A Model of Competitive Signaling*

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Abstract

Multiple senders compete over a fixed number of jobs. Senders' ability is unobservable, and determines their probability of success in different tasks. We study a signaling game with multiple senders each choosing one task to perform, and one receiver who observes all task choices and performances (success or failure), and matches senders to jobs. In order to analyze the effects of competition, we consider sequential equilibria that are supported by out-of-the-equilibrium-path beliefs consistent with trembles on strategies. We show that due to the competition among multiple senders, pooling in the most difficult task is the only outcome in the following two scenarios. First, if the more difficult tasks are more conspicuous in the case of both success and failure, then pooling in the most difficult task is the only pure strategy sequential equilibrium. Second, if the more difficult tasks are more conspicuous in the case of success and the competition is sufficiently high, then pooling in the most difficult task is the only sequential equilibrium. In addition, if senders have a lower overall likelihood of success in more informative (difficult) tasks, this unraveling towards conspicuousness could be inefficient.

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1 Introduction

In classical signaling models a principal aims to *classify* an agent according to his unobservable underlying type, and thus the reward to the agent is a continuous function of the likelihood that the principal attaches to him being a high type after observing his action.[1] However, there are many settings in which the objective of the principal is to *rank* the agents rather than to classify them. For instance, this is the case when a principal is deciding which among various subordinates to promote to a managerial position, or when students in high school take actions seeking to demonstrate their underlying abilities while competing for a fixed number of places in a higher education institution. In these settings, the rewards to the agents do not depend directly on the inference that the principal makes of their types, but on how he ranks them with respect to other agents. As we show in this paper, this difference can have significant consequences. The reason is that if the agent's reward just depends on the inference made by the principal, then his return from taking a more revealing action (a more conspicuous action) is just proportional to the resulting increase in the likelihood of being a high type that the principal's beliefs confer him. Thus, as long as beliefs vary continuously with the actions available to the agent, a small deviation by the agent can only have a small impact on his reward. On the other hand, if his reward depends on how the principal ranks him in relation to other candidates, then his return from taking a slightly more conspicuous action can be large.

The standard monotonic signaling models that build on Spence (1973) share the feature that despite the possible lack of intrinsic value of the signals used by agents in order to convey information to the principal, the availability of these signals can be socially efficient because in the robust equilibria[2] the higher type agents separate themselves from the lower type agents by sending more costly signals, and thus agents can be correctly classified by the principal. There is a myriad of environments in which agents' main motivation when choosing their behaviors is to convey information to an uninformed party. If we loosely analyze any such environment under the lens of standard monotonic signaling models then one conclusion might be that from a social welfare perspective the availability of the signals is good because in equilibrium it leads to a narrowing down of the information gap. However, some of these environments may not satisfy the key assumptions of this model. Broadly speaking the two key assumptions required for this result are that (1) the rewards to the agents are proportional to their expected marginal product and (2) the opportunity cost of sending any given signal is higher for the low types of agents than for the high types. And not surprisingly efficient separation does not

[1] In the classical model which assumes a competitive labour market, the expected marginal product of the agent.

[2] As selected by a large family of equilibrium refinements.

seem to be the main defining feature equilibrium behavior in these environments.

We present and discuss our model with reference to a recent public debate on the possible role of signaling in explaining the perpetuation of super string theory as the only contender to a Theory of Everything during the last 40 years. There are other applications of our model, but besides the clarity and simplicity of the metaphor, this story bares some connection to the standard job market signaling model. As we will argue, the assumptions of the standard model may not match well some of the stylized facts of this environment.

String theory arose in the 1960's and early 1970's as a theory of a class of subatomic particles called hadrons. In 1974 Scherk and Schwarz published a paper[3] in which they provided a mathematical argument suggesting that the theory could be developed into a theory of gravity. The theory thus became one among just a few plausible avenues towards a Theory of Everything and turned into a significant area of attention for the theoretical particle physics community. Four decades later string theory is the only candidate Theory of Everything within mainstream physics, but nevertheless the spectrum of attitudes towards it is quite broad, ranging from the contention that due to its elegance and mathematical beauty it is bound to be true[4], to the skepticism from people who like Roger Penrose (Penrose 2007) affirm that it is a case of fashion in science with little experimental support. Despite the substantial external and internal questioning of the promise of string theory, there are no significant mainstream efforts directed towards developing alternatives.

