

# SPATIAL ADVERTISEMENT IN POLITICAL CAMPAIGNS\*

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## Abstract

This paper characterizes the optimal advertising strategy of candidates in an election campaign, where groups of heterogeneous voters are targeted through media outlets. We discuss its effects on the implemented policy and connect it to polarization. It is well known that polarization has increased in the past decades. Additionally, we empirically establish that polarization displays electoral cycles. These cycles can emerge as candidates find it optimal to cater to different groups of voters and thus to adjust policies. Further, recent developments in campaign advertising, such as targeting voters more precisely, tend to increase polarization. Finally, we show that even greater spillovers of the campaign among voters can increase polarization.

**Keywords:** Mobilization, Media, Networks, Voting

**JEL Classification:** D85, D72, D83

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# 1 Introduction

Election campaigns increasingly invest in targeting certain subsets of voters via different media outlets. An article titled "Political Ads Take Targeting to the Next Level" in the Wall Street Journal highlights how this was done in the re-election campaign of Chris Christie, the governor of New Jersey, who wanted to reach Hispanic voters:<sup>1</sup>

"[it was] *discovered that viewers of "Dama y Obrero", a Spanish-language telenovela about a woman torn between two men, would likely be more receptive to his message than people who watch "Porque el Amor Manda," a romantic comedy.*"

This emphasizes that candidates carefully consider which groups of voters to target, in which media outlets to advertise their platforms.<sup>2</sup> At the same time, there has been a dramatic increase in *polarization*. [Poole and Rosenthal \(2000\)](#) show that Democrats and Republicans tend to favor very different policies, both in the House of Representatives as well as in Senate.

We therefore seek to connect the increase in polarization to the advertising strategy of candidates, who tailor their policy platforms to the voters they target. We empirically establish that polarization increases and that it displays electoral cycles, with parties alternatively catering to swing voters or partisans. In order to explain these patterns, we develop a model of advertising with heterogeneous voters. Targeted voters learn the policy platform, whereas non-targeted voters cast their vote based on a prior belief. Candidates cater to voters who are a priori least likely to turn out on their behalf, that is voters whose ideal policy has the greatest distance to the prior. If this prior belief is affected by past policies, it is optimal to target different sets of voters in each election, resulting in electoral cycles in polarization. Importantly, our framework cannot only help understand these cycles, but also the overall trend in polarization. Voters are targeted through media outlets and thus candidates are constrained in their advertising by the media network they face. In line with recent developments in the media landscape, we allow for advertising to become more precise. Candidates can target voters more specifically and tailor their policy to a more narrow subset of voters. This induces candidates to choose more extreme

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<sup>1</sup>See the Wall Street Journal article on <http://www.wsj.com/articles/political-ads-take-targeting-to-the-next-level-1405381606>

<sup>2</sup> An important goal of advertising is to provide information and [Freedman et al. \(2004\)](#) shows that campaigns are indeed crucial in informing voters.

policies, leading to an increase in polarization. Finally, we show that even if there are greater spillovers of the campaign among voters, polarization can increase.

To establish the trend and cycles in polarization, we use the DW-NOMINATE data set, created by [Poole and Rosenthal \(2000\)](#). It provides a measure of parties' policy positions the U.S. Senate and House of Representatives for each Congress. Based on these positions, we calculate the level of polarization, which is defined as the difference between the parties' policies and we establish that polarization has increased sharply since the 1970s. We show that, after removing the time trend, polarization exhibits electoral cycles. This implies that Democratic and Republican positions are closer together for one presidential election, only to be more divisive in the next election followed again by relatively more moderate policies.

In order to explain both the trend and cycles in polarization, we develop a model of targeting in networks with heterogeneous agents. In our model, two candidates compete to win an election. Both candidates are purely office-motivated and maximize their chance of winning by simultaneously selecting a policy platform they commit to as well as an advertising, or targeting, strategy. Candidates target voters through media outlets. That is, they advertise their platform in certain media outlets and voters that follow these outlets receive information about a candidate's platform. A media network describes which voters can access a given outlet, where we allow for voters to belong to multiple media outlet, see [Figure 1](#) as an example.

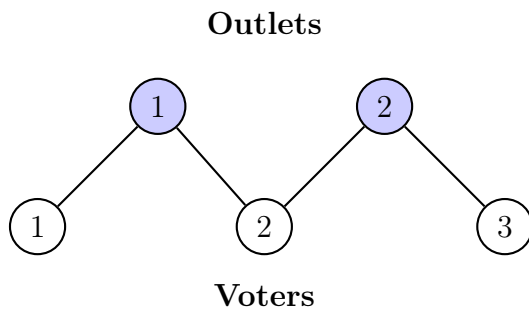


Figure 1: 3 Groups of Voters & 2 Outlets

Voters differ in what policy they prefer. That is, they have heterogeneous bliss points. A voter decides for a candidate according to a standard probabilistic voting model based on what he believes the candidates' platforms are. A voter is more likely to select the candidate whose platform he believes to be closest to his bliss point. Initially, every voter has a prior about the

platform each candidate will implement. If a voter is connected to an outlet which is targeted by a candidate, then he learns what the true policy platform of this candidate is. Otherwise, the voter sticks to his prior.

As voters differ in what policies they prefer, audiences of media outlets can be heterogeneous as well. Additionally, we allow for media outlets to differ in how many voters they reach. We show that candidates are more likely to target an outlet if it has a large audience. However, not only the size of the audience, but also its policy preference influences the choice of outlet. In particular, candidates are interested in targeting voters, whose bliss point is very different from the prior voters have about the candidate's choice. To see this, note that it is never optimal to cater to a group of voters that believes that the party implements their preferred policy. By targeting them, and thus selecting a policy that matches their position, the probability of this group to vote for the candidate remains the same as if the party did not disclose to them. However, selecting voters whose bliss point is very different from the prior they have yields an increase in the probability of voting for the candidate. By selecting these voters and a policy that coincides with their preferred policy, parties can increase the number of their supporters. The choice of such a policy does not alienate the voters whose bliss point is close to the prior as they do not learn about the true policy and therefore stick to the prior, based on which they cast the vote. Advertising thus allows candidates to increase the number of supporting voters, without losing those already in favor of the party. We take these considerations together: both the number of voters reached, as well as, their bliss points matter for the optimal target set.

Based on this we propose a new measure of centrality, which we refer to as *media centrality*. Media centrality captures the trade off between voters' bliss points and the number of voters targeted in a very simple way. This is, to the best of our knowledge, the first measure that explicitly highlights the trade off between an agent's characteristics, here their bliss points, and their position in the network.

We then extend our model to a dynamic setting, where we allow for the prior belief to change in each period. More precisely, we assume that the beliefs of voters regarding the candidate's platform adjust adaptively. If voters do not receive any information about a candidate's platform, they assume that it is going to be the same as last period's policy. We show that a candidate's platform changes with each election. A candidate caters to the voters whose preferred policy

has the greatest distance from last period's policy (controlling for the size of the audience). This leads to electoral cycles, a result that matches the electoral cycles we established in the data. We calculate for a given cycle the difference in policy platforms, that is the level of polarization.

This allows us to connect recent developments in the media networks to the policy set. We first focus on the increase in the number of media outlets. This has made it easier for voters to find programs that match their interests. As demographic characteristics predict viewership as well as voting behavior, it is now feasible to target a certain set of voters specifically. We show that such a change in the media network generally leads to an increase in polarization. Additionally, it has become easier for voters to be connected to a higher number of media outlets. In particular the internet has created many spillovers. Voters, although not directly targeted, might be able to observe a campaign ad because a friend posts a link on Twitter or Facebook, or a blog reports about it. In this sense, voters are connected to a higher number of media outlets than they have been previously, that is they have a higher number of links to various outlets. We show that this development can also lead to an increase in polarization. Thus, recent developments in the media landscape can be seen to have contributed to the rise in polarization.

**Related Literature** This paper contributes to various strands of literature, which we discuss in turn.

*Targeting in Networks* First, the paper contributes to the literature of targeting in networks. [Galeotti and Goyal \(2009\)](#), [Dziubiński and Goyal \(2013\)](#) and [Goyal and Vigier \(2013, 2014\)](#) analyze the targeting strategy of a monopolist, whereas [Goyal and Kearns \(2014\)](#) show how two competitors target a network of homogeneous consumers. [Groll and Prummer \(2015\)](#) show how two competitors target networks with heterogeneous nodes, but their set up does not lend itself to the analysis of arbitrary networks. Our work is also related to [Galeotti and Mattozzi \(2011\)](#), who consider a network of voters and are interested in how the network and homophily affects information dissemination by parties as well as polarization. Different from our model, they consider a model of word of mouth communication. Polarization follows from the assumption that parties are policy motivated and prefer extreme policies to more moderate ones. Political networks are also at the heart of the analysis in [Murphy and Shleifer \(2004\)](#). They consider a setting in which politicians use the network of voters to persuade them to support their policies.

The opinions of voters diverge depending on in which social community they are. Their setting does not allow for overlap between different social communities, which drives the divergence. [Bimpikis et al. \(2013\)](#) consider competition over networks in which firms advertise their products. They show that the most central agents are the most attractive targets, and highlight that there is a trade-off between attitude towards a firm and centrality, without being able to give a specific measure. Different from our model, their set up is one of opinion formation.<sup>3</sup>

*Advertising of Policy Platforms* Uncertainty about platforms motivates the literature of political advertising.<sup>4</sup> [Austen-Smith \(1987\)](#) assumes that voters face uncertainty about the true position of the party. This induces the party to advertise their position. However, if voters were rational, then this uncertainty would vanish. [Schultz \(2007\)](#) also discusses political advertising. Parties only inform a subset of voters about their platform and so in equilibrium there are informed and uninformed voters that differ in their decision, similar to our setting. [Callander and Wilkie \(2007\)](#) allow for candidates to lie when announcing policy platforms and they build on [Banks \(1990\)](#), where voters infer from the announced policy platform, which is a costly signal, what the true preferred platform of candidates is. They point out that the literature on election campaigns assumes either one of two extremes: either campaign promises are binding, or they are entirely meaningless. Our approach follows the first strand of literature, but allows for campaigns to only matter for a subset of voters, namely those who are informed.<sup>5</sup>

*Media* Our work also contributes to the literature on media and polarization. Different from much of this literature, see for example [Mullainathan and Shleifer \(2005\)](#), [Baron \(2006\)](#), [Gentzkow and Shapiro \(2006\)](#), our focus is not on the decision of the media to slant information.<sup>6</sup> Media

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<sup>3</sup>Work that deals explicitly with microtargeting, without targeting in networks, is [Schipper and Woo \(2014\)](#).

<sup>4</sup>A key assumption in our model is that there are voters that are not perfectly informed about the parties' platforms. That this can be optimal has been shown in [Alesina and Cukierman \(1990\)](#), where choosing higher ambiguity about the true preferences is beneficial. A similar assumption has been made in [Alesina and Holden \(2008\)](#), where parties do not know the true median and therefore choose different policy positions. [Aragones and Postlewaite \(2002\)](#) also provide a framework in which ambiguity emerges. In equilibrium voters do not know which policy will be implemented. In related work, [Aragones and Postlewaite \(2000\)](#) show that if candidates have reputation concerns, they can credibly commit to a policy platform which is believed by the candidates.

<sup>5</sup>[Coate \(2004\)](#) is complementary to [Schultz \(2007\)](#) in that it does not ask who is targeted for a given budget, but rather what the optimal budget for targeting is. In this setting voters again have different beliefs about the party, depending on whether information is revealed to them or not.

<sup>6</sup>Other work on media slant includes [Duggan and Martinelli \(2011\)](#) and [Chan and Suen \(2008\)](#).

is not an active player in our setting, rather, we take the media and its viewers as fixed.<sup>7</sup> We assume that politicians do not have the means to influence the voters' decisions about the newspapers they read or the TV programs they watch. Instead, our paper assumes that voters differ in their access to media. However, there is an impact of the different perceptions of candidates and parties on voter turnout. Empirically, [DellaVigna and Kaplan \(2007\)](#), [Gerber et al. \(2009\)](#), [Gentzkow \(2006\)](#), [Strömberg \(2004\)](#), [Gentzkow and Shapiro \(2004\)](#), [Gentzkow et al. \(2006\)](#), [Gentzkow et al. \(2011\)](#) and [Campante and Hojman \(2013\)](#) show that media exposure has an impact on voting behavior. Theoretically, [Bernhardt et al. \(2008\)](#) start with the observation that beliefs about political facts differ significantly between liberals and conservatives, implying that they get news from different outlets. These differences in information impact the voting decision, despite voters being rational to some extent. In particular, voters know that media are biased and update rationally, but cannot recover the full information. This implies that there is also no full information aggregation, which ultimately leads to polarization. [Gul and Pesendorfer \(2011\)](#) aim to connect increased media competition to higher polarization. They assume that voters prefer media that is in agreement with their views and that political information is only a byproduct of media consumption. If there is only one media outlet, then this media outlet remains neutral, which induces parties to choose the same position. As the number of media outlets increases, viewers select into media outlets according to their ideology, which allows the media to endorse candidates that choose more extreme position. Therefore, it becomes optimal for policy-motivated candidates to choose policies closer to their bliss points. Our work is also related to [Glaeser et al. \(2005\)](#). They assume that voters are divided into supporters of the left-wing and right-wing party. Supporters are more informed about the policy position of the party they support compared to that of the other party, which leads the parties to choose policies to the left and right of the median. In equilibrium, voters have correct beliefs. This model is used to explain why the Republican and Democratic party have become increasingly polarized along religious dimensions.

*Polarization* Additionally, our work aims to explain the increase in polarization. It is well-established that there has been an increase in elite polarization, see [McCarty et al. \(2006\)](#). They

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<sup>7</sup>This is different from [Garcia-Jimeno and Yildirim \(2015\)](#) who discuss the strategic interaction between politicians and media outlets.

show that polarization of the House of Representatives, as well as the Senate, has increased. Whether there has also been an increase in polarization in the electorate has been disputed, with [Abramowitz and Saunders \(2008\)](#), [Abramowitz and Stone \(2006\)](#) arguing that there has been an increase and [Fiorina and Abrams \(2008\)](#), [Fiorina et al. \(2008\)](#), [Fiorina \(2005\)](#), [Fiorina and Levendusky \(2006\)](#) stating that there has not. For an overview on polarization, its causes and consequences, see [Layman et al. \(2006\)](#). Our paper also contributes to the strand of literature that generates party polarization without policy motivated candidates. In [Glaeser et al. \(2005\)](#) this is due to supporters being better informed. In our model this is created by special interest groups, who always know the policy of the party they support.<sup>8</sup>

The remainder of this paper is structured as follows. We first describe the policy patterns we aim to explain in Section 2 and present our model of targeting in Section 3. We show what the equilibrium policy and the optimal targeting strategy are in Section 4. We connect the media network to the optimal policy in Section 5 and analyze trends and cycles in polarization in Section 6. Section 7 concludes. All proofs are collected in Appendix A.

## 2 Trends and Cycles in Polarization

We first analyze how polarization has changed over time. To do so, we use the DW-NOMINATE data, collected by Poole and Rosenthal based on roll call voting in U.S. Congress.<sup>9</sup>

**Data Description** Poole and Rosenthal collected data on roll-call voting in the House of Representatives and Senate from 1879 to 2013.<sup>10</sup> Their findings are summarized in [Poole and](#)

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<sup>8</sup>Other papers that generate party polarization without policy motivated candidates are [Palfrey \(1984\)](#), [Grosser and Palfrey \(2014\)](#) who allows for entry of a third party. In [Callander \(2005\)](#) and [Eyster and Kittsteiner \(2007\)](#) parties face heterogeneous districts. Similar to our setting, in [Eyster and Kittsteiner \(2007\)](#) voters are also not fully rational as they are not aware of the discrepancy between the party's position and the candidate's platform. In [Bernhardt et al. \(2007\)](#), [Bernhardt et al. \(2009\)](#) candidates have access to private polls, which induces them to choose different positions leading to polarization. [Snyder \(1994\)](#) assumes that parties are collections of self-motivated office holders who benefit from distinguishable platforms.

<sup>9</sup>For a detailed description of the data see Keith Poole's website [voteview.com](http://voteview.com).

<sup>10</sup>Roll-call votes show for each legislator whether he votes "yea" or "nay" on a given bill.



Rosenthal (2000). They use multidimensional scaling techniques in order to project the roll-call voting data on a one- or two-dimensional space. They assume that politicians have symmetric and single-peaked utility functions that are centered around an ideal point. Further, politicians vote probabilistically and assign a higher probability to their preferred outcome. Poole and Rosenthal perform a maximum likelihood estimation, simultaneously estimating the points of the roll-call votes and the ideal points of the politicians on the one- or two-dimensional space. The parameters that emerge from this estimation are those that make the real, observed roll-call votes as likely as possible. Poole and Rosenthal show that the one-dimensional policy space explains most of the votes and they argue that this dimension is the left-right, socio-economic dimension.<sup>11</sup>

The optimal parameters from the maximum likelihood estimation are then placed on a scale from -1 to 1, with -1 being liberal and 1 being conservative. Ideologically similar politicians are placed close to each other, ideologically different politicians are set further apart. Based on this, we can then calculate the average ideological position of each party over time and the difference in positions gives a measure of polarization. As the scale is bounded between -1 and 1, polarization must lie between 0 and 2. If parties' positions do not differ, polarization is zero.

**Polarization** We first show that there has been an increase in polarization, see Figure 2. The level of polarization is measured every second year, for each Congress. Figure 2 highlights that there has been an increasing divergence in the parties' policy positions both in the House of Representatives as well as in the Senate. Polarization was at an all time low after World War II but has steadily increased since then. In particular in recent years the level of polarization has increased steeply.

We are additionally interested in the the cyclical component of polarization. We therefore use an HP filter to de-trend the measure of polarization.<sup>12</sup> The cyclical components of polarization

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<sup>11</sup>More precisely, Poole and Rosenthal show that the one-dimensional model was correct 83% of the time. This means that the legislator's bliss point was closer to the outcome he voted for than to the outcome he did not vote for (both as estimated by DW-NOMINATE). The two-dimensional model is correct 85% of the time, higher dimensional models do not lead to an improvement. The second dimension is a regional and social dimension.

<sup>12</sup>We use an HP filter with a smoothing parameter of 6.25. The parameter of 6.25 was suggested for yearly data by Ravn and Uhlig (2002). To the best of our knowledge there does not exist a benchmark smoothing parameter for biannual data. We therefore report the results for a smoothing parameter of 6.25, but have used a

Figure 2: Polarization

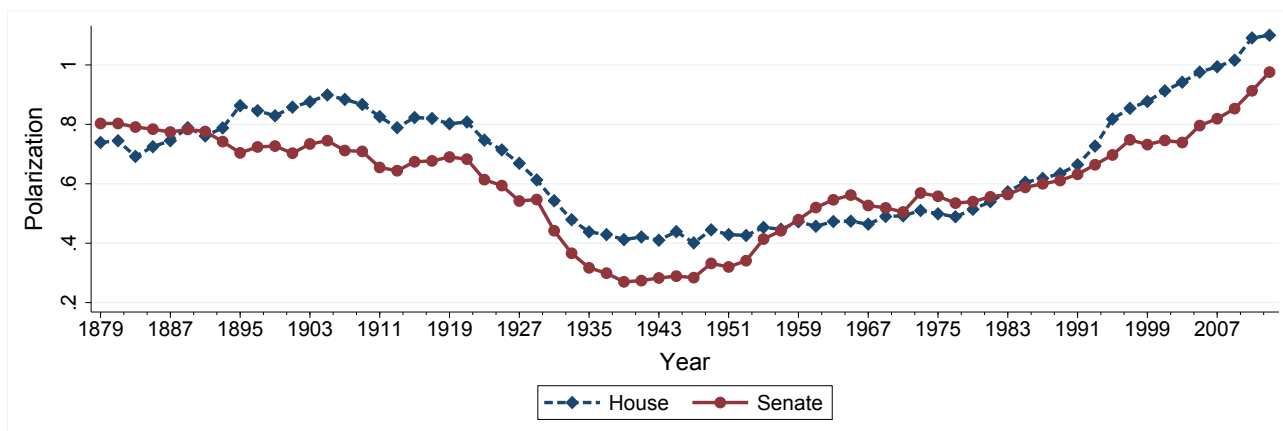
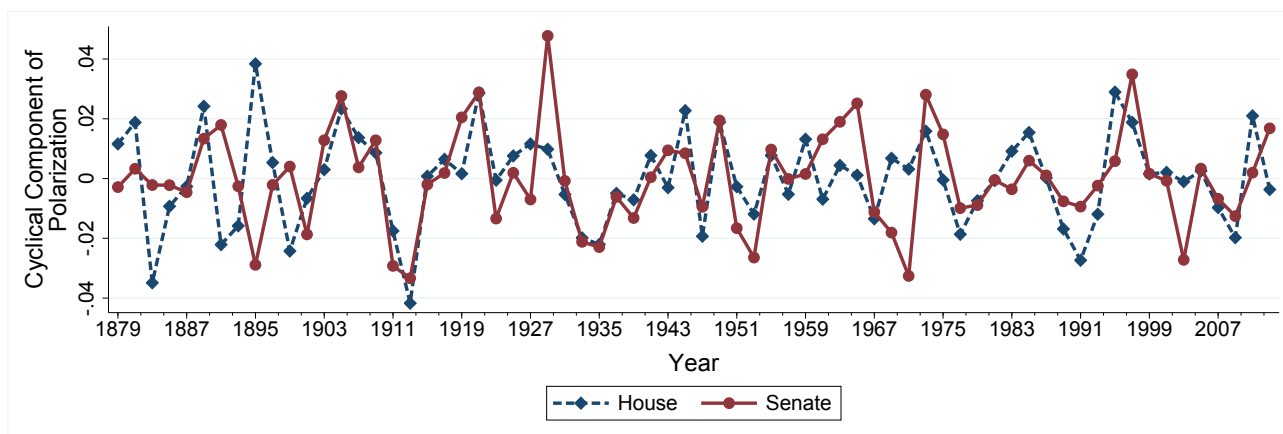


Figure 3: Cyclical Component of Polarization



Note: Cyclical component of polarization from Poole and Rosenthal's measure of polarization obtained by using an HP-filter with smoothing parameter 6.25.

for the Senate and House are given in Figure 3. It can be seen that the difference in parties' platforms fluctuates around the trend. In order to analyze this pattern in more detail we estimate various model specifications and select the best model based on the information criteria (AIC and BIC). The results are given in Table 1, which highlights that polarization is negatively correlated with the second lag. This implies that Democratic and Republican positions are closer together

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variety of smoothing parameters, both larger and smaller that have not yielded any different results. One reason for sticking to 6.25 are also the problems with too small of a smoothing parameter as pointed out by [Harvey and Trimbur \(2008\)](#). They show that choosing the parameter too small results in standard errors that are too small and that trends can wrongly absorb cycles.

