F. Romei (2020): "Household heterogeneity and the transmission of foreign shocks," *Journal of International Economics*, 103303.

Hines Jr, J. R. (2010): "Treasure islands," *Journal of Economic Perspectives*, 24, 103–26.

Kehoe, T. J. and D. K. Levine (2001): "Liquidity constrained markets versus debt constrained markets," *Econometrica*, 69, 575–598.

Krueger, D. and F. Perri (2006): "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory," *Review of Economic Studies*, 73, 163–193.

Lane, P. R. and G. M. Milesi-Ferretti (2007): "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004," *Journal of international Economics*, 73, 223–250.

Lucas, R. (1990): "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review*, 80, 92–96.

Mendoza, E. G., V. Quadrini, and J.-V. Rios-Rull (2009): "Financial integration, financial development, and global imbalances," *Journal of Political economy*, 117, 371–416.

Ranciere, R., M. N. A. Throckmorton, M. M. Kumhof, M. C. Lebarz, and M. A. W. Richter (2012): *Income inequality and current account imbalances*, 12-18, International Monetary Fund.

UNCTAD (2019): "The State of Commodity Dependence 2019," Tech. rep., United Nations conference on trade and development.

Uribe, M. and S. Schmitt-Grohé (2017): *Open economy macroeconomics*, Princeton University Press.

A Appendix to the Empirical Analysis

A.1 Data

We collect data on countries' external balance and income inequality from various sources. Data on income inequality are drawn from the World Income Inequality Database released by UNU-WIDER. The Gini index for net income is the measure of income inequality we use in our main specification. We exclude from the sample of countries tax havens, according to the definition in Hines Jr (2010), due to the difficulty in interpreting data involving these jurisdictions and to the large magnitude of the flows involved. We categorize countries as advanced or emerging economies. We use the classification of advanced economies from the IMF World Economic Outlook of 1999. We classify as emerging market economies all other countries for which we have data and that are not classified as least developed countries by UNCTAD. Data on income inequality from UNU-WIDER are largely unavailable for least developed countries.[14] We draw data for the current account balance and for national account variables from the dataset collected by Uribe and Schmitt-Grohé (2017). We use data on saving by the public sector from the World Economic Outlook.[15] We compute saving by the private sector residually, as the difference between aggregate saving and saving by the public sector. Moreover, we control for the old-age dependency ratio using data from the UN World Population Prospects 2019.[16] We construct dummies for countries' reliance of exports on mineral commodities and on fuel, using the classification in UNCTAD (2019). Finally, we collect data for countries' GDP in US dollars from the dataset of Lane and Milesi-Ferretti (2007), which we use to weight individual countries in the regressions described in the next section.

A.2 List of Countries

The advanced economies included in our sample are the following ones: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Israel, Italy, Japan, Korea, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Taiwan, United King-

[14] We provide in Appendix A.2 below the full list of countries and their classification.

[15] We use the series on General Government Primary Net Borrowing and Lending.

[16] Following Eurostat, we define this variable as the ratio of the population aged 65 and above to the population between 15 and 64 years of age.

dom.

The emerging economies included in our sample are the following ones: Argentina, Armenia Belarus, Bolivia, Botswana, Brazil, Bulgaria, Chile, Colombia, Croatia, Czech Republic, Côte d'Ivoire, Dominican Republic, Ecuador, El Salvador, Estonia, Georgia, Guatemala, Honduras, Hungary, India, Kenya, Kyrgyz Republic, Latvia, Lithuania, Macedonia, Mexico, Moldova, Nicaragua, Paraguay, Peru, Poland, Romania, Russia, Serbia, Slovenia, South Africa, Tajikistan, Turkey, Ukraine, Uruguay, Uzbekistan, Venezuela.

The total number of countries is 65.

A.3 Robustness Exercises

We discuss here the results of the empirical analysis conducted when extending the time sample considered beyond 1997-2007. Figures 8 9 graphically present the results for the two subsamples of advanced and emerging economies, respectively. The two figures present coefficient estimates and 99% confidence bands for the parameter $\beta_{1,j}$, in a rolling window of regressions over ten-year samples whose initial year is reported on the horizontal axis. In the four regressions of each figure the dependent variables are given respectively by the ratios to GDP of the trade balance (top-left panel), aggregate savings (top-right panel), investment (bottom-left panel) and private savings (bottom-right panel). Results appear to be broadly robust over the whole sample considered, strengthening as time progresses.