The process whereby string theory rose into the mainstream has been the subject of scholarly papers (Hedrich 2007; Schroer 2008; Zapata 2009), popular science books[5], and several mass media articles[6]. Some of these also address the inexistence of mainstream competing alternatives and consider the social mechanisms that may contribute to explain it. One particular argument that has surfaced within this category is that because theoretical physics is specially competitive, young scholars are forced to specialize in string theory in order to signal their talent and skill.[7] Cast in these terms we can think of the be-

[3]J. Scherk and J. H. Schwarz, "Dual Models for Nonhadrons", *Nuclear Physics B*, 81 (1974):118-144.

[4]"I can only speak for myself, though I suspect that most others working in this field would agree. I believe that we have found the unique mathematical structure that consistently combines quantum mechanics and general relativity. So it must almost certainly be correct." (Schwarz 1998)

[5]See for example *The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory* by Brian Greene and the controversial *Not Even Wrong: The Failure of String Theory and the Search for Unity in Physical Law* and *The Trouble with Physics* published in 2007 by Peter Woit and Lee Smolin respectively.

[6]See for example *Unstrung* an article by Jim Holt published in October 2006 in The New Yorker, or an interview with Roger Penrose published in the September 2009 issue of Discover Magazine.

[7]The following quotation, taken from a mass media article authored by a physicist, exemplifies this ar-

havior in this example as discussed by the skeptics as a form of inefficient pooling. There are two questions raised by this example:

1. Why is there inefficient pooling ? In the context of this example, a standard monotonic signaling model explains the separation between those theoretical physicists that choose string theory from those that don't. We could also look with more detail at the visible choices made by different graduate students and postdocs within string theory and then build a model for the way in which these differences efficiently sort them according to unobservable underlying characteristics. As discussed above, the main motive in standard monotonic signaling models is the ability of differences in costly behavior to express differences in underlying unobservable characteristics. Indeed the prediction of this class of models is the Riley outcome or efficient separation. The main contention of the claim however is that *too many people* are becoming string theorists as a product of the drive towards signaling. This stylized fact is not explained (or addressed) by the standard models.[8]

2. Why does the pooling take place in String Theory, among all other possible fields?

In this paper we study the possible role of competition in explaining this type of inefficient pooling (a pattern of behavior whereby agents pool in a single research task). With that purpose in mind we formulate a signaling game where multiple senders can choose among different tasks with the objective of being as well ranked as possible in terms of their privately known types by the principal. They can either succeed or fail at the task and their performance is correlated with their ability. These tasks are ordered by the informativeness of success, such that for a fixed sender a success in a task with a higher index implies a higher likelihood of him having a higher ability. We study the set of sequential equilibria of the game as the number of competing senders (the competitiveness of the environment) increases. We show that restricting attention to sequential equilibria that are supported by out-of-the-equilibrium-path beliefs that satisfy a monotonicity condition, the only sender behavior that is part of some equilibrium entails every sender

gument: *"One reason that only one new theory has blossomed is that graduate students, postdocs and untenured junior faculty interested in speculative areas of mathematical physics beyond the Standard Model are under tremendous pressures. For them, the idea of starting to work on an untested new idea that may very well fail looks a lot like a quick route to professional suicide. So some people who do not believe in string theory work on it anyway. They may be intimidated by the fact that certain leading string theorists are undeniably geniuses. Another motivation is the natural desire to maintain a job, get grants, go to conferences and generally have an intellectual community in which to participate. Hence, few stray very far from the main line of inquiry." -Peter Woit, American Scientist* Woit *(2002)*

[8] Whether the stylized fact is treu or not in this case is not important to argument; as we discuss above we use the story as just one example of a wide range of situations in which arguably this kind of large scale pooling may be taking place.

choosing the most informative task. If it is the case that agents have a lower overall likelihood of success in more informative tasks then this unraveling towards conspicuousness is inefficient.

The paper is structured as follows. In Section 2 we introduce our main arguments using a very simple example with just two types of senders and two tasks. In Section 3 we consider a general environment allowing for a broader class of applications with multiple types of senders and tasks. Section 4 characterizes the equilibrium refinements we use and the equilibrium behavior that the model predicts. We also show that the restriction on out-of-the-equilibrium path beliefs behind our uniqueness result is implied by Banks and Sobel's divinity refinement for sufficiently high levels of competition. The last section concludes.