Table 1: Cyclical Components of Polarization

	Senate	House of Representatives
1st Lag	0.840*** (0.132)	1.414*** (0.0927)
2nd Lag	-0.412*** (0.116)	-0.636*** (0.104)
Observations	68	68

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$ , standard errors in parentheses

**Note:** Cyclical Components in polarization from Poole and Rosenthal’s DW-NOMINATE scores of first dimension obtained by using an HP-filter with smoothing parameter 6.25. Processes selected according to BIC and AIC were an ARMA (2,1) for the Senate and an ARMA (2,2) for the House of Representatives.

for one presidential election, only to be more divisive in the next election followed again by relatively more moderate policies.<sup>13</sup>

We have thus established that polarization has increased from a value of roughly .6 in the 1970s to a level of above 1, but that it has not done so smoothly, rather there are fluctuations around the trend. These fluctuations in a given year can explain up to 5% of the deviation from the trend.<sup>14</sup> Based on these observations, we aim to develop a model that can help understand both the trend and cycles in polarization. We argue that a possible explanation for these patterns is political advertising that affects implemented policies.

### 3 Model

We introduce a model of probabilistic voting on a network. Two candidates,  $A$  and  $B$ , maximize the probability of being elected by choosing strategically a policy platform and how to advertise

<sup>13</sup>We show that these difference in polarization are indeed caused by parties selecting different platforms in Appendix B. Both Democrats and Republicans in the Senate and the House choose policies that exhibit electoral cycles. That is both parties alternate between catering to swing voters and their partisans.

<sup>14</sup>We take both the average of polarization in the House and Senate for the years later than 1990 and show that both for the House and the Senate the fluctuations can on average explain about 1.5 % of the trend. If we take into account the entire time series, then the cycles explain about 2% of the trend.

it in a media network. The targeted audience will observe the policy platform chosen by the politician, but others will decide how to vote based on their prior beliefs about the platform that the candidates intend to implement.

**Voters** Policy platforms are contained in the unit interval  $[0, 1]$ . A voter's utility only depends on the policy implemented and on the identity of the politician implementing it. In particular, the utility of a voter with bliss point  $x$ , who believes that candidate  $c$  will implement policy  $y$  amounts to

$$U(c, y|x) = \begin{cases} u(y|x) & \text{if } c = A \\ u(y|x) + \theta & \text{if } c = B \end{cases}.$$

Voters have an idiosyncratic bias  $\theta$  in favor of politician  $B$ . To simplify later discussions, we assume, as is standard, a quadratic loss function for the baseline policy preferences of voters – so that

$$u(y|x) = 1 - (y - x)^2.$$

The quadratic loss function implies that  $u(y|x)$  is single-peaked, concave and symmetric in  $y$ .

We further assume, as is common in the literature, that the idiosyncratic biases are uniformly, independently and identically distributed across voters on  $[-1, 1]$  – so that  $\theta \sim U[-1, 1]$ . Thus a voter with bliss point  $x$  would prefer voting for politician  $A$  when facing platforms  $y_A$  and  $y_B$  if and only if

$$u(y_A|x) - u(y_B|x) > \theta.$$

This specification makes the problem symmetric for party  $B$ .<sup>15</sup> The probability that a voter with bliss point  $x$  votes for  $A$  when facing platforms  $y_A$  and  $y_B$  then simply amounts to

$$P_A(y_A, y_B|x) = \frac{1}{2} [1 + u(y_A|x) - u(y_B|x)], \tag{1}$$

while the probability of voting for  $B$  amounts to  $P_B(y_A, y_B|x) = 1 - P_A(y_A, y_B|x)$ .<sup>16</sup>

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<sup>15</sup> Our results hold for any quadratic loss function, that is if we consider a utility function with  $u(y|x) = \alpha_0 - \alpha_1(y - x)^2$  and adjust the uniform distribution appropriately to  $\theta \sim U[-\alpha_1, \alpha_1]$ , then the candidates' optimization problem is unchanged. Put differently, we only need to adjust the uniform distribution and ensure the difference between the utility of voting for  $A$  and  $B$  has full support.

<sup>16</sup> This specification implies that  $P_A(y, y|x) = P_B(y, y|x) = 1/2$ , for any platform  $y \in [0, 1]$ .

**Communities, Outlets and the Media Network** Voters are partitioned into  $k$  *communities*. Let  $K$  denote the set of communities, and suppose that a measure 1 of voters belongs to every community. In community  $i \in K$ , the bliss points of voters are distributed according to a cumulative distribution  $G_i(x)$  which admits a density function  $g_i(x)$  with support on  $[0, 1]$ . Denote by  $E_i(X)$  its expectation.

There are  $m$  possible media outlets where politicians can advertise the platform they select. Let  $M$  denote the set of media outlets. The *media network*  $\{K, M, N\}$  is a bipartite network that describes which communities have access to a given outlet. Formally,  $N \subseteq K \times M$  and community  $i$  observes outlet  $j$  if and only if  $ij \in N$ . Denote the neighborhood of media outlet  $j$  by

$$K(j) = \{i \in K \mid ij \in N\}$$

Thus,  $K(j)$  consists of all communities that can observe a given outlet  $j$ . Without loss of generality, assume that every community observes at least 1 outlet – or formally  $K = \cup_{j \in M} K(j)$ .<sup>17</sup>

**Special Interest Groups** We assume that interest groups are affiliated with exactly one party as is commonly the case.<sup>18</sup> Therefore, special interest groups only decide whether to make a donation to "their" party or whether to not donate at all, but will never donate to the other party. If a special interest group does not donate, then we say the interest group abstains. Formally, an interest group affiliated with party  $c$  chooses  $c_c = \{donate, abstain\}$ . The utility from abstention is given by  $\bar{u}$ .<sup>19</sup> Thus, a special interest group connected to candidate  $A$  faces the following utility:

$$U_A(c_A, y_A|x) = \begin{cases} u(y_A|x) & \text{if } c_A = donate \\ \bar{u} + \theta & \text{if } c_A = abstain \end{cases} .$$

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<sup>17</sup>As the model allows for an arbitrary number of communities any measure of voters could observe any subset of outlets and so there is no loss in assuming that all communities have measure one.

<sup>18</sup> There is plenty of evidence that interest groups tend to favor one party. This can be readily seen in the pattern of contributions by labor unions and organizations such as the National Rifle Association. Systematic evidence for this hypothesis is provided by [Poole and Romer \(1985\)](#), [Grier and Munger \(1991\)](#), [Kroszner and Stratmann \(1998\)](#) . Theoretically, it has been established in [Felli and Merlo \(2007\)](#).

<sup>19</sup>We do not explicitly incorporate costs. Our results are unchanged if we take costs into account as long as the utility function of the special interest groups is separable in costs and benefits.

We impose the same assumptions on  $\theta$  as previously and we obtain a probability of donating to candidate  $c$  for an affiliated interest group that is given by

$$P_c(y_c|x) = \frac{1}{2} [1 + u(y_c|x) - \bar{u}] \quad (2)$$

Once special interest groups have decided whether to make a contribution, they give a *donation*  $D$ . We assume that this holds for the special interest groups of both parties. Further, the bliss points of the interest groups have a cumulative distribution  $G_c(x)$ , which admits a density function  $g_c(x)$  with support on  $[0, 1]$ .<sup>20</sup>

We let the interest groups of candidate  $A$  and  $B$  have bliss points  $E_A(X)$  and  $E_B(X)$ , respectively, with  $E_A(X) < E_B(X)$ . We further assume that every community  $i$  is more moderate,  $E_A(X) \leq E_i(X) \leq E_B(X)$ . This implies, as is common in the literature that the special interests are more extreme, compared to the average voter in every community.<sup>21</sup>

**Candidates: Targeting and Policy Setting** Candidates simultaneously select the platform that they are going to implement and the media outlets in which to advertise it, so as to maximize their expected vote share as well as donations.<sup>22</sup> Candidates can target an arbitrary number of outlets.<sup>23</sup>

A strategy for a candidate  $c \in \{A, B\}$  consist of a pair  $\{x_c, T_c\}$ . We denote by  $x_c \in [0, 1]$  the *policy* set by the candidate and by  $T_c \subseteq M$  the *target* subset of media outlets in which the candidate chooses to advertise. If an outlet is targeted by candidate  $c$ ,  $c$  commits to a platform  $x_c$ .

If a community is connected to any outlet targeted by  $c$ , every voter belonging to that community knows that the candidate will set platform  $x_c$ . If a community is not targeted however, voters stick to their *prior belief* about the candidates' policy which is denoted by  $\pi_c$

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<sup>20</sup>This assumes that there are interest groups of measure 1. This assumption is without loss of generality as we allow for the donations  $D$  to vary.

<sup>21</sup>A possible justification for this assumption is that interest groups generally care only about one issue, whereas voters care about multiple policy dimensions.

<sup>22</sup>Maximizing expected vote share, expected plurality (the vote share relative to that of the other party) and maximizing the probability of winning are equivalent in our setting. For a general discussion of the relation between these concepts, see for example [Banks and Duggan \(2005\)](#).

<sup>23</sup>We show in Appendix C an example that highlights some of the complications arising if the number of outlets that can be targeted is restricted.

and consists of a single policy in  $[0, 1]$ .<sup>24</sup> Allowing for uncertainty regarding the media network and the distribution of bliss points in other communities yields similar results in a model with rational voters. We provide an example of this in Appendix C, which also reflects the results of the literature on campaign advertising where rational voters cannot recover policy platforms fully (Schultz (2007), Coate (2004)).

For any subset of media outlets  $T \subseteq M$ , define its *coverage*  $K(T)$  as the set of communities that are linked to at least one outlet in  $T$ . Formally, this set is defined as  $K(T) = \bigcup_{j \in T} K(j)$ . Denote its cardinality by  $k(T)$ , and define voters' *posterior beliefs* of community  $i$  as

$$y_c^i(T) = \begin{cases} x_c & \text{if } i \in K(T) \\ \pi_c & \text{if } i \notin K(T) \end{cases}$$

Different from voters, special interest groups always know about the platforms parties commit to. We assume that the candidate's objective function is increasing in both the expected vote share as well as in donations and that it is linear in both elements. The problem faced by candidate  $c \in \{A, B\}$  is then given by

$$\max_{x_c, T_c} \left( \sum_{i \in K} \frac{1}{k} \int_0^1 P_c(y_A^i(T_A), y_B^i(T_B)|x) dG_i(x) + D \int_0^1 P_c(x_c|x) dG_c(x) \right), \quad (3)$$

where the first part denotes the expected vote share and the second part gives donations. An *equilibrium* is thus defined by a strategy profile  $\{x_c, T_c\}_{c \in \{A, B\}}$  which solves (3) for every candidate  $c \in \{A, B\}$ .

## 4 Equilibrium Policy and Targeting

A candidate faces the problem of which outlets to include in the target set and what policy to set for the targeted communities as well as the interest groups. In order to characterize the optimal strategy set by the politician we first simplify expression (3). Given the voting probabilities in expression (1), the maximization problem of candidate  $c$  will have the same solution as the

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<sup>24</sup>Alternatively, we can interpret the policy space as that of a policy dimension. As we have a probabilistic voting model, it is straightforward to extend our framework to a multi-dimensional policy space, where the analysis would then be carried out for each policy dimension separately.

following problem which abstracts from the competitor's strategy

$$\max_{x_c, T_c} \int_0^1 u(x_c|x) \sum_{i \in K(T_c)} g_i(x) dx + \int_0^1 u(\pi_c|x) \sum_{i \in K \setminus K(T_c)} g_i(x) dx + D \int_0^1 u(x_c|x) g_c(x) dx. \quad (4)$$

The first part of expression (4) maximizes the utility of the communities that are contained in the target set, whereas the second part refers to the communities not contained in the target set, which we denote by  $K \setminus K(T_c)$ . The problem that candidates face is therefore to determine which outlets to target depending on their audience as well as to choose the optimal policy for the outlets that are targeted, while also taking into account that the special interest groups can always observe the policy. For ease of exposition we initially set  $D = 0$ . This allows us to focus on the characteristics of the targeted voters and to provide several benchmark results.

We first show that the optimal policy coincides with the average of the bliss points of the communities in the targeted outlets. We denote this expected bliss point by  $E(X|T_c)$ . We then establish that the optimal targeting strategy prescribes to select the target set with the highest *media centrality*  $W_c(T)$ . Formally, media centrality is defined as

$$W_c(T) = k(T) [E(X|T) - \pi_c]^2, \quad (5)$$

where  $k(T)$  denotes the number of communities in the target set. This highlights that the most valuable targets are media outlets with a large coverage which have viewers whose ideal policies are not aligned with the prior beliefs on the politician's platform. This is summarized in Proposition 1.

**Proposition 1.** *In every equilibrium  $\{x_c, T_c\}$ , for every candidate  $c \in \{A, B\}$ :*

- (1) *The policy set  $x_c$  amounts to the average bliss point in the target's coverage,  $x_c = E(X|T_c)$ .*
- (2) *The outlets targeted,  $T_c$ , maximize media centrality  $W_c(T)$ .*

Candidates select a policy platform that caters to a subset of voters whose bliss point has the greatest distance to the prior. Unless all communities have identical expected bliss points, candidates never choose to target all outlets and all voters, even though there is no cost to targeting. Candidates can essentially segment voters according to their bliss points (subject to the constraint of the media network). They choose a policy that matches the preferences of the voters they target. This induces these voters to cast the vote in their favor. At the same time, the voters who are not targeted do not learn about the platform and vote based on their



belief. As the non-targeted voters have a preferred policy close to their prior, this still leads them to vote for the candidate with a high probability. Therefore, candidates can improve their expected vote share above what they would obtain were they to disclose in all outlets with a policy that is the average of all voters. But it is not straightforward to show how many voters will be targeted. On the one hand adding voters to the target set allows candidates to take their policy preferences into account. This increases their probability of voting on the candidate's behalf. But voters with a bliss point that is further from the prior than that of the newly added voters are now less likely to vote for the candidate as the policy has moved away from their bliss point. The optimal target set balances this trade off and the way this is done is highlighted in the following paragraph.

**Characterization of Target Set** This discussion aims to further characterize the optimal target set, which will be of use when we connect the implemented policy to the structure of the media network. We write in what follows  $T' - T$  if we refer to the *communities* contained in target set  $T'$ , but not in  $T$ . Thus, we denote the expected bliss points of these communities by  $E(X|T' - T)$  and their cardinality by  $k(T' - T)$ .<sup>25</sup> In order to gain some intuition, fix any two target sets satisfying  $T' \supset T$  and let the expected bliss points of these target sets lie below the prior,  $\max\{E(X|T), E(X|T')\} < \pi_c$ . First we establish that target  $T'$  has a lower media centrality than a given target set  $T$  whenever  $E(X|T' - T)$  and  $E(X|T)$  lie to different sides of the prior. If this is the case, the candidate prefers target set  $T$  to  $T'$ , that is to not include additional voters. Voters in communities contained in the target set  $T'$ , but not in  $T$  are more likely to vote for the candidate if they are not targeted. Their prior is closer to their preferred policy than the actual policy if they were targeted,  $E(X|T' - T) \geq \pi_c > E(X|T')$ , and thus targeting reduces the probability with which they vote for this candidate. Additionally, the voters in target set  $T$  are also less likely to vote for the candidate if they observe the policy associated with  $T'$  as the new policy is further away from their preferred one. This is formalized in Lemma 1.

**Lemma 1.** *Let  $\max\{E(X|T), E(X|T')\} < \pi_c$  and  $E(X|T' - T) \geq \pi_c$ . Then,  $W_c(T) > W_c(T')$ .*

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<sup>25</sup>Formally, the distribution of ideal points in  $K(T)$  that do not belong to the coverage  $K(S)$  of media outlets  $S \subseteq M$  is defined as  $G(x|T - S) = \frac{1}{|K(T) \setminus K(S)|} \sum_{i \in K(T) \setminus K(S)} G_i(x)$ . We denote the expectation of  $G(x|T - S)$  by  $E(X|T - S)$ . For convenience, we also let  $k(T - S)$  denote  $|K(T) \setminus K(S)|$ .

Outlets may be added to a given target only if  $E(X|T')$  and  $E(X|T' - T)$  lie to the same side of the prior  $\pi_c$ . The next result presents sufficient conditions for media centrality to be higher in  $T'$  than in the original target set  $T$ .

**Lemma 2.** *Let  $E(X|T) < \pi_c$ . If either of the following conditions hold,  $W_c(T') > W_c(T)$ :*

(1)  $E(X|T' - T) \leq [\pi_c + E(X|T)] / 2$ ;

(2)  $W_c(T) > W_c(T'')$  for some  $T'' \subset T$  with  $E(X|T - T'') \in (E(X|T' - T), \pi_c)$ .

The first part establishes that including communities whose bliss point is closer to  $E(X|T)$  than to the prior unambiguously increases media centrality. The second part of the result shows that if adding more moderate communities increases the support for a politician, then adding communities with more extreme bliss points also increases support. Thus, if media centrality increases by including an outlet, then any remaining outlet which has an expected bliss point below the one included should also be targeted.

When considering a target outlet  $T'$  such that  $E(X|T' - T) \in ((\pi_c + E(X|T)) / 2, \pi_c)$ , no further simplification is possible. Two forces drive the politician in opposite direction – on the one hand more outlets guarantee more coverage, on the other hand they dilute the effect of disclosure as the new policy is closer to the prior.

## 5 Media & Policy Platforms

We aim to capture recent developments in information technologies that have broadened the supply of entertainment and made it easier to access various programs and analyze their effect on the policy set. In particular the internet has created many spillovers. Voters, although not directly targeted, might be able to observe a campaign ad because a friend posts a link on Twitter or Facebook, or a blog reports about it. In this sense, voters are connected to a higher number of media outlets than they have been previously. Put differently, voters have a higher number of links to various outlets. But not only the number of outlets voters are connected to has changed, but also the way in which voters are connected to media outlets. Given the increase in the number of media outlets it has become easier for everyone to find programs that match their interests. As demographic characteristics predict viewership as well as voting behavior,

this has made it possible to target a certain set of voters specifically. In order to analyze the effect of this development, we focus on partitions and the impact they have on the implemented policy.