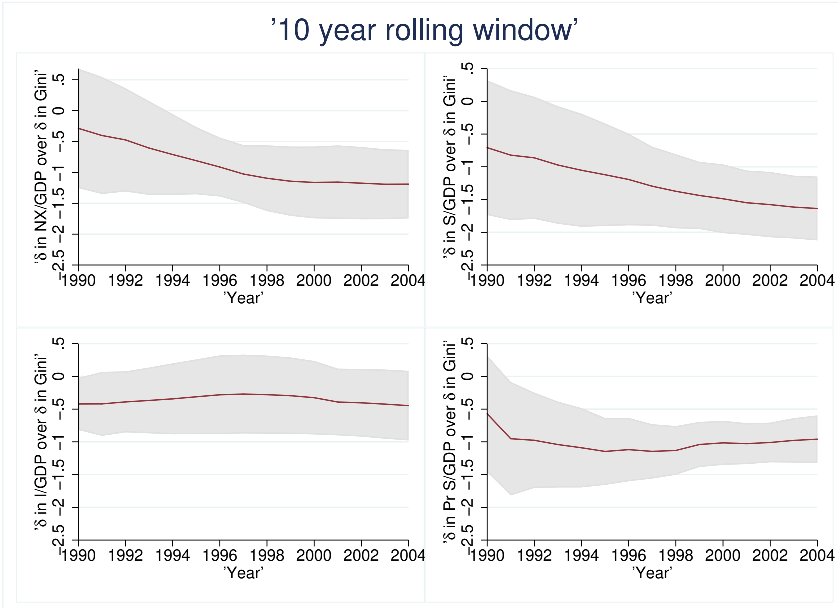

Figure 8: Rolling window for Advanced countries

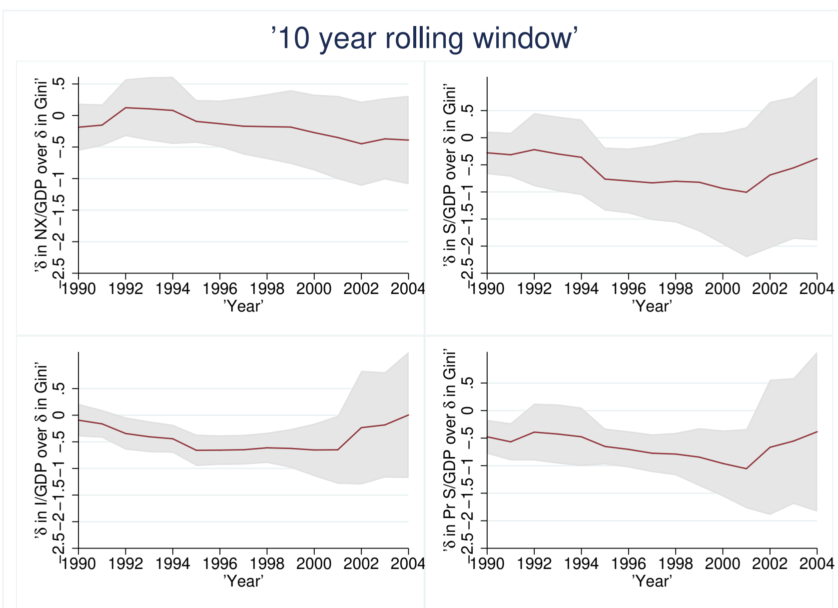

Figure 9: Rolling window for Emerging economies

Figure 10: Correlation between income inequality and financial development

B Appendix to the Open Economy Planner Problem

We present here proofs and additional material that supplement the presentation of the solution to the open-economy planner problem. In particular, we show here that:

1. no solution to the problem exists if there are too large differences in income inequality across the two countries: $\epsilon_1 < \tilde{\epsilon_2}$;

2. there exists an unique allocation $(\hat{\epsilon}, \hat{\tau})$ for which the high-endowment incentive compatibility constraints bind in both countries and $\hat{\epsilon} < \epsilon^*$;

3. it is never the case that the incentive compatibility constraint of the high-endowment household binds in country 2 but not in country 1;

4. the solution to the full planner problem may be given by the solution to a simplified problem with fewer constraints, under some conditions;

5. the incentive compatibility constraints of the low-endowment households never bind in equilibrium.