1.1 Related Literature

The current textbook example of costly signaling in economics was introduced by Spence (1973) and it discusses the role of costly education choices by job candidates in credibly communicating to potential employers their productivities. The importance of this classical article lies in the fact that Spence's model gave rise to a a large class of signaling games, -monotonic signaling games-, which have become the canonical way of thinking about costly signaling in economics. The unique equilibrium prediction of all monotonic signaling games is least cost separation among types (the Riley outcome), under any of the several refinements stronger than Cho and Krep's $D1$ criterion that were introduced during the 80s. So in terms of the discussion above, the now standard family of models of signaling in economics focus on the fine level of costly signaling processes: the efficient communication of unobservable characteristics by way of costly signals.The main difference between our paper and the extensive monotonic signaling literature is that our model focuses on a different phenomenon associated to signaling, which accounts for *keeping up with the Joneses* types of behavior.

Our paper is related to the broad literature on social status and conspicuous behavior. This literature explains a variety of behaviors as resulting from the human drive to gain better standing in hierarchies. Beginning with Veblen (1899), this theme has surfaced over the past century by way different concepts and applications: Veblen effects, snob effects, positional goods and status goods to name a few. More recently Frank (1987); Frank and Cook (1995); Frank (2005) and Basu (1989) provide general frameworks which allow them to offer explanations for a large variety of socioeconomic phenomena including wage dispersion, initiation rituals, wastefulness and individual patterns of consumption in time, in terms of conspicuousness. While most of the recent theoretical and empirical papers in this literature are related to consumption (Bagwell and Bernheim 1996; Corneo and Jeanne 1997; Ravina 2007; Charles, Hurst and Roussanov 2009), there are also a number of articles motivated by other applications. Glazer and Konrad (1996) presents a model philanthropy, and Bloch, Rao and Desai (2004) apply the idea to explain wedding celebrations in India. In general all this literature implicitly builds upon the arguments of Spence (1973) and Akerlof (1976) that show how information can be conveyed in equilibrium and make explicit the necessary correspondence between the underlying costs of actions and the meanings that these can convey.[9] One area in which it has many applications is conspicuous behavior emerging in competitions for social status. Some concrete examples include the lavish wedding ceremony traditions in rural India as discussed in Bloch, Rao and Desai (2004), the high expenditure on luxury items to signal wealth as documented in Charles, Hurst and Roussanov (2009), and conspicuous generosity as the basis for extensive food sharing in small scale societies, as discussed by Smith and Bliege Bird (2005). The phenomenon of conspicuousness has two distinct levels of detail. One level is about separation: how differences in status can be conveyed through small variations within a given category of behavior. The other level is about pooling: how due to competitive pressure this process of fine differentiation may lead to very intense devotion to that specific category of behavior by the whole community. Standard signaling models address the separation at the level of finer detail, but they do not explain the pooling at the broader level, which is what we do in our model. One other interesting feature of these examples is that they suggest that this aggregate convergence may have negative consequences as in the case of the generalized expenditure in luxury items or the lavish weddings in rural India, or very positive consequences as in the case of the resulting informal insurance system in the example of Smith and Bliege Bird (2005) about food sharing in small foraging societies.

One issue however is that in standard signaling models conspicuousness emerges as one among many possible patterns of stable behavior. Our paper shows that under a plausible restriction on off-the-equilibrium-path beliefs, conspicuous equilibria are the only ones that are robust to competition.[10]

[9]There are other models of multi-sender signaling (e.g. Bagwell and Ramey 1991), where the multiple senders signal a common state of nature, rather than their individual type.