**Partitions** We allow for the number of media outlets to increase, such that voters can be targeted more specifically and analyze how this affects the implemented policy. We keep the number of communities fixed and assign communities to new outlets without increasing the number of links. This is captured by a partition.

**Definition 1** (Partitioned Media Network). *Consider an outlet  $j \in M$ . A collection  $K^P(j)$  is a partition of  $K(j)$  if*

- (1)  $\bigcup_{K_i \in K^P(j)} K_i = K(j)$
- (2)  $\emptyset \notin K^P(j)$
- (3)  $K_i \cap K_{i'} = \emptyset$  for any  $K_i, K_{i'} \in K^P(j)$

A partition splits up the communities belonging to an outlet and assigns them to new outlets. This leads to the creation of a new network and we denote by  $K'(\cdot)$ ,  $E'(\cdot)$  and  $W'(\cdot)$  the new operators. The created outlets are denoted by  $j_i \in M^P$ , which leads to a new set of outlets  $M' = M \cup M^P$ . All the partitioned communities  $K_i \in K^P(j)$  belong to the neighborhood of some  $j_i$  and so we can write  $K_i = K'(j_i)$ .

We first consider the case where an outlet contained in the target set is partitioned. Note that it can always be the case that the expected bliss point of the optimal target set after the partition lies on a different side of the prior, that is it can always be the case that  $E(X|T) < \pi < E'(X|T')$ , where  $T'$  denotes the target set after the partition. An example of this is given in Figure 4. We consider three types of nodes, left nodes with  $E_L(X) = 1/4$ , right nodes with bliss point  $E_R(X) = 3/4$  and medium points with  $E_M(X) = 2/3$ . The prior is given by  $\pi = 1/2$ . If only two outlets, outlet 1 and 2 exist, it is optimal to target outlet 1. Once outlet 3 is created, which only caters to a right community previously connected to outlet 1, it is instead optimal to target outlets 2 and 3. In what follows we rule out that the target set switches to a different side of the prior. Additionally, we focus on the case where the communities connected to any outlet  $j_i$  only belong to this outlet, that is there is no overlap with other outlets. Formally,  $K'(j_i) \cap K'(j) = \emptyset \forall j_i \in M^P, j \in M'$ . This is essentially a restriction on how we partition. We can then show that

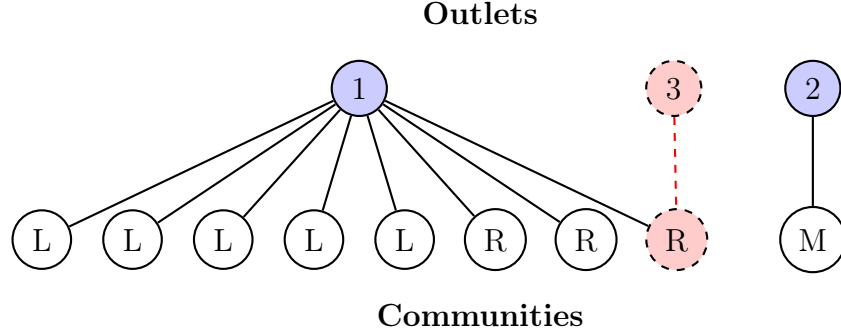


Figure 4: Partition: Target Set Switches

the target set after the partition contains weakly fewer communities.

**Lemma 3.** *Let  $K'(j_i) \cap K'(j) = \emptyset \forall j_i \in M^P, j \in M'$ . If  $\max\{E(X|T), E'(X|T')\} < \pi, j \in T$ , and  $K(j)$  is partitioned, then  $K'(T') \subseteq K(T)$  and  $E'(X|T') \leq E(X|T)$ .*

A partition of an outlet contained in the target set leads to fewer targeted communities. Partitioning allows the candidates to target communities more specifically and therefore communities with a high bliss point will be omitted from the target set. This leads to the expected bliss point of the target set to become more extreme. To see this recall Lemmas 1 and 2, where we have shown that if communities are omitted from the target set, their bliss point lies above  $(\pi + E(X|T))/2$  and below  $\pi$ , which results in the bliss point decreasing, that is  $E'(X|T') \leq E(X|T)$ .

If the outlet that is partitioned does not lie in the target set, then there can be fewer or more communities in the target set, the policy can move closer to the prior or further away. This depends on the specific network characteristics.

**Adding Links** We turn now to what happens when a community forms an additional link to an outlet. We first focus on the case where a community  $i$  contained in target set  $T$  forms an additional link to outlet  $l$ . As before, we write  $K'(\cdot), E'(\cdot)$  etc. to denote the operators of the new network. For our further analysis, we denote by  $j \in T$  the outlets to which  $i$  belongs in the original network, that is  $i \in K(j)$ . Again, it can be the case that the new target set has a bliss point to the other side of the prior, that is  $E(X|T) < \pi < E'(X|T')$ . We provide an example of this in Appendix C.

If the bliss points of the target set does not switch then the new target set depends on whether the link is formed to an outlet already contained in the target set (that is,  $l \in T$ ) or whether the link is formed to a previously non-targeted outlet (that is,  $l \notin T$ ). We establish that if  $l \in T$  the target set either remains the same or outlets are omitted from the target set. In this case it can never be the case that an outlet is added to the target set (that is, the target set cannot increase). If  $l \notin T$ , then it can still be the case that the target set remains the same (it never shrinks), but it can also be that  $j$  is omitted from the target set and replaced by outlet  $l$ .

Additionally, the change in the target set has an impact on the expected bliss point of the target set and thus also on the policy selected by the candidate.

**Lemma 4.** *Let  $\max\{E(X|T), E'(X|T')\} < \pi$  and  $i \in K(T)$*

*(1) If  $l \in T$ , then  $T' \subseteq T$  and  $E(X|T) \geq E'(X|T')$ .*

*(2) If  $l \notin T$  and  $j \in T'$ , then  $T = T'$ . If  $j \notin T'$ ,  $l \in T'$ .*

If a community, that is already connected to a targeted outlet, forms a link to another outlet in the target set, then it is intuitively plausible that the target set remains unchanged.<sup>26</sup> It is more surprising that the addition of such a link can in fact lead to fewer outlets in the target set. Note that not only outlet  $j$  might be dropped from the target set, but other outlets might be dropped as well. To see this consider the example in Figure 5 with  $\pi = 1/2$ ,  $E_L(X) = 1/4$ ,  $E_R(X) = 1/2$ .

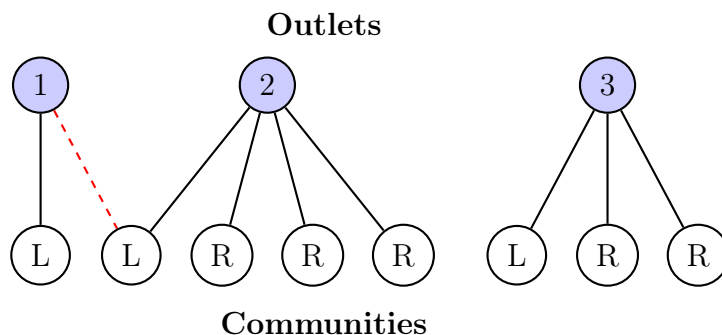


Figure 5: 3 Communities in 2 outlets

<sup>26</sup>An example of this is the case of outlets that have homogeneous communities as their audience. Forming a link has no influence on the expected bliss point of a given neighborhood of the media outlet and therefore the target set will remain the same. If the communities are heterogenous, but the heterogeneity is sufficiently small, the same logic applies.

Initially, the target set contains all outlets. But after adding the red, dashed link, not only outlet 2 is no longer targeted, but even outlet 3 is dropped from the target set. Again, by Lemma 2, it must be the case that the bliss point decreases, if outlets are dropped from the target set. Similar to the case of the partition, if an already targeted community joins another targeted media outlet, then the candidates can potentially tailor their policy to a more narrow subset of voters. This allows candidates to cater to the most extreme voters (relative to the prior), without losing the vote of the communities that are closer to the prior, as these are no longer informed about the true platform.

If a community forms a link to an outlet that does not belong to the target set, then by the same logic as before, it can be that  $T = T'$  when the additional link has no impact on the target set. But it can also be that outlet  $j$  is replaced by outlet  $l$ , an example of this is given in Appendix C.

Last, if a community that is not contained in the target set forms a link, then the target set can increase or decrease, the policy can move closer to the prior or further away. This is similar to the case of the partition, with the added complication that communities can form a link to an outlet contained in the target set or an outlet outside the target set.

This section highlights that it is far from obvious how a change in the media network affects the target set and with it the policy set. In particular, if there is a change in the media network that affects communities and outlets that are not contained in the target set, then the target set and with it the policy can change in an arbitrary way. In particular, the policies can increase or decrease relative to the prior and they can lie to the other side of the prior. We can however show how changes within a target set affect the implemented policies.

## 6 Polarization

We now analyze the differences in the platforms that candidates  $A$  and  $B$  set. We refer to this discrepancy as *polarization*. Suppose first that interest groups do not contribute to the election campaign. Then, the policies of both candidates can coincide.

**Homogeneous Candidates** We analyze the benchmark case of *homogeneous* candidates, that is the two candidates are ex-ante identical with,  $\pi_c = \pi$  for all  $c \in \{A, B\}$ . We first establish that in this special case, with special interest groups not donating, candidates select the same policy and same target set. The expression *symmetric equilibrium* as usual refers to an equilibrium in which both candidates play the same strategy.

**Proposition 2.** *When candidates are homogeneous:*

- (1) *Symmetric equilibria always exists.*
- (2) *Both candidates win with probability 1/2 in any equilibrium.*
- (3) *In any equilibrium candidates generically target the same communities  $K(T_A) = K(T_B)$  and set the same policy  $x_A = x_B$ .*

The generic uniqueness result relies on the observation that no two media coverages can generate the same support for a politician generically. The observation follows as, for any two distinct media coverages  $K(T) \neq K(S)$ , any small perturbation to bliss points implies that  $E(X|T) \neq E(X|S)$  with probability 1.

One implication of this result is that platforms can differ if voters have different beliefs about candidates.<sup>27</sup> But platforms generally differ, even if voters have the same prior about a candidate, in the presence of special interest groups.

If special interest groups do not contribute to the campaign, the policy set corresponds to the expected bliss point in the target set,  $x_c = E(X|T_c)$ . This is no longer the case if special interest groups donate a positive amount. Then, the policy weights the expected bliss point in the target set as well as the preferences of the special interest groups and is given by

$$x_c = \frac{k(T_c)E(X|T_c) + DE_c(X)}{k(T_c) + D}. \tag{6}$$

We generally do not know where the platforms of parties  $A$  and  $B$  lie relative to each other, but we can show that if donations are high enough, then the platform selected by candidate  $B$  lies to the right of  $A$ 's policy. Party  $A$  becomes left-wing, party  $B$  right-wing. We define two target

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<sup>27</sup>In principle one could analyze polarization by assuming that the priors differ without special interest groups. We refrain from doing so as this implies that a party that is perceived to be right-wing will choose a left-wing policy and vice versa. This is not the case if we take special interest groups into account.

sets  $T_{max}$  and  $T_{min}$ , where

$$T_{max} \in \arg \max_T k(T)E(X|T)$$

$$T_{min} \in \arg \min_T k(T)E(X|T).$$

**Proposition 3.** *The policy platform of candidate A always lies strictly below that of candidate B,  $x_A \leq x_B$ , for any prior  $\pi_A, \pi_B$ , if  $D > \bar{D}$  where*

$$\bar{D} = \sqrt{k(T_{max})k(T_{min})} \sqrt{\frac{E(T_{max}) - E(T_{min})}{E_B(X) - E_A(X)}}. \quad (7)$$

Equation (7) highlights that if donations are sufficiently high, namely above a threshold value  $\bar{D}$ , then party A's policies are always to the left of B's policies, *independently of the priors*. The threshold is decreasing in the difference in preferences of the interest groups. Additionally, a higher number of communities in the target sets lead to a higher threshold as they diminish the influence of the interest group. Last, if the difference in preferences of the communities in the target sets are small, then the threshold is lower.

**Dynamic Framework** Given that polarization is an inherently dynamic phenomenon, we analyze how polarization changes over time.<sup>28</sup> In particular, we allow for the prior beliefs to change over time. If the prior were the same in each election, then parties would select the same policy platform. However, we know that the policies change in each presidential election, as we have established empirically the existence of electoral cycles. Our model can generate these cycles if we allow for beliefs to adjust with each election. In particular, cycles emerge if voters change their beliefs adaptively, that is today's prior equals last period's policy. We denote the prior about candidate  $c$  in period  $t$  by  $\pi_c^t$  and we index the policy in each period with  $t$ . We impose the following assumption.

**Assumption 1.** *Adaptive Learning:  $\pi_c^t = x_c^{t-1}$  for  $t > 1$ .*

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<sup>28</sup>We provide in Appendix C an analysis of how polarization is affected by a change in the media network in a static framework. The previous section has shown that a change in the media network can affect the policies in an arbitrary way. This carries over to polarization and so polarization can increase or decrease. The dynamic framework on the other hand allows us to make sharper predictions.



We further set the discount factor for the candidates to zero. This captures the fact that a candidate cares first and foremost about winning a given election. Put differently, long-term concerns of how the party positions itself are of limited importance. This implies that candidates maximize expected vote share and campaign contributions for one period.

We restrict attention to the case where the number of media outlets is at least two, that is  $m \geq 2$  and we ask how policy platforms change over time.<sup>29</sup> We first show that it is never optimal to set the same policy in each period.

**Proposition 4.** *It is never optimal for a party to select the same policy in each election period.*

Selecting exactly the same policy as in the last period prevents parties from segmenting voters. Voters believe that the policy today will be the same as last period's policy. By selecting a policy that coincides with the prior, candidates do not single out a subset of voters where they increase the probability that these communities vote for them. This cannot be optimal as we have already established that it is never optimal to target all voters. Note that it does not imply that a given *outlet* cannot be targeted in two consecutive periods. Rather, the policy and with it the target set cannot remain unchanged over time.

This result does not depend on the assumption of adaptive expectations, but only requires that voters adjust their belief sufficiently over time. Thus, our model provides one possible motive of why there are electoral cycles.

We can further show that these platforms cycle between exactly two policies *in the long run*. We establish that for each  $t > \bar{t}$ , the optimal platform can take one of two possible values and parties fluctuate between two possible target sets, which we denote by  $T_L$  and  $T_R$ . The policies are then given by  $x_c^t \in \{x_{cL}, x_{cR}\}$  with  $x_{cL} < x_{cR}$ . However, the electoral cycles are not necessarily unique. There can be two cycles with distinct target sets and *distinct policies*. We denote the policies in the first cycle as  $x_{cL}$  and  $x_{cR}$ , those in the second one by  $x'_{cL}$  and  $x'_{cR}$ . There is cycling between  $x_{cL}$  and  $x_{cR}$  if one of the two platforms is reached and cycling between  $x'_{cL}$  and  $x'_{cR}$  if either  $x'_{cL}$  and  $x'_{cR}$  is set at some point.

**Lemma 5.** *A candidate's electoral cycle has the following properties*

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<sup>29</sup>In case of one media outlet, both candidates always target this outlet. Each party selects the same policy over time, but parties' platforms differ over time due to the presence of the special interest groups.

(1) In the long run, a candidate's platform cycles between exactly two policies.

(2) For any two distinct cycles, it must hold that  $x_{cL} < x'_{cL} < x_{cR} < x'_{cR}$ .

The target sets are characterized by the number of communities they contain as well as the expected bliss point. However, the size of the target set and the expected bliss points are not correlated. To see this more clearly consider an example with two outlets. We write  $T_1$  ( $T_2$ ) if only the first (second) outlet is targeted and  $T_{12}$  contains both media outlets. There are two cases, namely (i)  $x_1 < x_2 < x_{12}$  and (ii)  $x_1 < x_{12} < x_2$ . An example of case (i) is depicted in Figure 6 where for some parameter values it can be shown to be the case that  $x_1 < x_2 < x_{12}$ .<sup>30</sup> Additionally, it holds that  $k(T_{12}) > k(T_1)$  and  $k(T_{12}) > k(T_2)$  and so the a target set with a more extreme bliss point can be larger or smaller than a target set with a less extreme bliss point.

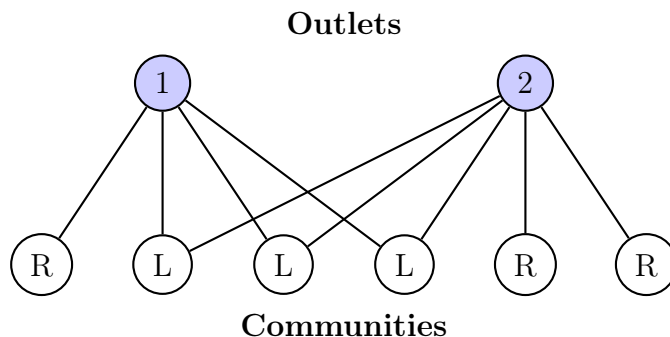


Figure 6:  $E(X|T_1) < E(X|T_{12}) < E(X|T_2)$

Therefore, our proof is solely based on how the optimal policies change over time. For a given initial policy,  $x^1$ , we distinguish between two possibilities of how the target sets change in the following periods, namely (i)  $x^1 < x^2$  and  $x^2 > x^3$  or (ii)  $x^1 < x^2 < x^3$ . We focus on the latter case and can show that  $T^4$  either equals  $T^2$ , in which case we have established that there is cycling between exactly two platforms, or  $x^4 \in (x^1, x^2)$ . We then turn to the optimal target set in period 5. If this equals  $T_3$ , we have again established that there is cycling between two policies. Otherwise,  $x^5 \in (x^2, x^3)$ . Therefore, it must be the case that  $x_{cL}$  lies in a bounded subset of the policy space, namely  $x_{cL} \in (x^1, x^4)$ . Similarly,  $x_{cR} \in (x^2, x^5)$ . This holds true more generally, that is  $x_{cL}$  lies in a bounded subset of the policy space and so does  $x_{cR}$ . It further holds that for all  $t > t'$  either  $x_c^t \leq x_c^{t+2}$  and  $x_c^{t+1} \leq x_c^{t+3}$  or  $x_c^t \geq x_c^{t+2}$  and  $x_c^{t+1} \geq x_c^{t+3}$ . This

<sup>30</sup>This holds for example for candidate  $B$  if  $E_L(X) = 1/4, E_R(X) = 5/6, E_X(B) = 1, D = 2$ .

implies that from a certain time period onwards, each policy in the cycle either moves to the left or the right, that is each policy chosen lies weakly to the left or the right of the one two periods before. As the policy space is bounded, this implies that in the long run, there is cycling between exactly two alternatives. However, this cycle does not necessarily have to be unique for each party. It can be the case that there are two cycles which are shifts of each other, that is one cycle contains policies that lie to the left of the policies of the other cycle.

As we have established that in the long run, there are exactly two target sets for each candidate, we can now define polarization over a cycle.

**Definition 2** (Polarization). *Polarization is the average distance between parties' platforms for an electoral cycle,  $\Delta_P = \frac{1}{2}(x_{BL} + x_{BR} - x_{AL} - x_{AR})$ .*

It is straightforward to see that polarization is increasing in  $D$  and  $E_B(X)$  and decreasing in  $E_A(X)$ . The level of polarization also depends on the network structure. We have shown that a partition as well as adding links affect the target set as well as the policy set. Therefore, these changes also have an implication on the difference between policies and therefore polarization. In order to analyze the impact of a partition or of adding links, we restrict attention to symmetric media outlets. This allows us to connect the features of the media network to polarization.

**Symmetric Media Outlets** A natural starting point to show the impact of media network structure on polarization are *symmetric* media outlets.

**Assumption 2** (Symmetry). *Media outlets as well as preferences of special interest groups are symmetric.*

**A.1** *For any outlets  $j, j' \in M$ , there exist outlets  $j'', j''' \in M$  such that*

$$k(j - j') = k(j'' - j''') \quad \text{and} \quad 1 - E(X|j - j') = E(X|j'' - j''')$$

**A.2** *Interest groups are biased to the same extent,  $E_B(X) = 1 - E_A(X)$ .*

Assumption [A.1](#) implies that for each media outlet that lies to the left of  $1/2$ , there exists a symmetric media outlet to the right of one half. It presumes that the distribution of voters among media outlets is the same on the left and on the right of  $1/2$ . This symmetry then also carries over to the preferences of the interest groups (Assumption [A.2](#)).

Based on these assumptions we can then establish that there always exists an equilibrium, in which the policies are symmetric.