B.1 No solution with too large differences in income inequality: $\epsilon_1 < \tilde{\epsilon_2} < \epsilon^*$

We show that no solution to the open economy planner problem exists if there are too large differences in income inequality across the two countries: $\epsilon_1 < \tilde{\epsilon_2}$. First, if $\epsilon_1 < \epsilon^*$ no solution with $\tilde{\epsilon} \neq \epsilon_1$ satisfies the two incentive compatibility constraints in country 1. This result follows because in this region of values for $\tilde{\epsilon}$ the incentive constraints of the two households in country 1 are increasing and decreasing in $\tilde{\epsilon}$, so that any value $\tilde{\epsilon} \neq \epsilon_1$ violates either of two constraints. This argument is identical to the one made in the closed economy allocation, for the case where $\epsilon \in (0, \epsilon^*)$.

Second, we show that $\tilde{\epsilon} = \epsilon_1$ does not satisfy the two incentive compatibility constraints in country 2. $\tilde{\epsilon_2}$ is the lowest value of $\tilde{\epsilon}$ that satisfies the incentive compatibility constraint of the high endowment household in country 2. Hence, $\epsilon_1 < \tilde{\epsilon_2}$ implies that $\tilde{\epsilon} = \epsilon_1$ cannot be a solution, as it would violate this household's incentive compatibility constraint. Hence, no solution to the open economy planner problem exists when $\epsilon_1 < \tilde{\epsilon_2} < \epsilon^*$. ∎

B.2 There exists an unique allocation $(\hat{\epsilon}, \hat{\tau})$ for which the high-endowment incentive compatibility constraints bind in both countries and $\hat{\epsilon} < \epsilon^*$

Define $(\hat{\epsilon}_i, \hat{\tau}_i)$ as the locus of values for $(\tilde{\epsilon}, \tau)$ that equate the value of the currently high income household in each country i to its autarky value. The locus $(\hat{\epsilon}_1, \hat{\tau}_1)$ is decreasing in the $(\tilde{\epsilon}, \tau)$ space for $\tilde{\epsilon} < \epsilon^*$, since the value function of the currently high income household in country 1 is increasing in both of its arguments. Conversely, the locus $(\hat{\epsilon}_2, \hat{\tau}_2)$ is increasing in the $(\tilde{\epsilon}, \tau)$ space for $\tilde{\epsilon} < \epsilon^*$. The two loci each intersect the $\tau = 0$ axis twice, at $\hat{\epsilon}_i = \epsilon_i$ and $\hat{\epsilon}_i = \tilde{\epsilon}_i$. For the intersection between the two loci to exist in the space where $(\hat{\epsilon}, \hat{\tau}) > 0$ it must be the case that $\tilde{\epsilon}_1 > \tilde{\epsilon}_2$. This condition holds when $\epsilon_1 \in (\tilde{\epsilon}_2, \epsilon_2)$, as assumed. Hence, the intersection of the two loci $(\hat{\epsilon}, \hat{\tau})$ exists and it is unique for $\tilde{\epsilon} < \epsilon^*$ given the monotonicity of the two loci in the $(\tilde{\epsilon} < \epsilon^*, \tau)$ space. ∎

Figure 6 graphically presents this setting.

B.3 Incentive compatibility constraint never binds in country 2 alone

We show here that it is never the case that, at the solution to the open economy planner problem (16), the Incentive compatibility constraint of the high-endowment household binds in country 2 and not in country 1.

Suppose there exists an allocation $(\tilde{\epsilon}, \tau)$ where the high-endowment household incentive compatibility constraint binds in country 2 but not in country 1. First, we show that this allocation must feature $\tilde{\epsilon} > \hat{\epsilon}$ and $\tau > \hat{\tau}$, where $\hat{\epsilon}, \hat{\tau}$ are the ones characterized in appendix B.2. To see this, suppose instead that $\tau < \hat{\tau}$. It must then be the case that $\tilde{\epsilon} < \hat{\epsilon}$ for the high-endowment incentive constraint in country 2 to bind. However, the high-endowment incentive constraint in country 1 is not satisfied for $\tilde{\epsilon} < \hat{\epsilon}$ and $\tau < \hat{\tau}$ which contradicts the hypothesis that this constraint does not bind.