[10]Midjord (2012) sets forth a model with similar features to study information revelation in job interviews in which a principal has the ability to noisily verify the information submitted by the candidates (they state whether they are high types or low types). In this model pooling onto stating that they are high types is the only equilibrium that survives as competition rises. A formal analogy can be drawn between research tasks in our model that differ in the quality that success in them provides about senders' underlying abilities and the noisy verification process in Midjord (2012). One crucial assumption of their model that is appropriate in the context of telling the truth or lying in job interviews but which does not fit the applications of our paper is that high types always pass the verification process. This assumption implies that in all sequential equilibria the beliefs of the principal after observing a failed verification test must be that the candidate

2 An Example

There are two senders, indexed by $i \in \{1,2\}$. The types of the senders are independently drawn from $\{t_L, t_H\}$ where $t_L < t_H$. Types are private information and represent senders' unobservable abilities. The type of a sender is t_L with probability $p(t_L)$ and t_H with probability $p(t_H) = 1 - p(t_L)$. After senders observe their own types, they simultaneously choose one task[11] m from the closed interval $M = [0, m^z]$, where $m^z < 1$. Senders undergo small trembles when choosing their tasks: With probability $1 - \varepsilon$ for small ε, the sender carries out the task m that he has chosen, and with probability ε he carries out a task chosen uniformly at random in a small neighborhood $(m - \gamma, m + \gamma) \bigcap M$ of the intended task m.[12]

Senders can either succeed or fail at the task that they carry out. A sender of type t achieves success with probability $s(t,m) \in (0,1)$ in task m. Let $s_H^0 = s(t_H, m = 0)$ and $s_L^0 = s(t_L, m = 0)$, where $s(t_H, m = 0) < s(t_L, m = 0)$ are the probabilities of success at the easiest task, and for any given task $m \in [0, m^z]$ let

$$s(t_L, m) = s_L^0 - m, \;\; s(t_H, m) = s_H^0 - \delta m$$

That is, higher tasks are more difficult for all types of senders, and their success probabilities decrease linearly. We normalize the rate at which the low type's success probability decreases to 1 and denote the rate at which the success probability of the high type decreases by δ. We make the following assumptions on δ in this example.

(A1) *Increasing success probability ratio:* $\delta < \frac{S_H^0}{S_L^0}$.

(A2) *Increasing failure probability ratio:* $\frac{1-S_H^0}{1-S_L^0} < \delta$.

In the context of this example $\delta < \frac{S_H^0}{S_L^0}$ guarantees that the success probability of the high type is higher than that of the low type across the whole task space. But most importantly it is a necessary and sufficient condition for $\frac{s(t_H,m)}{s(t_L,m)}$ to be increasing in m. Given any prior distribution of probability p on a high type, the probability of a high type con-

in question is a low type. This implication is important for their subsequent analysis and determines the techniques that they use. Given the applications in the scope of our paper, our model allows the beliefs of the receiver after observing successes or failures in any task to be non-degenerate.

[11] We use the term task as opposed to the standard term *message* in the signaling literature in the light of the fact that our setting and assumptions are quite different.

[12] This assumption determines the off-the-equilibrium path beliefs of the receiver in a small neighborhood around the tasks chosen by senders in equilibrium. One plausible interpretation is that as there are uncountably many different tasks, the receiver cannot perfectly distinguish between them.

ditional on a success in task m is,

$$p(t_H|s,m) = \frac{ps(t_H,m)}{ps(t_H,m)+(1-p)s(t_L,m)} = \frac{p}{p+(1-p)\frac{s(t_L,m)}{s(t_H,m)}} \tag{1}$$

Thus, **(A1)** implies that more difficult tasks are more conspicuous, in the sense that success in more difficult tasks is more revealing of a sender's type being high than success in less difficult tasks.[13] Similarly $\frac{1-S_H^0}{1-S_L^0} < \delta$ implies that failure in more difficult tasks is more revealing of a sender's type being high than success in less difficult tasks. Given **(A1)**, a sufficient condition for **(A2)** to hold is that $\delta > 1$. We discuss in detail the plausibility of these assumptions in Section 3.

After observing the tasks carried out by the senders and their performance, the receiver selects the sender which she deems more likely to be a high type, and if she considers them to be equally likely to be high types then she selects either sender with probability $\frac{1}{2}$. From now on, we refer to the receiver as "she" and to the sender as "he." Throughout what follows we will often be referring to the beliefs of the receiver before she observes the tasks chosen by the senders (her *prior beliefs*), her beliefs after she observes the tasks chosen by the senders, but before he observes their performance[14] (her *interim beliefs*), and her beliefs after she observes the tasks chosen by the senders and their performances (her *posterior beliefs*).

Senders do not value the tasks themselves and/or their performance, and only care about being selected by the receiver. We normalize the utility that a sender derives from being selected to 1, and assume that a sender gets utility 0 in case that he is not selected.