**Lemma 6** (Symmetric Policy Platforms). *If media outlets and interest groups are symmetric, there exists an equilibrium in which policies are symmetric, that is  $x_{AL} = 1 - x_{BR}$  and  $x_{AR} = 1 - x_{BL}$  for  $D \geq 0$ .*

Lemma 6 shows that there exists an equilibrium in which policies are symmetric. However, it can still be the case that there are asymmetric equilibria. As an electoral cycle is not necessarily unique, see Lemma 5, it can be the case that the two parties end up with different policy cycles. This results in asymmetric equilibria, where  $x_{AL} \neq 1 - x_{BR}$  and  $x_{AR} \neq 1 - x_{BL}$ . But even if policies are symmetric, it might not necessarily be the case that the target sets are the same. To see this consider Figure 7, which gives the symmetric bliss points associated with 4 different target sets, *without* taking the special interest groups into account. We rank the bliss points with  $E(X|T_{-2}) < E(X|T_{-1}) < E(X|T_1) < E(X|T_2)$ .

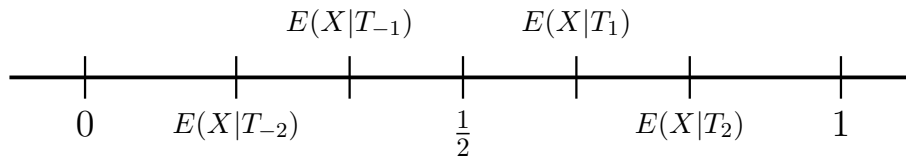


Figure 7: Bliss Points with Symmetric Media Outlets

Once we take the special interest groups into account we obtain the actual policies which are as given in Figure 8. Note that it is not necessarily the case that the ordering of the bliss

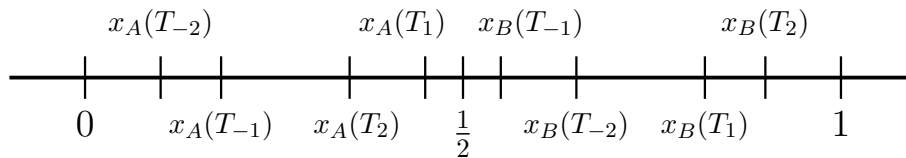


Figure 8: Policies with Symmetric Media Outlets

points carries over to the policies. Rather, it can be the case that a target set associated with a higher bliss point will lead to a lower policy than another target set with a lower bliss point. To

see this recall that policies are given by  $x_c(T) = \frac{\sum_{i \in T} E_i(X) + D E_c(X)}{k(T) + D}$ . Therefore, not only the bliss points in the target set matters, but also the number of communities that belong to the target set. If a target set contains only few communities, then the impact of the special interest groups is larger. They have a weight of  $D$  and each community contains a measure one of voters. If there is only one community in the target set, then the candidates set policies that cater greatly to the special interest groups. If the number of the communities is high, then the influence of the interest group is limited and the implemented policy places a higher weight on voter's preferences. From this it follows that it can be the case that despite the communities in target set  $T_2$  having a higher bliss point than those in  $T_1$ , the policy associated with  $T_2$  lies below the one selected when the target set is  $T_1$ . This is possible if the number of communities in  $T_2$  is sufficiently low compared to those in  $T_1$ , resulting in the interest group gaining more influence if  $T_2$  is targeted than if  $T_1$  is chosen. Formally, if  $k(T_1) > k(T_2)$  and  $E(X|T_1) < E(X|T_2)$ , then it can be the case that  $x_A(T_2) < x_A(T_1)$ . By symmetry, for candidate  $B$  the policies associated with target sets  $T_{-1}$  and  $T_{-2}$  have a reversed ranking compared to the bliss points as well.

This then allows us to discuss what target sets candidates select. First, parties might select the same target sets. Suppose, for example, that party  $A$  selects  $T_{-1}$  and  $T_1$ . Then, by symmetry, there exists an equilibrium in which  $T_{-1}$  and  $T_1$  are also optimal for party  $B$ . But it can also be the case that both candidates select different target sets. Suppose, for example, that party  $A$  cycles between  $T_{-2}$  and  $T_1$ . Then, by symmetry,  $T_{-1}$  and  $T_2$  are also optimal for party  $B$ . This still implies that the policy platforms are symmetric, as target set  $T_{-i}$  contains the same number of communities as target set  $T_i$  and the bliss points are symmetric around  $1/2$ , that is  $E(X|T_{-i}) = 1 - E(X|T_i)$ .

Based on this we can simplify the level of polarization. We denote by  $T_{BL}$ , the left target set of candidate  $B$ , by  $T_{BR}$  his right one. Polarization is then given by

$$\Delta_P = \frac{1}{2} \left( \frac{2 \left( \sum_{i \in K(T_{BL})} E_i(X) \right) - k(T_{BL})}{k(T_{BL}) + D} + \frac{2 \left( \sum_{i \in K(T_{BR})} E_i(X) \right) - k(T_{BR})}{k(T_{BR}) + D} + D (E_B(X) - E_A(X)) \left( \frac{1}{k(T_{BL}) + D} + \frac{1}{k(T_{BR}) + D} \right) \right), \quad (8)$$

where we omit the target sets of candidate  $A$  due to symmetry. This allows us to characterize how different symmetric equilibria affect the level of polarization.

**Lemma 7.** *In equilibria with symmetric policies, higher levels of polarization are associated with an increase in  $\sum_{i \in K(T_{BL})} E_i(X)$  and  $\sum_{i \in K(T_{BR})} E_i(X)$  and with a decrease in  $k(T_{BL})$  and  $k(T_{BR})$ .*

Lemma 7 highlights which features of an equilibrium impact the level of polarization.<sup>31</sup> In particular, if a symmetric equilibrium arises in which the bliss points in the target sets candidate  $B$  selects are higher, then polarization increases. Higher bliss points in the target sets for  $B$  imply lower bliss points in the target sets selected by candidate  $A$  and thus the gap between policies chosen grows, which results in higher polarization. At the same time, polarization is more moderate, in an equilibrium in which target sets contain a higher number of voters. Recall that polarization arises due to the presence of the special interest groups. A higher number of voters in the target set diminishes the influence of the interest group on the policy set and therefore moderates the policies selected. This ultimately leads to a decrease in polarization. However not only the overall number of communities contained in the target sets matters, but also how the many communities are contained in the left and right target set, respectively.

To simplify our further analysis, we assume that each outlet is contained in at least one of the two target sets of each party. That is, a given outlet must either be included in the left or right target set.

**Assumption 3** (Exclusion of Non-Targeting). *For any two target sets  $T_L$  and  $T_R$ , with  $E(X|T_L) < E(X|T_R)$ , there does not exist an outlet  $j \in M$  such that  $E(X|j - T_L) > \frac{x_{cL} + x_{cR}}{2}$  and  $E(X|j - T_R) < \frac{x_{cL} + x_{cR}}{2}$ .*

Recall that Lemma 2 established that any outlet with communities that have a bliss point closer to the policy than the prior should be added to the target set. Here, the prior is last period's policy. Therefore, if the bliss points of the communities in a given outlet are closer to the policy associated with the left target set, then the outlet should be added to this target set. Otherwise, the outlet should be added to the right target set. However, it can be the case that there are outlets that are never added to a target set as they have a sufficiently large overlap with other outlets. An example of this is given in Figure 9. The majority of left communities already belongs to  $T_L$  and the same holds true for the right communities that are also connected to

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<sup>31</sup>This is not a comparative static result, rather we illustrate features of the equilibrium.

outlets already contained in  $T_R$ . These type of outlets would not be contained in any target set and we rule that this type of outlets exist. We essentially focus on media networks in which each outlet has a sufficiently large voter base that is only connected to it.<sup>32</sup>

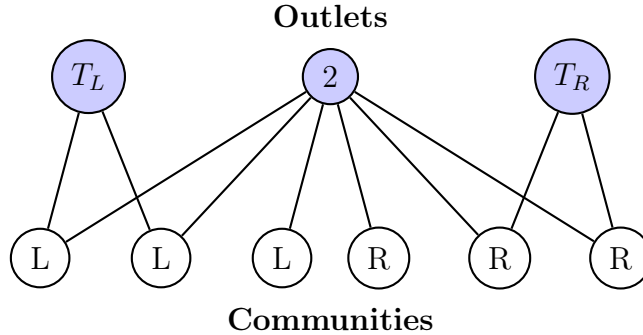


Figure 9: Exclusion Non-Targeting

One implication of Assumptions 2 and 3 is that the target sets associated with the more moderate policies contain weakly more voters than those with the more extreme policies. This is summarized in Lemma 8.

**Lemma 8.** *Let  $D > \bar{D}$ . Then, the number of voters in  $B$ 's left target set is weakly larger than the number of voters in his right target set,  $k(T_{BL}) \geq k(T_{BR})$ .*

By symmetry it then also must be the case that there are more voters in  $A$ 's right target set than in his left target set. The result follows from the fact that  $B$ 's left target set lies to the right of  $\frac{1}{2}$ . By Lemma 2, it must be that any outlet with a bliss point to the left of this target set must be contained in  $T_{BL}$ . As half of the communities lie to the left of  $1/2$  due to the symmetry of the media outlets, the number of voters in  $T_{BL}$  must be weakly larger than the number of voters in  $T_{BR}$ .

Symmetric equilibria together with Assumption 3 allow us to analyze the effect of a change in the media network on polarization. We know that each outlet is contained in a target set

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<sup>32</sup> Note that imposing this assumption on the long run policies is less restrictive than doing so for two arbitrary target sets. More precisely, this assumption will not hold for arbitrary target sets. To see this consider three policies that are chosen over time with  $x^1 < x^2 < x^3$ , with each policy becoming next period's prior. Then the target set associated with  $x^2$  does not contain outlets with communities that have a bliss point below  $x^1$ . Similarly, the target set associated with  $x^3$  does not contain outlets with communities that have a bliss point below  $x^2$ . This implies that there are outlets not be contained in the target set associated with  $x^2$  or  $x^3$ .

and therefore, we can use the results developed in Section 5. There we have shown that as long as the changes in the media network occur within a target set, we can analyze their impact on policy. This carries over to the effect on polarization.

**Partitions & Polarization** We now discuss the impact of a partition on polarization if candidates select the same target sets before turning to the case of different, but symmetric target sets. We consider two outlets,  $j_-$  and  $j_+$  that are symmetric, that is  $k(j_-) = k(j_+)$  and  $E(X|j_-) = 1 - E(X|j_+)$  and let the communities connected to these outlets be *partitioned symmetrically*.

**Definition 3** (Symmetric Partitions). *Two symmetric outlets  $j_-$  and  $j_+$  are partitioned symmetrically if for every outlet  $j_{-i}$ , there exists an outlet  $j_{+i}$ , such that  $k(j_{-i}) = k(j_{+i})$  and  $E(X|j_{-i}) = 1 - E(X|j_{+i})$ .*

By focussing on symmetric partitions, we preserve the symmetry of the underlying media outlets.

If the partitioning has no effect on the target set, then the policies set are unchanged and so is polarization. However, it can be the case that an outlet  $j_i$  is omitted from one of the target sets and then polarization is affected.<sup>33</sup>

If Assumptions 2 and 3 hold and both parties select the same target sets, then partitioning weakly increases polarization. If parties select different target sets, then partitioning can lead to an increase or decrease in polarization. We fix the preferred policies of the interest groups associated with party  $A$  and  $B$  at zero and one, respectively. We distinguish between two cases, namely (i) both partitioned outlets are contained in  $T_{BL}$  and (ii) one partitioned outlet is contained in  $T_{BL}$ , the other in  $T_{BR}$ . We can focus on these two cases as it is never optimal to include two symmetric outlets in  $B$ 's right target set. We assume that  $B$ 's policy associated with his left target set is above  $1/2$  and it is lower than the policy associated with his right target set. Then, it cannot be the case that two symmetric outlets which by definition have a bliss point of  $1/2$  are contained in  $T_{BR}$ , see Lemma 1. Therefore, the partitioned outlets cannot both be contained in  $T_{BR}$ . We show that if both partitioned outlets are contained in  $B$ 's left

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<sup>33</sup> It can be the case that several outlets are omitted from the target set. This does not change the analysis as long as for any omitted outlet  $j'$ ,  $K'(j') \cap K(j) = \emptyset \forall j', j \in M'$ . By partitioning appropriately, we can always ensure that this condition holds and thus we can focus on outlet  $j_i$ .



target, then polarization can increase or decrease. If outlets in both target sets are partitioned, then polarization unambiguously increases. This is summarized in Proposition 5.

**Proposition 5.** *Let  $K'(j_i) \cap K'(j) = \emptyset \forall j_i \in M^P, j \in M'$  and let partitions be symmetric.*

*(1) If both candidates select the same target sets, then partitioning weakly increases polarization.*

*(2) Consider a symmetric equilibrium where candidates select different target sets and  $E_B(X) = 1$ .*

*(a) Let  $j_-, j_+ \in T_{BL}$ . If  $E(X|j_i)$  is sufficiently high partitioning increases polarization, otherwise polarization weakly decreases.*

*(b) If  $j_- \in T_{BL}$  and  $j_+ \in T_{BR}$ , polarization weakly increases.*

Independently of whether candidates select the same target sets or different ones, it can always be the case that the target sets are unaffected by the partition. Then, polarization is unchanged as well. If partitioning does affect the target sets, then the induced change and its effect on polarization depends on the type of equilibrium.

Suppose first that both candidates select the same target sets. Then it can either be the case that communities switch target sets, that is communities originally in the left target set move to the right one and vice versa. Alternatively, it can be the case that communities move from  $B$ 's right target set to his left one. We first show that these changes in the target set are indeed the only ones that can occur, before turning to the analysis of how these changes affect polarization.

Note that it must be the case that one partitioned outlet is contained in  $T_{BL}$ , and the other in  $T_{BR}$ . If both parties select the same target sets, then these target sets must be equal sized as otherwise the policies would not be symmetric. Thus, for each outlet in the left target set it must be the case that its symmetric counterpart is contained in the right target set. We can then rule out that both partitioned outlets are contained in  $B$ 's right target set after the partition,  $T'_{BR}$ . Again, as the outlets are symmetric, their bliss point is  $1/2$  and so it can never be optimal to include both outlets in  $T'_{BR}$ . We further show that if both partitioned outlets are in  $B$ 's left target set after the partition, then none of these outlets is contained in  $T'_{BR}$ . This implies that the number of communities in the target sets is unaffected by the partition, that is  $k(T_{BL}) + k(T_{BR})$  equals  $k(T'_{BL}) + k(T'_{BR})$ . We have thus established that target sets either change by communities switching from the left to the right target set and vice versa or that communities move from the left to the right target set.

We now connect the changes in the target sets to polarization. Suppose first that outlets

switched. This implies that the number of communities in the target set after the partition equals the number of communities in the target set before the partition. Further, both candidates still select the same target sets, that is target sets changed symmetrically. In case of symmetric target sets, the actual bliss point in the target sets has no impact on polarization as the measure of polarization simplifies to

$$\Delta_P = \frac{D}{k(T) + D} (E_B(X) - E_A(X)). \quad (9)$$

Equation 9 shows that only the number of communities in the target sets matters for the level of polarization and as this number is unchanged, polarization is unaffected by the change in the target sets. We then turn to the case where the partitioned outlet moves from the right target set to the left. This leads to an increase in the number of communities contained in the left target set and a decrease in the number of communities in the right target set, again from the perspective of candidate  $B$ . For candidate  $A$  the change is reversed with the right target set increasing and the left one decreasing. This implies that candidates no longer select the same target sets. We can show that the policy associated with the right targeted sets becomes more extreme whereas the policy associated with the left target set can be more or less extreme. However, we can show that overall, polarization increases as  $B$ 's right policy increases more than  $B$ 's left policy can decrease. Thus we have established the first part of Proposition 5, namely that partitioning weakly increases polarization if candidates select the same target sets.

We then turn to the effect of a partition on polarization if candidates do not select the same target sets. If one partitioned outlet is contained in  $T_{BL}$ , and the other in  $T_{BR}$ , then the logic is exactly the same as if candidates select the same target sets. We therefore focus on what happens if both partitioned outlets are contained in  $B$ 's left target set, and by symmetry,  $A$ 's right target set. If both outlets are contained in  $T_{BL}$ , then an outlet might be omitted from the left target set and added to the right target set. If this happens, polarization increases if the bliss point of the communities added to the right target set is sufficiently high. Otherwise polarization decreases. There are two opposing forces at play. On the one hand, increasing the number of communities in the right target set decreases polarization, as the influence of the special interest groups is reduced. At the same time adding communities that have an extreme bliss point to the right target set induces a more extreme policy. If the second effect outweighs the first one, polarization increases.

To summarize, polarization can decrease if only moderate voters select new outlets. It tends to increase if more extreme voters are moving away from the media outlets they consumed previously and instead connect to outlets that offer programs specifically targeted to them. If only outlets with moderate voters are partitioned, then this makes it easier to target these voters specifically and reduces polarization. If however, due to the partition, also more extreme voters can be targeted specifically, then polarization increases.

**Adding Links & Polarization** We again restrict attention to the case of symmetric media outlets and symmetric equilibria.

We focus on symmetric outlets  $j_-$  and  $j_+$  where  $E(X|j_+) = 1 - E(X|j_-)$  and  $k(j_+) = k(j_-)$ . The same conditions hold for  $l_-$  and  $l_+$ . Similarly, communities  $i_-$  and  $i_+$  are symmetric if  $E_{i_+}(X) = 1 - E_{i_-}(X)$ . We aim to preserve the symmetry of the media network and therefore add links *symmetrically*.

**Definition 4** (Symmetrically Added Links). *If a community  $i_+ \in K(j_+)$  forms an additional link to outlet  $l_+$ , the community  $i_- \in K(j_-)$  forms a link to outlet  $l_-$ .*

We then consider how polarization is affected by symmetrically adding links.

**Proposition 6.** *Suppose communities  $i_-$ ,  $i_+$  form an additional link. Then, polarization can increase or decrease.*

The key departure in the analysis of adding links relative to the case of partitions is that more communities can now belong to both target sets. This feature per se decreases polarization. Also, eventually it must be the case that polarization decreases as in a network where all voters observe every outlets, polarization is minimized. Nevertheless, it can be the case that polarization increases. To see this we again distinguish between the case where candidates select the same target set and the case of a symmetric equilibrium with different target sets. If candidates select the same target sets, then we further need to distinguish whether the links were formed within the target sets or across target sets. We can show that if communities form links within the target sets, then the target sets never change and polarization is unaffected. Next, if communities form links across target sets, then it can be that (i) target sets remain unchanged, (ii) outlets switch between target sets symmetrically and (iii) candidates no longer choose the same target

sets. If target sets are unchanged, then polarization *decreases* as the overall number of targeted communities,  $k(T_{BL}) + k(T_{BR})$  has increased. The same holds true if outlets switch between target sets symmetrically, again as  $k(T_{BL}) + k(T_{BR})$  increases and both target sets are of equal size. This is no longer true if candidates choose different target sets after the links are added. In this case it can then occur that polarization increases. In particular, it can be that communities switch from the right target set to the left target set, which also in the case of adding links can lead to an increase in polarization.

Proposition 6 highlights that there is a non-monotonicity in how adding links affects the level of polarization. This implies that even if new technologies have made it possible for voters to better observe politicians campaigns, this does not necessarily lead to policies that cater to a greater set of voters. Instead, candidates adjust their campaign strategies, which ultimately can result in a higher level of polarization.

## 7 Conclusion

The paper discusses the increase in polarization and establishes the presence of electoral cycles. In order to explain these policy patterns, we develop a framework to analyze targeting in a network with heterogeneous agents and provide a new measure of network centrality that explicitly takes the characteristics of the agents into account. Based on this we show how candidates' advertising strategies and ultimately, the policies they set are influenced by the features of the network. We extend the model to a dynamic setting and show that due to advertising, policies exhibit an electoral cycle, that is with each election they fluctuate. We then ask how the features of the media network influence the level of polarization. Changes in the media landscape have made it possible to target messages very narrowly to a certain subset of voters, which generally induces an increase in polarization. Additionally, we show that if communities are connected to a higher number of media outlets then, counterintuitively, in certain environments polarization increases.