Second, we can show that an allocation with $\tilde{\epsilon} > \hat{\epsilon}$ and $\tau > \hat{\tau}$ is never optimal for the open-economy planner. This follows because, by concavity of the utility function, the two sums $V(\tilde{\epsilon}, \tau) + V(-\tilde{\epsilon}, \tau)$ and $V(\tilde{\epsilon}, -\tau) + V(-\tilde{\epsilon}, -\tau)$ which form the objective of the planner are both decreasing in $\tilde{\epsilon}$ and, similarly, the two sums $V(\tilde{\epsilon}, \tau) + (\tilde{\epsilon}, -\tau)$ and $V(-\tilde{\epsilon}, \tau) + (-\tilde{\epsilon}, -\tau)$ are both decreasing in τ. Hence, it is always optimal for the planner to reduce both $\tilde{\epsilon}$ and τ if feasible while satisfying the incentive compatibility constraints. Hence, it is never the case that the high-endowment household incentive compatibility constraint only binds in country 2 at an optimum. ∎

B.4 Simplified planner problem

We describe here a simplified version of the full open economy planner problem defined in (16). In this simplified problem the incentive compatibility constraint of the high-endowment household in country 1 holds with equality and the the incentive compatibility constraint of the high-endowment household in country is omitted, as are both low-endowment households' constraint.

First, we define the problem. Second, we discuss properties of its solution. Third, we show under what conditions its solution amounts to the solution of the full planner problem. Fourth, we show under what condition for the simplified problem's solution the first best allocation solves the full planner problem.

B.4.1 Definition of simplified planner problem

The simplified problem is defined as follows:

$$\begin{aligned} &\max_{\tilde{\epsilon},\tau} W(\tilde{\epsilon},\tau) = V\left(\tilde{\epsilon},\tau\right) + V\left(-\tilde{\epsilon},\tau\right) + V\left(\tilde{\epsilon},-\tau\right) + V\left(-\tilde{\epsilon},-\tau\right), \\ &\text{s.t. } V\left(\tilde{\epsilon},\tau\right) = V_{1,AUT,H}. \end{aligned} \tag{24}$$

B.4.2 Solution of simplified planner problem

We prove here that the solution of the simplified planner problem can be characterized as the solution of two loci. One locus is the incentive compatibility constraint of the high-endowment household in country 1. The other locus represents the first-order optimality conditions of the problem, and we show it to be increasing in the space $(\tilde{\epsilon}, \tau)$ and including the point $(0, 0)$.

The solution to the simplified problem is characterized by the solution of a system of three equations in three unknowns. The three unknowns are $\tilde{\epsilon}$, τ and a lagrange multiplier λ. The three equations are given by the two first-order necessary conditions and the incentive compatibility constraint:

$$\begin{aligned} W_\tau - \lambda V_\tau &= 0, \\ W_{\tilde{\epsilon}} - \lambda V_{\tilde{\epsilon}} &= 0, \\ V\left(\tilde{\epsilon},\tau\right) &= V_{1,AUT,H}, \end{aligned} \tag{25}$$

where W_x and V_x denote, respectively, the partial derivatives of the planner objective and of the household value function with respect to the variable x, which is given by either τ or $\tilde{\epsilon}$.

The third of the three equations above defines the locus of points in the space $(\tilde{\epsilon}, \tau)$ that represents the incentive compatibility constraint of the high-endowment household in country 1. The subsystem of the first two equations gives rise to another locus, which we now show to be increasing in the same space and including the point $(0, 0)$.