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## Appendix

**Proof Proposition 1:** We first establish that for any target set, both parties disclose the average bliss point in the target's coverage. Fix  $T_c$  the target set of politician  $c$ , and recall that his policy decision needs to solve

$$\max_{x_c} \frac{1}{2} \left[ \int_0^1 u(x_c|x) \sum_{i \in K(T_c)} g_i(x) dx + \int_0^1 u(\pi_c|x) \sum_{i \in K \setminus K(T_c)} g_i(x) dx \right]$$

Our functional form assumption and requirement on voting behaviour implicit in the model imply that the problem is equivalent to solving

$$\max_{x_c} \frac{1}{2} \int_0^1 (1 - (x_c - x)^2) \sum_{i \in K(T_c)} g_i(x) dx$$

Necessary conditions for an interior optimum then require that

$$\int_0^1 (x_c - x) \sum_{i \in K(T_c)} g_i(x) dx = 0$$

By our functional form assumptions, the condition is also sufficient for an optimum, and fully characterizes the equilibrium policy for any given target decision. Simple manipulations then establish that

$$\begin{aligned} x_c &= \left[ \int_0^1 x \sum_{i \in K(T_c)} g_i(x) dx \right] / \left[ \int_0^1 \sum_{i \in K(T_c)} g_i(x) dx \right] = \\ &= \frac{1}{k(T_c)} \sum_{i \in K(T_c)} \int_0^1 x g_i(x) dx = E(X|T_c) \end{aligned}$$

which establishes the first part of the proposition.

Begin by calculating the payoff of any possible targeting strategy. Consider a target set  $T$ , and observe that, at the optimal policy, the preference of candidate  $c$  are a positive affine transformation of

$$\begin{aligned} &\int_0^1 u(E(X|T)|x) \sum_{i \in K(T)} g_i(x) dx + \int_0^1 u(\pi_c|x) \sum_{i \in K \setminus K(T)} g_i(x) dx = \\ &\int_0^1 [1 - (E(X|T) - x)^2] \sum_{i \in K(T)} g_i(x) dx + \int_0^1 [1 - (\pi_c - x)^2] \sum_{i \in K \setminus K(T)} g_i(x) dx \end{aligned}$$

If so target set  $T$  is better than target set  $S$  if and only if

$$\begin{aligned} &\int_0^1 (E(X|T) - x)^2 \sum_{i \in K(T)} g_i(x) dx + \int_0^1 (\pi_c - x)^2 \sum_{i \in K \setminus K(T)} g_i(x) dx < \\ &\int_0^1 (E(X|S) - x)^2 \sum_{i \in K(S)} g_i(x) dx + \int_0^1 (\pi_c - x)^2 \sum_{i \in K \setminus K(S)} g_i(x) dx \end{aligned}$$

However, the definition of variance and simple manipulations establish that

$$\begin{aligned}\int_0^1 (E(X|T) - x)^2 \sum_{i \in K(T)} g_i(x) dx &= k(T) \int_0^1 (E(X|T) - x)^2 dG(x|T) \\ &= k(T) (E(X^2|T) - E(X|T)^2)\end{aligned}$$

Similarly, it also follows that

$$\begin{aligned}\int_0^1 (\pi_c - x)^2 \sum_{i \in K \setminus K(T)} g_i(x) dx &= \int_0^1 (\pi_c^2 - 2\pi_c x + x^2) \sum_{i \in K \setminus K(T)} g_i(x) dx \\ &= k(M - T) (\pi_c^2 - 2\pi_c E(X|M - T) + E(X^2|M - T))\end{aligned}$$

Using the three previous observations then implies that

$$\begin{aligned}&k(T) (E(X^2|T) - E(X|T)^2) + k(M - T) (\pi_c^2 - 2\pi_c E(X|M - T) + E(X^2|M - T)) \\ &< k(S) (E(X^2|S) - E(X|S)^2) + k(M - S) (\pi_c^2 - 2\pi_c E(X|M - S) + E(X^2|M - S))\end{aligned}\quad (10)$$

Now, observe that for any  $T$ , we have that

$$k(T)E(X^2|T) + k(M - T)E(X^2|M - T) = kE(X^2|M),$$

as we are averaging over all communities in  $K$ . Further,

$$\begin{aligned}k(M - S)E(X|M - S) - k(M - T)E(X|M - T) &= \sum_{i \in K \setminus K(S)} E_i(X) - \sum_{i \in K \setminus K(T)} E_i(X) \\ &= \sum_{i \in K(T)} E_i(X) - \sum_{i \in K(S)} E_i(X) \\ &= k(T)E(X|T) - k(S)E(X|S),\end{aligned}$$

where the second equality folds by adding and subtracting  $\sum_{i \in K} E_i(X)$ . Based on the last two simplifications, inequality (10) becomes

$$\begin{aligned}k(M - T) (\pi_c^2 - 2\pi_c E(X|M - T)) - k(T)E(X|T)^2 &< k(M - S) (\pi_c^2 - 2\pi_c E(X|M - S)) - k(S)E(X|S)^2 \\ k(S) (E(X|S)^2 + \pi_c^2) - k(T) (E(X|T)^2 + \pi_c^2) &< 2\pi_c (k(M - T)E(X|M - T) - k(M - S)E(X|M - S)) \\ k(S) (E(X|S)^2 + \pi_c^2) - k(T) (E(X|T)^2 + \pi_c^2) &< -2\pi_c (k(T)E(X|T) - k(S)E(X|S)) \\ k(S) (E(X|S) - \pi_c)^2 &< k(T) (E(X|T) - \pi_c)^2\end{aligned}$$

As the same logic applies to every target set it follows that, it is optimal to choose that target set that maximizes  $k(T) (E(X|T) - \pi_c)^2$ . We can use the concept of media centrality to compare target sets. The following lemma will be useful for this.

**Lemma 9.** Consider any two target sets  $T, T' \subseteq M$  such that  $T \subseteq T'$ . Then  $W_c(T) < W_c(T')$  if and only if

$$k(T) [E(X|T' - T) - E(X|T)]^2 < k(T') [E(X|T' - T) - \pi_c]^2. \quad (11)$$

**Proof Lemma 9:** Observe first that, since  $T \subseteq T'$ ,

$$W(T') = k(T') [E(X|T') - \pi_c]^2 = k(T') \left[ \frac{k(T)E(X|T) + k(T' - T)E(X|T' - T)}{k(T')} - \pi_c \right]^2.$$

If so,  $W(T) < W(T')$  is equivalent to

$$k(T) [E(X|T) - \pi_c]^2 < k(T') \left[ \frac{k(T)E(X|T) + k(T' - T)E(X|T' - T)}{k(T')} - \pi_c \right]^2.$$

Simple manipulations then show that this is equivalent to

$$k(T) [E(X|T' - T) - E(X|T)]^2 < k(T') [E(X|T' - T) - \pi_c]^2.$$

**Proof Lemma 1:** We assume without loss of generality that  $E(X|T') < \pi_c$ . A symmetric logic applies to the converse scenario. Observe that  $E(X|T' - T) \geq \pi_c$  implies that  $E(X|T) < \pi_c$ , as

$$E(X|T') = \frac{k(T)E(X|T) + k(T' - T)E(X|T' - T)}{k(T) + k(T' - T)} < \pi_c.$$

Moreover, this implies that

$$\pi_c - E(X|T) > \pi_c - E(X|T') > 0. \quad (12)$$

For a candidate **not** to be willing to add outlets  $T' \setminus T$  to the target set it must be that

$$k(T') [\pi_c - E(X|T')]^2 < k(T) [\pi_c - E(X|T)]^2.$$

Moreover by (12) the latter condition is equivalent to

$$k(T')^{1/2} [\pi_c - E(X|T')] < k(T)^{1/2} [\pi_c - E(X|T)].$$

Writing explicitly the expected target set, then yields the following

$$k(T')^{-1/2} \sum_{i \in K(T')} [\pi_c - E_i(X)] < k(T)^{-1/2} \sum_{i \in K(T)} [\pi_c - E_i(X)].$$

Thus the politician does not target outlets  $T' \setminus T$  if

$$\sum_{i \in K(T')} [\pi_c - E_i(X)] < \left( \frac{k(T')}{k(T)} \right)^{1/2} \sum_{i \in K(T)} [\pi_c - E_i(X)]. \quad (13)$$

But, as  $\pi_c - E(X|T' - T) \leq 0$  is equivalent to  $\sum_{i \in K(T') \setminus K(T)} [\pi_c - E_i(X)] \leq 0$ , we have that

$$\sum_{i \in K(T')} [\pi_c - E_i(X)] \leq \sum_{i \in K(T)} [\pi_c - E_i(X)].$$

If so, condition (13) must hold as well since  $k(T') > k(T)$ .

**Proof Lemma 2:** (1) First consider the case in which  $E(X|T' - T) \leq E(X|T)$ . If so,

$$E(X|T') = \frac{k(T)E(X|T) + k(T' - T)E(X|T' - T)}{k(T')} \leq E(X|T),$$

and therefore  $W_c(T) \leq W_c(T')$ , as it is possible to both increase the number of the communities targeted – as  $k(T') \geq k(T)$  – without decreasing the difference between the expected bliss point and the prior – as  $\pi_c - E(X|T') \geq \pi_c - E(X|T)$ .

Next consider the case in which  $E(X|T' - T) > E(X|T)$ . It is better to disclose also in outlet  $j$  if

$$k(T') \left[ \frac{k(T)E(X|T) + k(T' - T)E(X|T' - T)}{k(T')} - \pi_c \right]^2 \geq k(T) [E(X|T) - \pi_c]^2.$$

this inequality by the previous proof is equivalent to

$$k(T' - T) (E(X|T' - T) - \pi)^2 \geq k(T) (E(X|T) - \pi) (E(X|T) + \pi - 2E(X|T' - T)). \quad (14)$$

However by assumption we have that  $E(X|T) + \pi > 2E(X|T' - T)$  and  $E(X|T) < \pi$ . Thus the RHS of (14) is negative, whereas the LHS of (14) is positive; and thus inequality of (14) must hold.

(2) By part (1), including outlets  $T' \setminus T$  to the target set increases the candidate's payoff if  $E(X|T' - T) < E(X|T) < \pi_c$ . Thus, suppose that  $E(X|T' - T) > E(X|T)$ . By assumption  $T$  has a higher media centrality than  $T''$  and thus by Lemma 9 we have that

$$k(T) [\pi_c - E(X|T - T'')]^2 > k(T'') [E(X|T'') - E(X|T - T'')]^2.$$

We want to establish that

$$k(T') [\pi_c - E(X|T' - T)]^2 > k(T) [E(X|T) - E(X|T' - T)]^2. \quad (15)$$

Observe that, as  $E(X|T - T'') \in (E(X|T' - T), \pi_c)$ , we have that

$$[\pi_c - E(X|T' - T)]^2 > [\pi_c - E(X|T - T'')]^2,$$

Similarly, as  $E(X|T' - T) > E(X|T)$ , we have that

$$[E(X|T) - E(X|T - T'')]^2 > [E(X|T) - E(X|T' - T)]^2.$$

But, from the previous inequalities we have that

$$\begin{aligned} k(T') [\pi_c - E(X|T' - T)]^2 &> k(T') [\pi_c - E(X|T - T'')]^2 \\ &> \frac{k(T')k(T'')}{k(T)} [E(X|T'') - E(X|T - T'')]^2 \\ &= \frac{k(T')k(T)}{k(T'')} [E(X|T) - E(X|T - T'')]^2 \\ &> k(T) [E(X|T) - E(X|T - T'')]^2 \\ &> k(T) [E(X|T) - E(X|T' - T)]^2. \end{aligned}$$

where the equality follows as  $k(T'')E(X|T'') = k(T)E(X|T) - k(T - T'')E(X|T - T'')$ . This completes the proof by establishing (15).

**Proof of Lemma 3** A partition of the communities of outlet  $j$  can either lead to (i) no change at all (ii) an outlet dropped from the target set.

Suppose first that the communities in the target set remain the same. Then, it can also not be optimal to add an outlet to the target set. Note that none of the other outlets have changed, that is for any outlet  $o \notin T$ , Lemma 9 remains unchanged. Then, it can also not be optimal to add an additional outlet,  $o \notin T$  to the target set.

Suppose next that an outlet is dropped from the target set, that is there are fewer communities in the target set  $K(T') \subset K(T)$ . By the definition of media centrality, it must hold that  $E(X|T') < E(X|T)$  as  $k(T') < k(T)$ .

We then show that it cannot be optimal for an outlet  $o \notin T$  to be added once outlets have been dropped. Formally, if  $o \notin T$ , then  $o \notin T'$ . We denote the outlets that are dropped by  $j_i$ . Recall that  $K(j_i) \cap K(o) = \emptyset$ . Given that it has not been optimal to add outlet  $o$  to target set  $T$  it must hold that

$$k(T) (E(X|o - T) - E(X|T))^2 > (k(T) + k(o - T)) (E(X|o - T) - \pi)^2 \quad (16)$$

This implies that it is also not optimal to add outlet  $o$  to target set  $T'$ ,

$$k(T') (E(X|o - T') - E(X|T'))^2 > (k(T') + k(o - T')) (E(X|o - T') - \pi)^2 \quad (17)$$



Note first that given  $K(j_i) \cap K(o) = \emptyset$ ,  $E(X|o - T) = E(X|o - T')$  and  $k(o - T) = k(o - T')$ . This implies that RHS of inequality (16) is larger than the right hand side of inequality (17). Additionally, it has to hold that

$$k(T') (E(T') - \pi)^2 > k(T) (E(T) - \pi)^2 \quad (18)$$

If  $E(X|o - T) > \pi$ , then by Lemma 1,  $o \notin T$ . We therefore focus on  $E(X|o - T) < \pi$ . By Lemma 2 for  $o \notin T$ , it must be that  $E(X|T) < E(X|o - T)$ . This implies that

$$E(X|T') < E(X|T) < E(X|o - T) < \pi$$

By Lemma 13 C.1 together with inequality 18, this shows that the LHS of inequality (17) is greater than the LHS of inequality (16) which establishes that inequality (17) is fulfilled.

**Proof Lemma 4:** Note first that we have ruled out that the target set switches, as

$$\max\{E(X|T), E'(X|T')\} < \pi.$$

We maintain this assumption and proceed as follows. We first consider case 1, that is a link if formed within the target set. Recall that  $j \in T$  denotes an outlet such that  $i \in K(j)$  – which must exist by assumption. We first show that for  $j \in T'$ , it must be that  $T = T'$ .

**Lemma 10.** *If  $j \in T \cap T'$ , then  $T = T'$ .*

**Proof:** If  $j \in T'$ , it must be that

$$W'(T') = W(T').$$

Assume by contradiction that  $T' \neq T$ . If so, it must be that

$$W(T') < W(T).$$

But as  $j \in T$  it must be that

$$W(T) = W'(T).$$

But this implies that

$$W'(T') < W'(T),$$

which contradicts the optimality of  $T'$ .

Next we show that if  $j \in T \setminus T'$ , it there does not exists an outlet  $o \notin T$ , but  $o \in T'$ .

**Lemma 11.** *Let  $j \in T \setminus T'$  and  $l \in T$  then  $T' \subseteq T$ .*

**Proof:** To see this consider three outlets, call them  $\{1, 2, 3\}$ . To begin with assume that their coverages do not overlap and that  $j = T \setminus T'$ . We then proceed by contradiction, that is it is not optimal to include  $j$  in the new target set, but it is optimal to include  $o$ . Denote by  $n_1$  the number of groups in outlet 1, by  $n_3$  the number of groups in outlet 2 that only belong to outlet 2 and by  $n_4$  the number of groups in outlet 3. The bliss points are given along the same lines that is  $x_1$  is the expected bliss point in outlet 1,  $x_2$  the bliss point of the group that forms a new connection,  $x_3$  is the bliss point of all other groups in outlet 2,  $x_4$  the bliss point in outlet 3. Suppose that suppose that  $l = 1$  and  $j = 2$ . If  $T = \{1, 2\}$ , but  $3 \in T'$ , then has to be the case that

$$(n_1 + n_3 + 1) \left( \frac{x_2 + n_3 x_3}{1 + n_3} - \pi \right)^2 > n_1 \left( \frac{x_2 + n_3 x_3}{1 + n_3} - x_1 \right)^2 \quad (19)$$

$$(n_1 + 1) \left( x_3 - \frac{n_1 x_1 + x_2}{n_1 + 1} \right)^2 > (n_1 + n_3 + 1) (x_3 - \pi)^2 \quad (20)$$

$$(n_1 + 1 + n_3) \left( x_4 - \frac{n_1 x_1 + x_2 + n_3 x_3}{n_1 + 1 + n_3} \right)^2 > (n_1 + 1 + n_3 + n_4) (x_4 - \pi)^2 \quad (21)$$

$$(n_1 + 1 + n_4) (x_4 - \pi)^2 > (n_1 + 1) \left( x_4 - \frac{n_1 x_1 + x_2}{n_1 + 1} \right)^2 \quad (22)$$

The first equation requires  $W(1, 2) > W(1)$ ; the second requires  $W'(1, 2) < W'(1)$ ; the third requires  $W(1, 2, 3) < W(1, 2)$ ; and the last requires  $W'(1, 3) > W'(1)$ . The last condition holds as  $T' \neq \{1, 2, 3\}$  since

$$W(1, 2, 3) = W'(1, 2, 3) \leq W(1, 2) = W'(1, 2).$$

To make the problem more tractable I define the following coefficients

$$\begin{aligned} A_0 &= 1 + n_1, \\ A_1 &= n_1 + 1 + n_3, \\ A_2 &= n_1 + 1 + n_4, \\ A_3 &= n_1 + 1 + n_3 + n_4. \end{aligned}$$

Equation 19 holds if and only if

$$\frac{1}{n_3} \left( A_1 \pi - n_1 x_1 - x_2 - \sqrt{n_1 A_1} (\pi - x_1) \right) > x_3.$$

Equation 20 holds if and only if

$$x_3 > \frac{1}{n_3} \left( A_1\pi - n_1x_1 - x_2 - \sqrt{\frac{A_1}{A_0}}((1+n_1)\pi - n_1x_1 - x_2) \right).$$

Equation 21 holds if and only if

$$x_4 > \frac{1}{n_4} \left( A_3\pi - n_1x_1 - x_2 - n_3x_3 - \sqrt{\frac{A_3}{A_1}}(A_1\pi - n_1x_1 - x_2 - n_3x_3) \right).$$

Equation 22 holds if and only if

$$x_4 < \frac{1}{n_4} \left( A_2\pi - n_1x_1 - x_2 - \sqrt{\frac{A_2}{A_0}}(A_0\pi - n_1x_1 - x_2) \right).$$

For the last two equations to hold, it must be that

$$\begin{aligned} \left( A_2\pi - \sqrt{\frac{A_2}{A_0}}(A_0\pi - n_1x_1 - x_2) \right) &> \left( A_3\pi - n_3x_3 - \sqrt{\frac{A_3}{A_1}}(A_1\pi - n_1x_1 - x_2 - n_3x_3) \right) \\ \frac{1}{\sqrt{\frac{A_3}{A_1}} - 1} \left( (A_2 - A_3 + \sqrt{A_1A_3} - \sqrt{A_0A_2}) \pi + \left( \sqrt{\frac{A_2}{A_0}} - \sqrt{\frac{A_3}{A_1}} \right) (n_1x_1 + x_2) \right) &> n_3x_3 \end{aligned}$$

As we also have a lower bound on  $x_3$  is must be that

$$\begin{aligned} \frac{1}{\sqrt{\frac{A_3}{A_1}} - 1} \left( (A_2 - A_3 + \sqrt{A_1A_3} - \sqrt{A_0A_2}) \pi + \left( \sqrt{\frac{A_2}{A_0}} - \sqrt{\frac{A_3}{A_1}} \right) (n_1x_1 + x_2) \right) & \quad (23) \\ &> \left( A_1\pi - n_1x_1 - x_2 - \sqrt{\frac{A_1}{A_0}}(A_0\pi - n_1x_1 - x_2) \right), \end{aligned}$$

which never holds and which gives the contradiction we were looking for.