The two first-order necessary conditions can be summarized by the following equation:

$$\frac{W_{\tilde{\epsilon}}}{W_\tau} = \frac{\lambda V_{\tilde{\epsilon}}}{V_\tau}. \tag{26}$$

Under the assumption of CRRA utility, it is possible to collect terms containing either τ or $\tilde{\epsilon}$

on the two sides of the equation, as follows:

$$LHS\left(\tau\right)=RHS\left(\tilde{\epsilon}\right), \tag{27}$$

where, respectively:

$$\begin{aligned}LHS\left(\tau\right)&=\frac{1}{1+\tau}\frac{\left(1+\tau\right)^{-\gamma}-\left(1-\tau\right)^{-\gamma}}{\left(1+\tau\right)^{1-\gamma}+\left(1-\tau\right)^{1-\gamma}}\\ RHS\left(\tilde{\epsilon}\right)&=\frac{\left(1-\beta\pi\right)\left(1+\tilde{\epsilon}\right)^{1-\gamma}+\left(\beta-\beta\pi\right)\left(1-\tilde{\epsilon}\right)^{1-\gamma}}{\left(1+\tilde{\epsilon}\right)^{1-\gamma}+\left(1-\tilde{\epsilon}\right)^{1-\gamma}}\frac{\left(1+\tilde{\epsilon}\right)^{-\gamma}-\left(1-\tilde{\epsilon}\right)^{-\gamma}}{\left(1-\beta\pi\right)\left(1+\tilde{\epsilon}\right)^{-\gamma}-\left(\beta-\beta\pi\right)\left(1-\tilde{\epsilon}\right)^{-\gamma}},\end{aligned} \tag{28}$$

and γ is the coefficient of relative risk aversion.

Both $LHS\left(\tau\right)$ and $RHS\left(\tilde{\epsilon}\right)$ are decreasing in their own arguments. Hence, the locus of points that characterizes the solution to this equation is increasing in the $\left(\tilde{\epsilon},\tau\right)$ space. We provide details on both sides of the equation below. First, it is easier to see that $LHS'\left(\tau\right)$ for $\tau>0$ when rewriting the expression as:

$$LHS\left(\tau\right)=\frac{1-\left(\frac{1+\tau}{1-\tau}\right)^{\gamma}}{\left(1+\tau\right)^{2}\left[1+\left(\frac{1+\tau}{1-\tau}\right)^{\gamma-1}\right]}, \tag{29}$$

whose numerator and denominator are respectively decreasing and increasing in τ.

Second, rewrite $RHS\left(\tilde{\epsilon}\right)=f\left(\tilde{\epsilon}\right)g\left(\tilde{\epsilon}\right)$, where the two functions f and g denote the two fractions in the expression in (28). The two functions are, respectively, positive and negative, and both are (weakly) decreasing. Hence, for $RHS'\left(\tilde{\epsilon}\right)<0$, it must be the case that

$$\frac{f'\left(\tilde{\epsilon}\right)}{f\left(\tilde{\epsilon}\right)}>-\frac{g'\left(\tilde{\epsilon}\right)}{g\left(\tilde{\epsilon}\right)}>0. \tag{30}$$

To show that this is the case, it is convenient to define two additional auxiliary functions, $z\left(\tilde{\epsilon}\right)=\left(\frac{1+\tilde{\epsilon}}{1-\tilde{\epsilon}}\right)^{\gamma}$, $x\left(\tilde{\epsilon}\right)=\left(\frac{1+\tilde{\epsilon}}{1-\tilde{\epsilon}}\right)^{\gamma-1}$, and the combination of parameters $\omega=\frac{\beta-\beta\pi}{1-\beta\pi}\in(0,1)$. We can thus rewrite:

$$\frac{f'}{f}=\frac{\left(\omega-1\right)z'}{\left(1-z\right)\left(1-\omega z\right)}\text{ and }\frac{g'}{g}=\frac{\left(\omega-1\right)x'}{\left(1+x\right)\left(1+\omega x\right)}. \tag{31}$$

Noticing that $\frac{z'}{x'}=\frac{\gamma}{\gamma-1}\frac{z}{x}$, we can rewrite the condition (30) as

$$\gamma\frac{z}{\left(1-z\right)\left(1-\omega z\right)}<\left(1-\gamma\right)\frac{x}{\left(1+x\right)\left(1+\omega x\right)}\text{ or }\frac{\gamma}{1-\gamma}<\frac{x}{z}\frac{1-z}{1+x}\frac{1-\omega z}{1+\omega x}. \tag{32}$$

For $\gamma > 1$, $\frac{\gamma}{1-\gamma} < -1$. The above condition is thus satisfied if $\frac{x}{z}\frac{1-z}{1+x}\frac{1-\omega z}{1+\omega x} > -1$. This is the case because $\frac{x}{1+x}, \frac{z-1}{z}, \frac{1-\omega z}{1+\omega x}$ all fall in the interval $(0, 1)$.