The first part of the argument relies on two assumptions, namely: (1) coverages have no overlap, (2)  $j = T \setminus T'$ . We remove these assumptions one after the other. We first consider the case in which all outlets possibly overlap. Denote: by  $n_1$  the number of groups that belong only to outlet 1 and by  $y_1$  the bliss point in these groups; by  $n_2$  the number of groups that belong both to outlet 1 and 2 and by  $y_2$  the bliss point in these groups; by  $n_3$  the number of groups that belong both to outlets 1 and 3 and by  $y_3$  the bliss point in these groups; by  $n_4$  the number of groups that belong only to all 3 outlets and by  $y_4$  the bliss point in these groups; by  $n_7$  the number of groups that belong both to outlets 2 and 3 and by  $y_7$  the bliss point in these groups. Finally denote by  $n_6 + 1$  the number of groups that belong only to outlet 2, by  $y_6$  the bliss point

in these groups except  $i$ , and by  $y_5$  the bliss point in group  $i$ . Again suppose that suppose that  $l = 1$  and  $j = 2$ . If  $T = \{1, 2\}$ , but  $3 \in T'$ , then equations 19-22 now become

$$\begin{aligned} \sum_{i=1}^7 n_i \left( \frac{y_5 + n_6 y_6 + n_7 y_7}{1 + n_6 + n_7} - \pi \right)^2 &> \sum_{i=1}^4 n_i \left( \frac{y_5 + n_6 y_6 + n_7 y_7}{1 + n_6 + n_7} - \frac{\sum_{i=1}^4 n_i y_i}{\sum_{i=1}^4 n_i} \right)^2 \\ \sum_{i=1}^7 n_i \left( \frac{n_6 y_6 + n_7 y_7}{n_6 + n_7} - \pi \right)^2 &< \sum_{i=1}^5 n_i \left( \frac{n_6 y_6 + n_7 y_7}{n_6 + n_7} - \frac{\sum_{i=1}^5 n_i y_i}{\sum_{i=1}^5 n_i} \right)^2 \\ \sum_{i=1}^8 n_i (y_8 - \pi)^2 &< \sum_{i=1}^7 n_i \left( y_8 - \frac{\sum_{i=1}^7 n_i y_i}{\sum_{i=1}^7 n_i} \right)^2 \\ \sum_{i=1-5,7,8} n_i \left( \frac{n_7 y_7 + n_8 y_8}{n_7 + n_8} - \pi \right)^2 &> \sum_{i=1}^5 n_i \left( \frac{n_7 y_7 + n_8 y_8}{n_7 + n_8} - \frac{\sum_{i=1}^5 n_i y_i}{\sum_{i=1}^5 n_i} \right)^2 \end{aligned}$$

Additionally, it must be the case that

$$\begin{aligned} \sum_{i=1}^7 n_i \left( \frac{\sum_{i=1}^7 n_i y_i}{\sum_{i=1}^7 n_i} - \pi \right)^2 &> \sum_{i=1-4,7,8} n_i \left( \frac{\sum_{i=1-4,7,8} n_i y_i}{\sum_{i=1-5,7,8} n_i} - \pi \right)^2 \\ \sum_{i=1}^8 n_i (y_6 - \pi)^2 &< \sum_{i=1-5,7,8} n_i \left( y_6 - \frac{\sum_{i=1-5,7,8} n_i y_i}{\sum_{i=1-5,7,8} n_i} \right)^2 \end{aligned}$$

To simplify notation I set

$$k_1 = \sum_{i=1}^4 n_i \quad \text{and} \quad x_1 = \sum_{i=1}^4 n_i y_i.$$

Similar to before I now define

$$\begin{aligned} B_0 &= k_1 + 1, \\ B_1 &= k_1 + 1 + n_6 + n_7, \\ B_2 &= k_1 + 1 + n_7 + n_8, \\ B_3 &= k_1 + 1 + n_6 + n_7 + n_8. \end{aligned}$$

Again, it has to be the case that

$$y_8 > \frac{1}{n_8} \left( B_3 \pi - x_1 - y_5 - n_7 y_7 - \sqrt{\frac{B_3}{B_1}} (B_1 \pi - x_1 - y_5 - n_7 y_7) + n_6 y_6 \left( -1 + \sqrt{\frac{B_3}{B_1}} \right) \right).$$

as well as

$$y_8 < \frac{1}{n_8} \left( B_2 \pi - x_1 - y_5 - n_7 y_7 - \sqrt{\frac{B_2}{B_0}} (B_0 \pi - x_1 - y_5) \right).$$

For this to hold it has to be the case that

$$\begin{aligned} & \left( B_2\pi - x_1 - y_5 - n_7y_7 - \sqrt{\frac{B_2}{B_0}}(B_0\pi - x_1 - y_5) \right) \\ & > \left( B_3\pi - x_1 - y_5 - n_7y_7 - \sqrt{\frac{B_3}{B_1}}(B_1\pi - x_1 - y_5 - n_7y_7) + n_6y_6 \left( -1 + \sqrt{\frac{B_3}{B_1}} \right) \right) \end{aligned}$$

which is equivalent to

$$\frac{1}{\sqrt{\frac{B_3}{B_1}} - 1} \left( (B_2 - B_3 + \sqrt{B_1B_3} - \sqrt{B_0B_2}) \pi + \left( \sqrt{\frac{B_2}{B_0}} - \sqrt{\frac{B_3}{B_1}} \right) (x_1 + y_5) - \sqrt{\frac{B_3}{B_1}} n_7y_7 \right) > n_6y_6.$$

We have again a lower bound  $y_6$ , namely

$$\begin{aligned} (n_6 + n_7)n_6y_6 & > (1 + n_6 + n_7)B_1\pi - (1 + n_6 + n_7)x_1 - (1 + n_6 + n_7)y_5 - (n_6 + n_7)n_7y_7 \\ & \quad - (1 + n_6 + n_7)\sqrt{\frac{B_1}{B_0}}(B_0\pi - x_1 - y_5) \end{aligned}$$

To simplify notation once again, we let

$$B_4 = \frac{1 + n_6 + n_7}{n_6 + n_7}$$

Then,

$$n_6y_6 > B_4B_1\pi - B_4x_1 - B_4y_5 - n_7y_7 - B_4\sqrt{\frac{B_1}{B_0}}(B_0\pi - x_1 - y_5)$$

and so it has to be the case that

$$\begin{aligned} & \frac{1}{\sqrt{\frac{B_3}{B_1}} - 1} \left( (B_2 - B_3 + \sqrt{B_1B_3} - \sqrt{B_0B_2}) \pi + \left( \sqrt{\frac{B_2}{B_0}} - \sqrt{\frac{B_3}{B_1}} \right) (x_1 + y_5) - \sqrt{\frac{B_3}{B_1}} n_7y_7 \right) \\ & \quad > B_4(B_1\pi - x_1 - y_5) - n_7y_7 - B_4\sqrt{\frac{B_1}{B_0}}(B_0\pi - x_1 - y_5) \\ & \frac{1}{\sqrt{\frac{B_3}{B_1}} - 1} \left( (B_2 - B_3 + \sqrt{B_1B_3} - \sqrt{B_0B_2}) \pi + \left( \sqrt{\frac{B_2}{B_0}} - \sqrt{\frac{B_3}{B_1}} \right) (x_1 + y_5) - \left( \sqrt{\frac{B_3}{B_1}} - 1 \right) n_7y_7 \right) \\ & \quad > B_4 \left( B_1\pi - x_1 - y_5 - \sqrt{\frac{B_1}{B_0}}(B_0\pi - x_1 - y_5) \right) \end{aligned}$$

Comparing this to equations 23, we can see that the left hand side has decreased and the right hand side has increased. Thus, as 23 did not hold, it cannot be that case that this equation holds. Given this result, we can restrict attention to the case without overlap.

Consider last the case where an additional outlet is also dropped from the target set. Then we simply need to redefine  $x_3$  and the logic remains the same and the results as well. This establishes that if the target set remains on the same side of the prior and an outlet is omitted, that it then cannot be optimal for an outlet that was originally not included in the target set to be included in the new target set.

We have thus established that if a community establishes an additional link to a an outlet contained in the target set, then  $T' \subseteq T$ . It is clear that if the target sets are identical with  $T' = T$ ,  $E(X|T) = E'(X|T')$ . Consider next the case that  $T' \subset T$ . Then  $k(T) \geq k(T')$ . Given optimality of  $T'$ , it must be that  $W(T) = W'(T) < W'(T')$ , where  $W(T) = k(T) (E(X|T) - \pi)^2$  and  $W'(T') = k(T') (E'(X|T') - \pi)^2$ . For  $W(T) < W'(T)$  and  $k(T) \geq k(T')$  it then must be the case that  $(E(X|T) - \pi)^2 < (E'(X|T') - \pi)^2$ . As we have assumed that the bliss points of the target sets do not switch  $E(X|T), E'(X|T') < \pi$ ,  $\pi - E'(X|T') > \pi - E(X|T)$ , which is equivalent to  $E(X|T) > E'(X|T')$ .

We then turn to the case where a group that belongs to a targeted outlet forms a link to an outlet that does not belong to the target set. There we establish that next lemma.

**Lemma 12.** *Let  $l \notin T$  and  $i \in K(T)$ . If  $j \in T'$ ,  $T' = T$ . If  $j \notin T'$ ,  $l \in T'$*

**Proof:** We have already established in the proof to Lemma 10, that if  $j \in T'$ , it must be that  $T = T'$ . It remains to be shown what happens if  $j \notin T'$ . First note that if  $j \notin T'$ , then  $l \in T'$ . Suppose by contradiction that  $j \notin T'$  and  $l \notin T'$ . It must be the case that  $W(T \setminus j) < W(T)$ , otherwise it would not have been optimal to add outlet  $j$ . Note that  $W'(T \setminus j) = W(T \setminus j)$  and  $W'(T) = W(T)$  and so  $W'(T \setminus j) < W'(T)$ . This establishes that it cannot be optimal to only omit outlet  $j$  from the target set, without making any other changes to it. There are two possibilities given our assumption that  $l \notin T'$ . Either another outlet  $o \neq l$ ,  $o \in T$  is omitted from the target set or an outlet  $o \notin T$  is added to the target set. Consider first the case where an outlet is omitted from the target set. The media centrality of such a target set is  $W(T \setminus (j \cup o)) = W'(T \setminus (j \cup o))$ , as the change in the linking structure has not affected outlet  $o$ . It must be the case that  $W(T \setminus (j \cup o)) < W(T)$ . But this implies that  $W'(T \setminus (j \cup o)) = W'(T') < W'(T)$ , contradicting the optimality of  $T'$ . Consider next the case where  $o \notin T$  is added to the target set. The media centrality of such a target

set is  $W((T \cup o) \setminus j) = W'((T \cup o) \setminus j)$ , as again the added link does not affect  $o$ . It holds that  $W((T \cup o) \setminus j) < W(T)$  and thus  $W'((T \cup o) \setminus j) = W'(T') < W'(T)$ , contradicting the optimality of  $T'$ . Therefore, it must be the case that if  $j \notin T'$ , then  $l \in T'$ .

**Proof Proposition 2:** (2) Obviously, the probability of either of the two candidates winning the election must necessarily equal 1 – that is,  $P_A(x_A, x_B|x) + P_B(x_A, x_B|x) = 1$ . However, if candidate  $c$  enjoys a probability of winning that strictly exceeds  $1/2$ , then the other candidate must be losing with a probability that exceeds  $1/2$ . But if so, the strategy of the latter candidate cannot be optimal as, by mimicking candidate  $c$ , he could increase his odds of winning to  $1/2$ , when candidates are homogeneous. Thus, if  $\pi_A = \pi_B$ , both candidates must win with probability  $1/2$  in any equilibrium. Moreover, (1) must hold as candidates' best responses are independent of the competitor's strategy and identical.

(3) By Proposition 1, both candidates set the same policy when targeting the same media outlets. Thus, to prove the last part of claim it suffices to show that candidates generically select the same media coverage. Clearly, symmetric equilibria always exist as either candidate chooses the optimal target by solving

$$\max_{T \subseteq M} k(T) [E(X|T) - \pi]^2. \quad (24)$$

If (24) admits a single solution, any equilibrium is necessarily symmetric. To conclude that this is generically the case, denote by  $E_i$  the average bliss point  $E_i(X)$  in every community  $i \in K$ . Suppose that every  $E_i$  is drawn from a continuous distribution with no atoms and with support  $[l, h] \subseteq [0, 1]$ .<sup>34</sup> If so, for any  $T, S \subseteq M$  such that  $K(T) \neq K(S)$ , we have that

$$\Pr(E(X|T) = E(X|S)) = \Pr\left(k(S)\sum_{i \in K(T)} E_i = k(T)\sum_{i \in K(S)} E_i\right) = 0.$$

The second equality holds as  $k(S)\sum_{i \in K(T)} E_i = k(T)\sum_{i \in K(S)} E_i$  defines a hyperplane in  $[l, h]^k$  when  $K(T) \neq K(S)$ , and because any such hyperplane has measure zero in  $[l, h]^k$  when all the  $E_i$  are drawn independently from a common distribution with no atoms.

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<sup>34</sup>Implicitly, we are assuming that  $G_i$  is determined once  $E_i(X)$  has been set so to satisfy

$$E_i(X) = \int_0^1 x dG_i(x).$$

**Proof of Proposition 3** The policy platform of candidate  $c$  is given by

$$x_c = \frac{k(T_c)E(X|T_c) + DE_c(X)}{k(T_c) + D}$$

We want to ensure that

$$\frac{k(T_{max})E(X|T_{max}) + DE_A(X)}{k(T_{max}) + D} < \frac{k(T_{min})E(X|T_{min}) + DE_A(X)}{k(T_{min}) + D}$$

This can be rewritten as

$$\begin{aligned} & k(T_{max})k(T_{min})(E(X|T_{max}) - E(X|T_{min})) + D(k(T_{max})E(X|T_{max}) - k(T_{min})E(X|T_{min})) \\ & < D^2(E_B(X) - E_A(X)) + D(k(T_{max})E_B(X) - k(T_{min})E_A(X)) \end{aligned}$$

It is always the case that

$$D(k(T_{max})E(X|T_{max}) - k(T_{min})E(X|T_{min})) \leq D(k(T_{max})E_B(X) - k(T_{min})E_A(X))$$

as  $E_A(X) \leq E_i(X) \leq E_B(X)$ . Therefore, if

$$k(T_{max})k(T_{min})(E(X|T_{max}) - E(X|T_{min})) \leq D^2(E_B(X) - E_A(X))$$

it must be that  $x_A \leq x_B$ .

**Proof of Proposition 4** Let there be two outlets  $j, j' \in M$  with  $k(j - j') > 0$ . Consider an arbitrary target set  $T^t$ . Then the media centrality in  $t + 1$  is given by

$$(k(T^{t+1}) + D) \left( \frac{k(T^{t+1})E(X|T^{t+1}) + DE_c(X)}{k(T^{t+1}) + D} - \frac{k(T^t)E(X|T^t) + DE_c(X)}{k(T^t) + D} \right)^2$$

By setting  $T^{t+1} = T^t$ , media centrality is zero. Choosing any other target set leads to an increase in media centrality. Even if  $E(X|T^{t+1}) = E(X|T^t)$ , choosing different target sets that induce differences in  $k(T^{t+1})$  and  $k(T^t)$  lead to a higher media centrality. Finding these target sets is always possible as there exist at least two outlets with  $k(j - j') > 0$ .

**Proof of Lemma 5 (1)** Before turning to the actual proof, we establish a Lemma which helps understand how policies move over time.

**Lemma 13.** *Let one of the following conditions hold.*



$$\mathbf{C.1} \quad 0 < E(X|T) < E(X|T') < \pi' < \pi < 1$$

$$\mathbf{C.2} \quad 0 < E(X|T') < E(X|T) < \pi < \pi' < 1$$

$$\mathbf{C.3} \quad 0 < E(X|T) < \pi < \pi' < E(X|T') < 1$$

Then, if  $T$  is preferred to  $T'$  given prior  $\pi$ , it is also preferred given prior  $\pi'$ .

**Proof of Lemma 13**  $T$  is preferred to  $T'$  given prior  $\pi$ , that is

$$k(T) (E(X|T) - \pi)^2 > k(T') (E(X|T') - \pi)^2$$

Then it is straightforward to verify that it cannot be the case that

$$k(T') (E(X|T') - \pi')^2 > k(T) (E(X|T) - \pi')^2$$

Based on this, we can then turn to the actual proof. We aim to show that *in the long run* a politicians platform  $x_c^t \in \{E(X|T_L), E(X|T_H)\}$  for  $t > \bar{t}$ , with  $E(X|T_L) < E(X|T_H)$  and switches between these two policies that is for some  $t$ ,  $x_c^t = E(X|T_L)$ ,  $x_c^{t+1} = E(X|T_H)$  and  $x_c^{t+2} = E(X|T_L)$ .

Note first that for a given  $\pi$ , there exists a generically unique target set  $T$  that maximizes  $W_c(T)$ . To see this denote by  $\mathcal{P}(M)$  the power set of outlets  $M$ . The power set contains any possible target set. Every set has a maximum. This maximum is generically unique, by the same logic as used in the proof of Proposition 2. Denote by  $T^t$  the target set in period  $t$ . Such a target set leads to policy  $E(X|T^t)$ .

**Part 1:** Suppose first that  $E(X|T^1) < E(X|T^2) < E(X|T^3)$ . We are then interested in where  $E(X|T^4)$  lies.

We first establish that

$$E(X|T^1) < E(X|T^4) < E(X|T^3) \tag{25}$$

To see this note that it must have been optimal to select  $E(X|T^2)$  over  $E(X|T^3)$  as the platform given prior  $E(X|T^1)$ , that is

$$k(T^2) (E(X|T^2) - E(X|T^1))^2 > k(T^3) (E(X|T^3) - E(X|T^1))^2$$

Further, it must be the case that it is better to select platform  $E(X|T^3)$  over  $E(X|T^4)$  given prior  $E(X|T^2)$ ,

$$k(T^3) (E(X|T^3) - E(X|T^2))^2 > k(T^4) (E(X|T^4) - E(X|T^2))^2$$

Last, it must be the case that  $E(X|T^4)$  is preferred to  $E(X|T^2)$  for prior  $E(X|T^3)$ ,

$$k(T^4) (E(X|T^4) - E(X|T^3))^2 > k(T^2) (E(X|T^2) - E(X|T^3))^2$$

Taking these three equations together, it is straightforward to show that they hold if and only if  $E(X|T^1) < E(X|T^4) < E(X|T^3)$ . If  $E(X|T^4) = E(X|T^2)$ , that is the target sets and policies in period 2 and 4 are identical, then we have established that there is indeed cycling between two alternatives.

If, on the other hand,  $E(X|T^4) \neq E(X|T^2)$ , it has to be the case that

$$k(T^2) (E(X|T^2) - E(X|T^1))^2 > k(T^4) (E(X|T^4) - E(X|T^1))^2.$$

It has to be better to choose  $E(X|T^2)$  rather than  $E(X|T^4)$  given prior  $E(X|T^1)$ . This can only hold for

$$E(X|T^1) < E(X|T^4) < E(X|T^2). \quad (26)$$

We then have a sequence of policy platforms, where  $E(X|T^1) < E(X|T^4) < E(X|T^2) < E(X|T^3)$ .

Based on this we turn to  $E(X|T^5)$ . If  $E(X|T^5) = E(X|T^3)$ , then we have again established that there is cycling between two alternatives. If  $E(X|T^5) \neq E(X|T^3)$ , we again need to characterize where  $E(X|T^5)$  can lie. Similar to the previous step, it now has to hold that

$$\begin{aligned} k(T^3) (E(X|T^3) - E(X|T^2))^2 &> k(T^5) (E(X|T^5) - E(X|T^2))^2 \\ k(T^5) (E(X|T^5) - E(X|T^4))^2 &> k(T^3) (E(X|T^3) - E(X|T^4))^2 \end{aligned}$$

which by Lemma 13 C.2 is the case if and only if

$$E(X|T^4) < E(X|T^5) < E(X|T^3) \quad (27)$$

Additionally, it has to hold that

$$\begin{aligned} k(T^3) (E(X|T^3) - E(X|T^2))^2 &> k(T^4) (E(X|T^4) - E(X|T^2))^2 \\ k(T^4) (E(X|T^4) - E(X|T^3))^2 &> k(T^5) (E(X|T^5) - E(X|T^3))^2, \end{aligned}$$

which together with the previous equations is fulfilled if and only if

$$E(X|T^2) < E(X|T^5) < E(X|T^3) \quad (28)$$

We then have restricted the region in which  $E(X|T^5)$  can lie and established the following ordering

$$E(X|T^1) < E(X|T^4) < E(X|T^2) < E(X|T^5) < E(X|T^3) \quad (29)$$

We now consider  $E(X|T^6)$ . Again, if  $E(X|T^6) = E(X|T^4)$  we have established cycling between exactly two alternatives. Otherwise, it has to be the case that the following five equations have to be fulfilled

$$\begin{aligned} k(T^2) (E(X|T^2) - E(X|T^1))^2 &> k(T^5) (E(X|T^5) - E(X|T^1))^2 \\ k(T^3) (E(X|T^3) - E(X|T^2))^2 &> k(T^6) (E(X|T^6) - E(X|T^2))^2 \\ k(T^4) (E(X|T^4) - E(X|T^3))^2 &> k(T^2) (E(X|T^2) - E(X|T^3))^2 \\ k(T^5) (E(X|T^5) - E(X|T^4))^2 &> k(T^3) (E(X|T^3) - E(X|T^4))^2 \\ k(T^6) (E(X|T^6) - E(X|T^5))^2 &> k(T^4) (E(X|T^4) - E(X|T^5))^2 \end{aligned}$$

For these equations to hold it must be that  $E(X|T^1) < E(X|T^6) < E(X|T^5)$ . Further, it cannot be the case that  $E(X|T^4) < E(X|T^6)$ , by Lemma 13 C.1. It therefore must be the case that  $E(X|T^4) > E(X|T^6) > E(X|T^1)$ . We then have the following order,

$$E(X|T^1) < E(X|T^6) < E(X|T^4) < E(X|T^2) < E(X|T^5) < E(X|T^3) \quad (30)$$

Then, continuing this reasoning it must be  $E(X|T^2) < E(X|T^7) \leq E(X|T^5)$ ,  $E(X|T^1) < E(X|T^8) \leq E(X|T^6)$  etc. This implies that there are two bounded sets, with the policy chosen in odd periods moving closer to  $E(X|T^1)$  and the policy chosen in even periods moving closer to  $E(X|T^2)$ , that is the bliss points of the optimal target sets move to the left over time. It cannot be that one of the bliss points moves to the right, as  $E(X|T^6) < E(X|T^5)$ . This implies that for any target set with bliss point  $E(X|T^t) > E(X|T^2)$ ,  $E(X|T^{t+1}) < E(X|T^t)$ . Eventually, it must be the case that either  $E(X|T^t) = E(X|T^{t+2})$ , or that the last possible target set to the right of one of the boundaries, that is of  $E(X|T^2)$  or  $E(X|T^1)$  has been reached, which implies cycling between two alternatives.

**Part 2** Suppose next that  $E(X|T^1) < E(X|T^2)$ , but  $E(X|T^3) < E(X|T^2)$ . We need to distinguish between two cases, namely,  $E(X|T^3) < E(X|T^1)$  and  $E(X|T^1) < E(X|T^3) < E(X|T^2)$ .

Step 1 We first let  $E(X|T^3) < E(X|T^1)$ . We again ask where  $E(X|T^4)$  can lie. If  $E(X|T^4) = E(X|T^2)$ , the proof is completed. We therefore proceed to the case of  $E(X|T^4) \neq E(X|T^2)$ . By Lemma 13 C.1, it must be that  $E(X|T^4) < E(X|T^2)$ . We can further show that  $E(X|T^1) < E(X|T^4)$ . The following equations have to be fulfilled:

$$\begin{aligned} k(T^2) (E(X|T^2) - E(X|T^1))^2 &> k(T^3) (E(X|T^3) - E(X|T^1))^2, \\ k(T^3) (E(X|T^3) - E(X|T^2))^2 &> k(T^4) (E(X|T^4) - E(X|T^2))^2, \\ k(T^4) (E(X|T^4) - E(X|T^3))^2 &> k(T^2) (E(X|T^2) - E(X|T^3))^2, \end{aligned}$$

and they hold only if  $E(X|T^1) < E(X|T^4)$ . Therefore, it must be that

$$E(X|T^1) < E(X|T^4) < E(X|T^2),$$

which results in the following overall ordering,

$$E(X|T^3) < E(X|T^1) < E(X|T^4) < E(X|T^2) \tag{31}$$

We then turn to  $E(X|T^5)$ . If  $E(X|T^5) = E(X|T^3)$ , we have established that there is cycling between two policy platforms. Therefore, let  $E(X|T^5) \neq E(X|T^3)$ . Then there are two possible cases, namely (i)  $E(X|T^5) < E(X|T^4)$  and (ii)  $E(X|T^5) > E(X|T^4)$ .

**(i)**  $E(X|T^5) < E(X|T^4)$ : By Lemma 13 C.1, it must be that  $E(X|T^5) < E(X|T^3)$ . Continuing this reasoning, it must be that  $E(X|T^6) \leq E(X|T^4)$ . This implies that both platforms gradually move left. As  $0 \leq E(X|T)$ , at some point we must reach cycling between two alternatives.

**(ii)**  $E(X|T^5) > E(X|T^4)$ : By Lemma 13 C.1,  $E(X|T^5) > E(X|T^2)$ , which yields the following ordering,

$$E(X|T^3) < E(X|T^1) < E(X|T^4) < E(X|T^2) < E(X|T^5)$$

We then turn to  $E(X|T^6)$ , assuming that  $E(X|T^6) \neq E(X|T^4)$ . It cannot be the case that  $E(X|T^6) > E(X|T^5)$ , by Lemma 13 C.3. Then, it has to be the case that  $E(X|T^6) < E(X|T^5)$ . By Lemma 13 C.2 it must hold that  $E(X|T^6) > E(X|T^3)$ . Further, it must be the case that

$E(X|T^6) < E(X|T^4)$  as for  $E(X|T^6) > E(X|T^4)$  the following two equations are never fulfilled:

$$\begin{aligned} k(T^4) (E(X|T^4) - E(X|T^3))^2 &> k(T^6) (E(X|T^6) - E(X|T^3))^2 \\ k(T^6) (E(X|T^6) - E(X|T^5))^2 &> k(T^4) (E(X|T^4) - E(X|T^5))^2, \end{aligned}$$

Last, we show that  $E(X|T^1) < E(X|T^6) < E(X|T^4)$ . It must be the case that

$$\begin{aligned} k(T^2) (E(X|T^2) - E(X|T^1))^2 &> k(T^5) (E(X|T^5) - E(X|T^1))^2 \\ k(T^3) (E(X|T^3) - E(X|T^2))^2 &> k(T^6) (E(X|T^6) - E(X|T^2))^2 \\ k(T^4) (E(X|T^4) - E(X|T^3))^2 &> k(T^2) (E(X|T^2) - E(X|T^3))^2 \\ k(T^5) (E(X|T^5) - E(X|T^4))^2 &> k(T^3) (E(X|T^3) - E(X|T^4))^2 \\ k(T^6) (E(X|T^6) - E(X|T^5))^2 &> k(T^4) (E(X|T^4) - E(X|T^5))^2 \end{aligned}$$

These equations only hold for  $E(X|T^1) < E(X|T^6)$  and thus it must be

$$E(X|T^3) < E(X|T^1) < E(X|T^6) < E(X|T^4) < E(X|T^2) < E(X|T^5)$$

Based on this, the policy platform on the left  $E(X|T_L) \in (E(X|T^1), E(X|T^4))$  and  $E(X|T_R) \in (E(X|T^2), E(X|T^5))$  and  $E(X|T^{t+2}) \leq E(X|T^t)$ . As the set is bounded, eventually cycling between two platforms must be reached.

*Step 2* We now turn to the case where  $E(X|T^1) < E(X|T^3) < E(X|T^2)$ . Note that if  $E(X|T^4) < E(X|T^3) < E(X|T^2)$ , the setting is as in Part 1 and we have shown that this results in cycling between two alternatives. We therefore restrict attention to the case where  $E(X|T^4) > E(X|T^3)$ . By Lemma 13 C.1, it must be the case that  $E(X|T^4) > E(X|T^2)$  and thus  $E(X|T^1) < E(X|T^3) < E(X|T^2) < E(X|T^4)$ . By Lemma 13 C.2,  $E(X|T^5) \leq E(X|T^3)$  and by Lemma 13 C.3,  $E(X|T^5) \leq E(X|T^4)$ . Additionally,  $E(X|T^2) > E(X|T^5)$  as the following three equations must be fulfilled

$$\begin{aligned} k(T^3) (E(X|T^3) - E(X|T^2))^2 &> k(T^4) (E(X|T^4) - E(X|T^2))^2, \\ k(T^4) (E(X|T^4) - E(X|T^3))^2 &> k(T^5) (E(X|T^5) - E(X|T^3))^2, \\ k(T^5) (E(X|T^5) - E(X|T^4))^2 &> k(T^3) (E(X|T^3) - E(X|T^4))^2. \end{aligned}$$

They only hold for  $E(X|T^2) > E(X|T^5)$  which yields

$$E(X|T^1) < E(X|T^3) < E(X|T^5) < E(X|T^2) < E(X|T^4)$$

Again, there are two possibilities. It is either the case that  $E(X|T^6) > E(X|T^5)$ . By Lemma 13 C.1,  $E(X|T^6) > E(X|T^4)$ . It then must also be that  $E(X|T^7) \geq E(X|T^5)$ . This implies that both platforms gradually move right. As  $1 \geq E(X|T)$ , at some point we must reach cycling between two alternatives. Alternatively, it can be the case that at some point it becomes better to switch to a platform on the left again, namely  $E(X|T^t) > E(X|T^{t+1}) > E(X|T^{t+2})$ , which is covered by Part 1 for which we have shown that there has to be cycling between exactly two platforms.

**Proof of Lemma 5: (2)** We aim to show that  $E(X|T_L) < E(X|T'_L) < E(X|T_R) < E(X|T'_R)$ . First, by Lemma 13 it cannot be the case that

$$E(X|T'_L) < E(X|T_L) < E(X|T_R) < E(X|T'_R)$$

Further, it cannot be the case that

$$E(X|T_L) < E(X|T_R) < E(X|T'_L) < E(X|T'_R)$$

For these cycles to be optimal, the following equations have to hold

$$\begin{aligned} k(T_R) (E(X|T_R) - E(X|T_L))^2 &> k(T'_L) (E(X|T'_L) - E(X|T_L))^2, \\ k(T'_L) (E(X|T'_L) - E(X|T'_R))^2 &> k(T_R) (E(X|T_R) - E(X|T'_R))^2. \end{aligned}$$

Given the constellation of platforms these two equations can never hold simultaneously.

**Proof of Lemma 6** For  $D = 0$ , there always exists an identical policy cycle, if the initial prior is the same. We therefore focus on  $D > 0$ . For a given target set  $T_i$  there always exists a target set  $T_{-i}$  by symmetry, with  $E(X|T_i) = 1 - E(X|T_{-i})$  and  $k(T_i) = k(T_{-i})$ . Then, for any given target set the following holds

$$\frac{k(T_i)E(X|T_i) + DE_B(X)}{k(T_i) + D} = 1 - \frac{k(T_{-i})E(X|T_{-i}) + DE_A(X)}{k(T_{-i}) + D} \quad (32)$$

This can be simplified as follows

$$\frac{k(T_i)E(X|T_i) + k(T_i)(1 - E(X|T_i)) + DE_B(X) + DE_A(X)}{k(T_i) + D} = 1$$

$$\frac{k(T_i) + D}{k(T_i) + D} = 1$$

and establishes that equation (32) holds. This implies that for each policy that is available for candidate  $A$  there exists a symmetric policy for candidate  $B$ . Suppose that  $T_{AL}$  and  $T_{AR}$  are the target sets for candidate  $A$ . Then, there exists target sets  $T_{BL}$  and  $T_{BR}$  such that  $x_{AL} = 1 - x_{BR}$  and  $x_{AR} = 1 - x_{BL}$ . Additionally, if  $T_{AL}$  and  $T_{AR}$  are optimal, then it also has to be the case that  $T_{BL}$  and  $T_{BR}$  are optimal due to the symmetry. This completes the proof.

**Proof of Lemma 7** We take derivatives of equation (8) with respect to  $\sum_{i \in K(T_L)} E_i(X)$ ,  $\sum_{i \in K(T_R)} E_i(X)$  and  $k(T_L)$  and  $k(T_R)$ .

1. Change in  $\sum_{i \in K(T_L)} E_i(X)$

$$\frac{\partial \Delta}{\partial \sum_{i \in K(T_L)} E_i(X)} = \frac{2}{k(T_L) + D} > 0$$

2. Change in  $\sum_{i \in K(T_R)} E_i(X)$

$$\frac{\partial \Delta}{\partial \sum_{i \in K(T_R)} E_i(X)} = \frac{2}{k(T_R) + D} > 0$$

3. Change in  $k(T_L)$

$$\frac{\partial \Delta}{\partial k(T_L)} = \frac{-D - 2 \left( \sum_{i \in K(T_L)} E_i(X) \right) - D (E_B(X) - E_A(X))}{(k(T_L) + D)^2} < 0$$

4. Change in  $k(T_R)$

$$\frac{\partial \Delta}{\partial k(T_R)} = \frac{-D - 2 \left( \sum_{i \in K(T_R)} E_i(X) \right) - D (E_B(X) - E_A(X))}{(k(T_R) + D)^2} < 0$$

**Proof of Lemma 8** We want to show that  $k(T_L) \geq k(T_R)$ . If both candidates select the same target sets, it is clear that the target sets are of equal size. We therefore focus on the case where candidates select different target sets that yield symmetric policies. Due to symmetry, it must hold that the number of communities to the left of  $1/2$  equals the number of communities to the right of  $1/2$ . As  $D > \bar{D}$ , we know that for any possible target set  $T_L$ ,  $x_{BL} > \frac{1}{2}$ . To see this

note that for  $x_{BL} < \frac{1}{2}$ , by symmetry it must hold that  $\frac{1}{2} < x_{AR}$ . But then  $x_{AR} > x_{BL}$ , which yields a contradiction as  $D > \bar{D}$ . Suppose now by contradiction that the number of outlets in  $T_R$  is higher than the number of outlets in  $T_L$ . This can only be the case if for some outlet  $j_+$ , the symmetric outlet  $j_-$  is also contained in  $T_R$ . Otherwise, the number of communities in each target set would be half of all communities. By symmetry, it must be that  $E(X|j_- \cup j_+) = \frac{1}{2}$ . Lemma 1 establishes that it cannot be optimal to include outlets with  $E(X|j_- \cup j_+) < x_{BL}$ , which yields a contradiction and completes the proof.

**Proof of Proposition 5** It can always be the case that the target sets are not affected by the partition in which case polarization remains unchanged. In what follows we focus on what happens if target sets are changed by the partition. Consider first the case of candidates selecting the same target sets. Then it must be the case that  $k(T_L) + k(T_R)$  is unchanged as  $K'(j_i) \cap K(j) = \emptyset \forall j_i, j \in M'$ . We denote the bliss point of a subset of the partitioned communities by  $E(X|j_i)$ . Denote the bliss point of the target set with  $j_i$  omitted by  $E(X|T_1 - j_i)$  and the bliss point of the target set which is unchanged by  $E(X|T_2)$ . Then by Lemma 9 for  $j_i$  to be contained in both target sets it must be that both

$$\begin{aligned}
& (k(T_2) + D) \left( E(X|j_i) - \frac{k(T_2)E(X|T_2) + DE_c(X)}{k(T_2) + D} \right) \\
& < (k(T_2) + k(j_i) + D) \left( E(X|j_i) - \frac{k(T_1)E(X|T_1) + DE_c(X)}{k(T_1) + D} \right) \\
& (k(T_1 - j_i) + D) \left( E(X|j_i) - \frac{k(T_1 - j_i)E(X|T_1 - j_i) + DE_c(X)}{k(T_1 - j_i) + D} \right) \\
& < (k(T_1) + D) \left( E(X|j_i) - \frac{k(T_2)E(X|T_2) + k(j_i)E(X|j_i) + DE_c(X)}{k(T_2) + D + k(j_i)} \right)
\end{aligned}$$

We can show that these equations never hold simultaneously taking into account that by Lemmas 1 and 2, it must be that

$$\frac{k(T_2)E(X|T_2) + k(j_i)E(X|j_i) + DE_c(X)}{k(T_2) + D + k(j_i)} > E(X|j_i) > \frac{k(T_1)E(X|T_1) + DE_c(X)}{k(T_1) + D}$$

Thus, it must be the case that  $j_i$  either in  $T_L$  or  $T_R$ . Due to symmetry,  $j_{-i} \in T_L$ ,  $j_{+i} \in T_R$ . Then, there are three cases to consider.

1.  $j_{-i}, j_{+i} \in T'_L$
2.  $j_{-i}, j_{+i} \in T'_R$



3.  $j_{-i} \in T'_R, j_{+i} \in T'_L$

*Case 1:*  $j_{-i}, j_{+i} \in T'_L$  This implies that communities originally contained in  $T_R$  now belong to  $T_L$ . The new level of polarization is given by

$$\begin{aligned} & \frac{2 \left( \sum_{i \in K(T_L)} E_i(X) + \sum_{i \in K'(j_{+i})} E_i(X) \right) - k(T_L) - k(j_{+i})}{k(T_L) + k(j_{+i}) + D} \\ & + \frac{2 \left( \sum_{i \in K(T_R)} E_i(X) - \sum_{i \in K'(j_{+i})} E_i(X) \right) - k(T_R) + k(j_{+i})}{k(T_R) + k(j_{+i}) + D} \\ & + D (E_B(X) - E_A(X)) \left( \frac{1}{k(T_L) + D + k(j_{+i})} + \frac{1}{k(T_R) + D - k(j_{+i})} \right) \end{aligned}$$

It is straightforward to show that this term is larger than  $\frac{2D}{k(T_L)+D} (E_B(X) - E_A(X))$ , the old level of polarization and thus polarization increases.

*Case 2:*  $j_{-i}, j_{+i} \in T'_R$  This can never occur, see Lemma 8.

*Case 3:*  $j_{-i} \in T'_R, j_{+i} \in T'_L$  Target sets change symmetrically, as  $k(j_{-i}) = k(j_{+i})$  and  $E(X|j_{-i}) = 1 - E(X|j_{+i})$ . This implies that the level of polarization is unchanged, that is polarization still equals  $\frac{2D}{k(T_L)+D} (E_B(X) - E_A(X))$

We then turn to symmetric equilibria in which candidates select different target sets. Again, we can show that  $j_i$  is only contained in one target set, but not in both, which follows from the same equations as before. We assume first that  $j_{-i} \in T_L, j_{+i} \in T_R$ . Then, we distinguish again between the following two cases

1.  $j_{-i}, j_{+i} \in T'_L$
2.  $j_{-i} \in T'_R, j_{+i} \in T'_L$

*Case 1:*  $j_{-i}, j_{+i} \in T'_L$  Here again outlets are omitted from  $T_R$  and added to  $T_L$ .

$$\begin{aligned} & \frac{2 \left( \sum_{i \in K(T_L)} E_i(X) + \sum_{i \in K'(j_{+i})} E_i(X) \right) - k(T_L) - k(j_{+i})}{k(T_L) + k(j_{+i}) + D} \\ & + \frac{2 \left( \sum_{i \in K(T_R)} E_i(X) - \sum_{i \in K'(j_{+i})} E_i(X) \right) - k(T_R) + k(j_{+i})}{k(T_R) - k(j_{+i}) + D} \\ & + D (E_B(X) - E_A(X)) \left( \frac{1}{k(T_L) + D + k(j_{+i})} + \frac{1}{k(T_R) + D - k(j_{+i})} \right) \end{aligned}$$

Before partitioning the level of polarization was given by

$$\begin{aligned} & \frac{2 \left( \sum_{i \in K(T_L)} E_i(X) \right) - k(T_L)}{k(T_L) + D} \\ & + \frac{2 \left( \sum_{i \in K(T_R)} E_i(X) \right) - k(T_R)}{k(T_R) + D} \\ & + D (E_B(X) - E_A(X)) \left( \frac{1}{k(T_L) + D} + \frac{1}{k(T_R) + D} \right) \end{aligned}$$

and we can again establish that polarization has increased due to partitioning.

*Case 2:*  $j_{-i} \in T'_R$ ,  $j_{+i} \in T'_L$  Again, outlets switch between target sets. This implies that the number of communities in each set remains unchanged. It therefore suffices to show that

$$\begin{aligned} & \frac{2 \left( \sum_{i \in K(T_L)} E_i(X) + \sum_{i \in K'(j_{+i})} E_i(X) - \sum_{i \in K'(j_{-i})} E_i(X) \right) - k(T_L)}{k(T_L) + D} \\ & + \frac{2 \left( \sum_{i \in K(T_R)} E_i(X) - \sum_{i \in K'(j_{+i})} E_i(X) + \sum_{i \in K'(j_{-i})} E_i(X) \right) - k(T_R)}{k(T_R) + D} \\ & > \frac{2 \left( \sum_{i \in K(T_L)} E_i(X) \right) - k(T_L)}{k(T_L) + D} + \frac{2 \left( \sum_{i \in K(T_R)} E_i(X) \right) - k(T_R)}{k(T_R) + D} \end{aligned}$$

which always holds due to the symmetry and  $\frac{\sum_{i \in K'(j_{-i})} E_i(X)}{k(j_{-i})} > \frac{1}{2}$ . Otherwise it can never be optimal to add  $j_{-i}$  to  $T_R$ , see Lemma 1.