B.4.3 Simplified problem solution solves full problem only if $\tau_1^* < \hat{\tau}$.

Suppose the solution to the simplified problem $\tau_1^* > \hat{\tau}$. Since the high-endowment incentive constraint in country 1 binds, it must be that at the solution to the simplified problem $\tilde{\epsilon} < \hat{\epsilon}$. If this is the case, the high-endowment incentive constraint in country 2 is not satisfied, which would contradict the hypothesis that the solution to the simplified problem also solves the full open-economy planner problem (16). ∎

Figure 7 provides a graphical representation of this setting.

B.4.4 If the minimum of τ_1^* and $\hat{\tau} > 0$ is negative, the problem allows for the first-best allocation as a solution

We have shown in Appendix B.2 that it is always the case that $\hat{\tau} > 0$. Hence, consider the case where $\min\{\tau_1^*, \hat{\tau}\} = \tau_1^* < 0$. We show here that this solution implies $\tilde{\epsilon} < 0$.

We proceed by contradiction. Suppose that a couple $(\tilde{\epsilon} > 0, \tau_1^* < 0)$ solves the planner problem. If this were the case, it would be possible to choose a different couple with smaller $\tilde{\epsilon}$ and larger (but smaller in absolute value) τ while satisfying the incentive constraint of the high endowment household in country 1. This different couple would yield a higher value to the planner, as it is easy to show that the planner value is increasing in τ for negative values of it, and decreasing in $\tilde{\epsilon}$ for positive values of it.[17] We have thus reached a contradiction and a $(\tilde{\epsilon} > 0, \tau_1^* < 0)$ cannot solve the planner problem.

Suppose now that a couple $(\tilde{\epsilon} < 0, \tau_1^* < 0)$ satisfies the incentive constraint of the high endowment household in country 1. Then it must be the case that $(0, 0)$ strictly satisfies the constraint, too. The latter allocation is the unconstrained optimum of the planner's objective, and thus is the solution to the planner's problem in this setting. ∎

[17] The argument is similar to the one made in Appendix B.3, as reducing the absolute values of both τ and $\tilde{\epsilon}$ improves risk-sharing both across and within countries.

B.5 Incentive compatibility constraints of low-endowment households do not bind

We show here that in the open economy planner allocation, the incentive compatibility constraints of the low-endowment households do not bind.

We have shown above that the solution to the open-economy planner problem always features $\tau \geq 0$. If $\tau = 0$, the solution also features $\tilde{\epsilon} = 0$. The value of the full risk-sharing allocation is higher than the autarky one for all low-endowment households, so their incentive compatibility constraints never bind at this solution.

If $\tau > 0$, the incentive compatibility constraint of country 1's high-endowment household always holds with equality. It must then be the case that $\tilde{\epsilon} < \tilde{\epsilon}_1$, or the high-endowment household would be better off than in autarky. Hence, the low endowment household of country 1 is better off than in the closed-economy allocation, benefiting from the positive τ and the better degree of risk sharing (lower $\tilde{\epsilon}$). This implies that low endowment household of country 1 is better off than in autarky and its incentive compatibility constraint does not bind.

For the low-endowment household in country 2, we make use of the assumption that preferences are CRRA. Under this assumption, the ratio of values $\frac{V(-\tilde{\epsilon},\tau)}{V(\tilde{\epsilon},\tau)}$ is invariant in τ and increasing in $\tilde{\epsilon} < \epsilon^*$. Hence, if $\tilde{\epsilon} < \epsilon_2$,

$$\frac{V\left(-\tilde{\epsilon},\tau\right)}{V\left(\tilde{\epsilon},\tau\right)} > \frac{V_{L,AUT,2}}{V_{H,AUT,2}} \tag{33}$$

The above implies that if the incentive compatibility constraint in country 2 is satisfied for the high-endowment household, it is also satisfied for the low-endowment household, i.e.

$$V\left(\tilde{\epsilon},\tau\right) \geq V_{H,AUT,2} \rightarrow V\left(-\tilde{\epsilon},\tau\right) \geq V_{L,AUT,2} \quad \blacksquare \tag{34}$$