Next we consider the case where  $j_{-i}, j_{+i} \in T_L$ . Then, outlets can be omitted from  $T_L$  and added to  $T_R$  and polarization can increase or decrease. Polarization increases if and only if

$$\begin{aligned} & \frac{2 \left( \sum_{i \in K(T_L)} E_i(X) - \sum_{i \in K'(j_i)} E_i(X) \right) - k(T_L) + k(j_i)}{k(T_L) - k(j_i) + D} \\ & + \frac{2 \left( \sum_{i \in K(T_R)} E_i(X) + \sum_{i \in K'(j_i)} E_i(X) \right) - k(T_R) - k(j_i)}{k(T_R) + k(j_i) + D} \\ & + D (E_B(X) - E_A(X)) \left( \frac{1}{k(T_L) + D - k(j_{+i})} + \frac{1}{k(T_R) + D + k(j_{+i})} \right) \\ & > \frac{2 \left( \sum_{i \in K(T_L)} E_i(X) \right) - k(T_L)}{k(T_L) + D} + \frac{2 \left( \sum_{i \in K(T_R)} E_i(X) \right) - k(T_R)}{k(T_R) + D} \\ & + D (E_B(X) - E_A(X)) \left( \frac{1}{k(T_L) + D} + \frac{1}{k(T_R) + D} \right) \end{aligned}$$

Equating the left and right hand side of the inequality and solving for  $\sum_{i \in K'(j_i)} E_i(X)$  shows that polarization is increasing for  $\sum_{i \in K'(j_i)} E_i(X)$  sufficiently large.

**Proof of Proposition 6** For an example when polarization decreases, consider Figure 10. Let

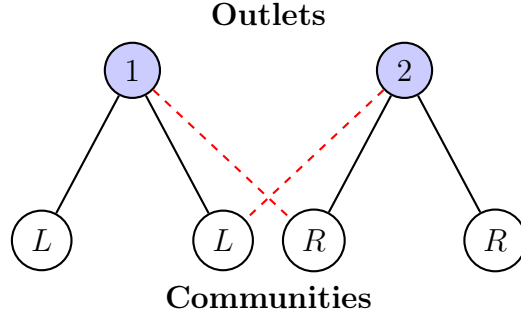


Figure 10: Polarization Decreases

$E_X(B) = 1$ ,  $E_R(X) = 3/4$ , by symmetry it follows that  $E_L(X) = 3/4$ . Let  $D = 2$ . The level of polarization is given by 1. Once links are added symmetrically, it is still optimal to set  $T'_L = \{1\}$  and  $T'_R = \{2\}$ . Polarization is now given by  $4/5$  and has thus decreased. Consider next an example where polarization increases. Let the network be as depicted in Figure 11. Let  $E_M(X) = 1/2$ . The other bliss points are as in the previous example. Then,  $T_L = \{1, 2\}$ ,  $T_R = \{3, 4\}$  and  $T_L = \{1, 2, 3\}$ ,  $T_R = \{4\}$ . Polarization is initially given by  $1/3$ , after links are added polarization is at  $7/16$ .

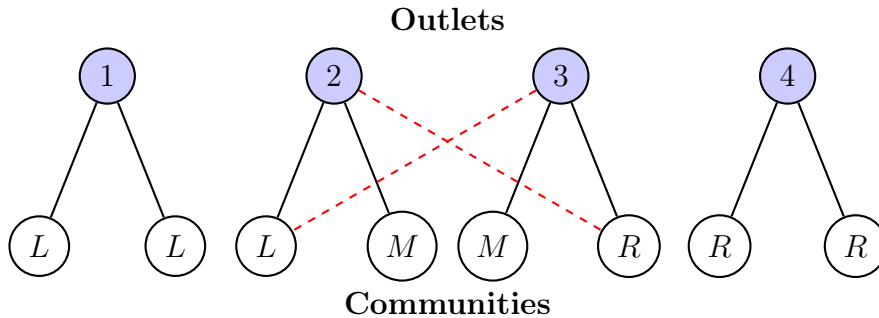
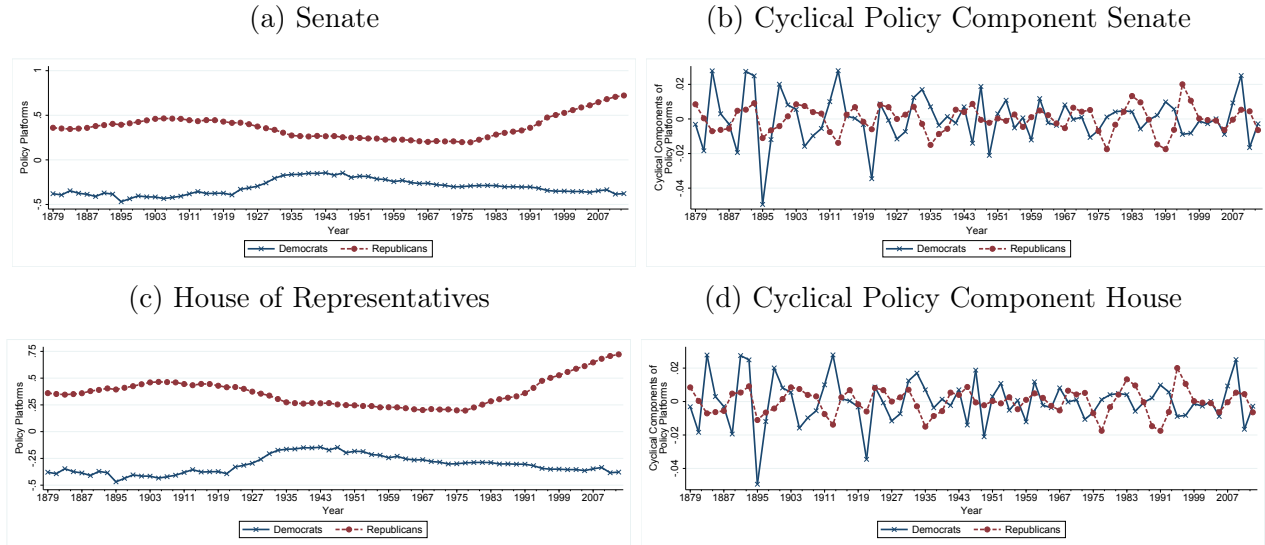


Figure 11: Polarization Increases

# Appendix B: Data

Figure 12: Cycling of Policies in Senate and House of Representatives



**Note:** The figures on the left hand side show the party means on the  $[-1,1]$  scale over time for the House and Senate. On the right side we show the deviations from the trend. We apply an HP filter with smoothing parameter 6.25 to obtain the cyclical components.

We also show that there are not only cycles in polarization, but parties' positions fluctuate themselves as well. In order to see this we consider the parties' average position both in the Senate and House, see Figures 12a and 12c. These figures highlight that in both chambers of Congress, Democrats choose a more left-wing position than Republicans. As there is a trend in the platforms of Democrats and Republicans, we again apply a HP filter, which identifies the cyclical component of the party platforms. The cycles in the platforms are depicted in Figures 12b and 12d. This highlights that in different years parties platforms are more central, in others they are more extreme. We can show that these cycles can explain about 2% of the trend on average, when taking into account years later than 1990.

But from the graphs it does not become clear how exactly the party positions fluctuate around the trend. In order to investigate this, we estimate various ARMA specifications and select the best model based on the information criteria (AIC and BIC).<sup>35</sup> The results, given in

<sup>35</sup>The best model for the House democrats is an ARMA (2,1), the best fit for all other platform changes is an ARMA(2,2).

Table 2: Autoregressive Processes

	House of Representatives		Senate	
	Dem. Position	Rep. Position	Dem. Position	Rep. Position
1st Lag	0.465*** (0.102)	1.024*** (0.143)	1.090*** (0.304)	1.412*** (0.0916)
2nd Lag	-0.287* (0.123)	-0.683*** (0.107)	-0.501** (0.190)	-0.688*** (0.0907)
Observations	68	68	68	68

\*  $p < 0.05$ , \*\*  $p < 0.01$ , \*\*\*  $p < 0.001$ , standard errors in parentheses

**Note:** The estimation results given here are the processes with the best AIC and BIC. For columns (1)-(4): ARMA(2,1), ARMA(2,2), ARMA (2,2), ARMA(2,2), respectively.

Table 2, show that there is a clear negative correlation with the second lag. That is, for a given presidential election year the position of the party in the next election year is significantly more conservative (liberal) and becomes in the next election cycle more liberal (conservative) again and thus, we observe electoral cycles.

## Appendix C: Further Examples and Extensions

**Non-Updating with Rational Agents** Suppose there are two outlets, outlet 1 and 2 and three communities. Let there be one community connected to outlet 1, community 1, and two communities, with the same expected bliss point, belonging to outlet 2. Assume that voters know the network structure. The communities in outlet 2 correctly believe that community 1 has bliss point  $1/2$ . Community 1 believes with probability  $1/2$  that if outlet 1 is not targeted then the policy implemented is given by  $1/4$  and with probability  $1/2$  it is  $3/4$ . The candidates know what the expected bliss point of the communities in outlet 2 is, namely whether it is  $1/4$  or  $3/4$ . The expected utility of voters in community 1 is then given by

$$1 - \frac{1}{2} \left( \frac{1}{4} - x \right)^2 - \frac{1}{2} \left( \frac{3}{4} - x \right)^2$$

Then, the loss for the candidates, if they target outlet 2 is given by  $1/16$ , independently of what the bliss point is (with candidates setting the bliss point as the policy). The loss if community 1 is targeted is  $1/8$  and so it is always optimal to target outlet 2. This example highlights that if there is uncertainty about the bliss points in the various communities, it is straightforward to construct an example in which voters do not learn anything if they are not targeted. The key assumption that we are therefore invoking is not that voters cannot infer what the policies are if they are not targeted, but rather, that voters form expectations about policies as if they were risk neutral. Note that in the example we gave,  $\pi = 1/2$ , that is if voters are not targeted the expected policy is given by  $1/2$ . Instead of explicitly setting up the optimization problem with the expected utility, we instead set up the problem using the expected policy.

**Unrestricted Targeting** To see some of the complications that arise when candidates can only target a limited number of outlets, consider the case where all communities have the same bliss point. Consider the media network depicted in Figure (13). Outlet 2 has the highest degree in that it caters to 4 distinct communities. Clearly, candidates will target such an outlet if they are constrained to not disclose their policy in more than one outlet.

However, if politicians can target two outlets, they are better off disclosing in outlets 1 and 3. Targeting outlets 1 and 2 covers 5 communities, whereas targeting outlets 1 and 3 covers 6. The example highlights the trade-offs implicit at the targeting stage when the communities are

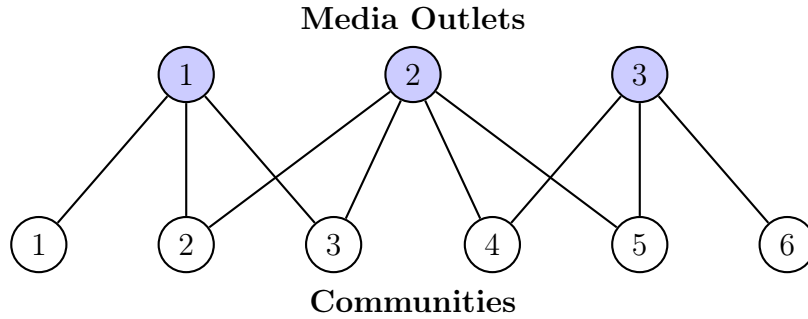


Figure 13: Maximal Target Set

homogenous and there are limits to full disclosure.

Due to the complications arising when only a limited number of outlets can be targeted, in we allow for an arbitrary number of outlets being targeted. Note that this does not imply that all outlets will be targeted if communities are heterogeneous.

**Adding Links: Target Set Switches** We assume that  $E(X|T) < \pi$ . Then, it can always be the case that  $E'(X|T') > \pi$  – that is, the expected bliss point of the new optimal target set might lie on the other side of the prior no matter how the additional link is formed. We provide an example in Figure 14 of such a switch in target sets if a link is added between a targeted outlet and a targeted community. The original network  $N$  is given by the black links the new link is the dashed red link. Suppose that  $\pi = 1/2$ , and consider two types of communities – namely left communities with  $E_L(X) = 1/4$  and right communities with  $E_R(X) = 2/3$ . It is straightforward to verify that under linking structure  $N$  it is optimal to disclose in outlets 1 and 2, whereas once the additional link is formed, it is optimal to disclose in outlets 2 and 3.

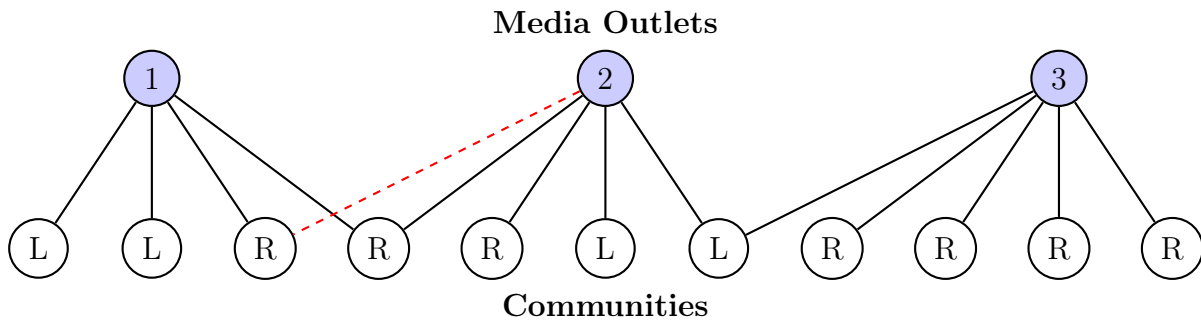


Figure 14: Target Set Switches

**Adding Links:**  $l \notin T$  We consider a community that is contained in the target set and forms a link to an outlet  $l$  not contained in the target set and show that if  $j$  is omitted, then outlet  $l$  can be added to the target set. This result is highlighted in Figure 15. The expected bliss points of the various groups are shown in the nodes.

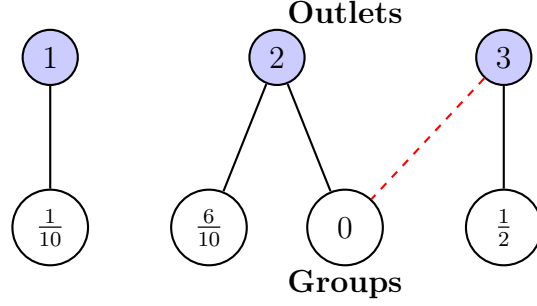


Figure 15: Target Set Changes

We set  $\pi = \frac{1}{2}$ . Then, the original target set,  $T$ , in absence of the red, dashed link contains outlets 1 and 2. If the group with expected bliss point of zero forms a link to outlet 3, then it is no longer optimal to include outlet 2, but instead it is better to include outlet 3, that is  $T' = \{1, 3\}$ .

**Comparative Statics:** Polarization is given by

$$\Delta = \frac{1}{2} \left( \frac{\sum_{i \in T_{1B}} E_i(X) + DE_B(X)}{k(T_{1B}) + D} + \frac{\sum_{i \in T_{2B}} E_i(X) + DE_B(X)}{k(T_{2B}) + D} - \frac{\sum_{i \in T_{1A}} E_i(X) + DE_A(X)}{k(T_{1A}) + D} - \frac{\sum_{i \in T_{2A}} E_i(X) + DE_A(X)}{k(T_{2A}) + D} \right)$$

This term is increasing in  $D$  and  $E_B(X)$  and decreasing  $E_A(X)$ .

1. Change in  $D$

$$\frac{\partial \Delta}{\partial D} = \frac{k(T_{1B})E_B(X) - \sum_{i \in T_{1B}} E_i(X)}{(k(T_{1B}) + D)^2} + \frac{k(T_{2B})E_B(X) - \sum_{i \in T_{2B}} E_i(X)}{(k(T_{2B}) + D)^2} - \frac{k(T_{1A})E_A(X) - \sum_{i \in T_{1A}} E_i(X)}{(k(T_{1A}) + D)^2} - \frac{k(T_{2A})E_A(X) - \sum_{i \in T_{2A}} E_i(X)}{(k(T_{2A}) + D)^2} > 0$$

as  $E_B(X) > E_i(X) > E_A(X)$  for any  $E_i(X)$ .

2. Change in  $E_A(X)$

$$\frac{\partial \Delta}{\partial E_A(X)} = -\frac{D}{k(T_{1A}) + D} - \frac{D}{k(T_{2A}) + D} < 0$$



### 3. Change in $E_B(X)$

$$\frac{\partial \Delta}{\partial E_B(X)} = \frac{D}{k(T_{1B}) + D} + \frac{D}{k(T_{2B}) + D} > 0$$

**Media Networks & Polarization: Constant Prior** Using assumptions [A.1](#) and [A.2](#), we show under what circumstances polarization increases or decreases if outlets are partitioned. This depends on whether the policy the candidate chooses lies to the left or the right of the prior. We consider the problem from candidate  $B$ 's perspective. If candidate  $B$  chooses a policy that lies to the right of his prior, then polarization is higher than if he chooses a policy to the left of his prior. Partitioning allows candidates to target the voters they aim more precisely. If candidate  $B$  chooses a target set to the right of the prior, then a partition allows to target more ideologically extreme voters and this leads to an increase in polarization. If on the other hand, the candidate selects a target set to his left, then a partition results in a policy that is more centrist and leads to a decrease in polarization. This is summarized in [Proposition 7](#). We denote by  $T_{BL}$  ( $T_{BR}$ ) the target set of candidate  $B$  if the policy lies to the left (right) of the prior. We further refer to the policy associated with the left target set as  $x_{BL}$ , that associated with the right target set as  $x_{BR}$ . We index the target sets and policies after the partition by prime. We distinguish between two cases, namely (i) both partitioned outlets are contained in target set and (ii) at least one outlet is not contained in the target set. We denote by  $E(X|j_i)$  the bliss point of the communities *added to the target set*.

**Proposition 7.** *Let  $K'(j_i) \cap K(j) = \emptyset \forall j_i, j \in M'$  and let partitions be symmetric. Consider a symmetric equilibrium.*

(1) *Let  $\mu_B$  be sufficiently low, such that  $\max\{x_{BL}, x'_{BL}\} < \mu_B$ .*

(a) *If  $j_-, j_+ \in T_{BL}$ , polarization weakly decreases.*

(b) *If at least  $j_+ \notin T_{BL}$  and  $E(X|j_i) > x_{BL}$ , polarization increases. Otherwise, polarization weakly decreases.*

(2) *If  $x_{BL} < \mu_B < x'_{BL}$ , polarization increases.*

(3) *Let  $\mu_B$  be sufficiently high, such that  $\max\{x_{BR}, x'_{BR}\} > \mu_B$ . If  $E(X|j_i) < x_{BR}$ , polarization decreases. Otherwise, polarization weakly increases.*

(4) *If  $x_{BR} > \mu_B > x'_{BR}$ , polarization decreases.*

Proposition 7 shows that partitioning can increase or decrease polarization depending on whether the candidate selects a moderate policy or an extreme one. Generally a partition allows for a more precise targeting of voters and a partition decreases polarization if candidates select a moderate policy, whereas it induces a rise in polarization if candidates select a more extreme policy. However, it can also be the case that a partition increases polarization in case of a moderate policy. This occurs if at least one of the partitioned outlets is not contained in the target set. Then, it can be the case that communities are added to the target set, which have a bliss point that lies above the previously implemented policy, leading to an increase in polarization. By the same logic it can also be the case that a partition decreases polarization if more ideologically extreme voters are targeted as at least one the partitioned outlets is not contained in the target set. Note again that it cannot be the case that both partitioned outlets are contained in  $B$ 's right target set. Further, it can always be the case that the target sets switch, that is before the partition it was better to choose a policy to the left of the prior whereas it is now better to choose a policy to the right of the prior.

**Proposition 8.** *Suppose communities  $i_-$ ,  $i_+$  form an additional link. Then, polarization can increase or decrease.*

An additional complication arising in the added links case, compared to the partitions, is that it not only matters whether the communities that form a link are contained in the target set or not, but also whether they form link to an outlet in the target set or out of the target set. This changes the expected bliss point of the target set and can lead to outlets being added to or omitted from the target set and ultimately to an increase or decrease in the target set. If the both communities that form an additional link are contained in the target set and the policy caters to more ideologically extreme voters, then polarization weakly increases, see Lemma 4. If on the other hand the target set is not affected by the addition of links, then polarization can increase or decrease, depending on the effects of the target set.

The proofs of both propositions are omitted as they follow exactly the proofs of Propositions 5 and 6.