CLAIM **1** *There are no separating equilibria.*

Proof of Claim 1: There are no separating equilibria since rather than revealing himself, a low type sender could do strictly better by imitating the high type. This results from the fact that the outcomes are probabilistic and that society treats the senders equally when it cannot distinguish them. ▮

CLAIM **2** *Under **(A1)** and **(A2)**, in all pure strategy equilibria of the game described above, senders pool at the highest task m^z.*

[13] Formally, holding the receiver's interim beliefs constant, the posterior beliefs of the receiver after observing a sender succeed in a higher task will place a greater probability on him being a hight type than after observing him succeed in an easier task.

[14] Formally, this is not a stage of the game, but it is a useful intermediate step for computing his posterior beliefs.

Proof of Claim 2: We rule out pooling at any other task by contradiction. Suppose there exists a pooling equilibrium at $m < m^z$. We consider a deviation to a slightly higher task $m' \in (m, m+\gamma)$ ($m' < m^z$) by a high type sender i. Let $\triangle = m' - m$.

Using the other sender's ex ante type distribution $p(t_H)$, $p(t_L)$ and the success probabilities conditional on types, we can calculate the other player's unconditional success probability, denoted as $s(m)$. To simplify the notation, let $f(m) = 1 - s(m)$ and $f(t, m) = 1 - s(t, m)$. If sender i chooses task m, his payoff is

$$s(t_H, m)f(m) + \frac{1}{2}s(t_H, m)s(m) + \frac{1}{2}f(t_H, m)f(m) \tag{2}$$

This is the case as sender i is selected for sure if he is the only one to succeed, is not selected if he is the only one to fail and is selected with probability $\frac{1}{2}$ if both senders' outcomes are the same.

Now consider a deviation to m'. By trembling in choosing tasks, the receiver holds the same prior for m and m'. Then by **(A1)**, when seeing one sender succeed at m and the other one at m', the receiver will pick the one having succeeded at m'. This is the case since as discussed in (1), the posterior probability of a high type is higher after seeing a success in a higher task m'. Similarly, due to **(A2)**, when both senders fail, the receiver will select the one failing at m'. So sender i's payoff from choosing m' is:[15]

$$(s(t_H, m) - \delta\triangle)f(m) + (s(t_H, m) - \delta\triangle)s(m) + (f(t_H, m) + \delta\triangle)f(m) \tag{3}$$

where sender i is selected for sure unless he fails and the other player succeeds. When $\triangle$ is small enough, (3)>(2), so the deviation is profitable and it follows that pooling at $m < m^z$ cannot be an equilibrium. An analogous argument shows that pooling at the most difficult task m^z is indeed an equilibrium. A sender, regardless of his type would never find it profitable to deviate to a slightly lower task. Deviations to tasks $m' < m - \gamma$ outside of the trembling region can be precluded in any sequential equilibrium by assuming that the receiver's interim beliefs upon observing a sender choose such a task place probability 1 on him being a low type. . ∎

The intuition for this result is as follows: The trembling when choosing tasks implies that for any task m' close enough to m , the receiver's interim beliefs on the type of a sender choosing m' will be equal to her beliefs when observing a sender choose m, and in particular they will be given by the prior distribution. And given our assumptions about δ, these interim beliefs imply that upon seeing the senders perform equally well in tasks m' and m, the receiver will conclude that the sender in task m' is more likely to be a

[15]Formally the expressions for the payoffs throughout the example are approximations due to the trembling in task choices, so all the statements hold for sufficiently small trembling probabilities.

high type than the sender in m. So while competing at the same task as the other sender, sender i only beats him for sure if i performs better than him, while when competing against him by choosing a different and more difficult task m', i beats him for sure as long as i performs at least as well as him. This discrete improvement in the number of events (according to relative performance) under which i is selected by the receiver, outweighs the small decline in i's probability of success. Specifically the gap between expressions (2) and (3) is large, that is, the difference between these payoffs is bounded away from 0. This implies that Claim 2 would continue to hold if there are small continuously increasing costs of performing the tasks, as long as their total costs never exceed the expected payoff from being selected by the receiver uniformly at random. So our assumptions imply that the competition between the senders leads to unraveling in their task choices.

Furthermore, note that the fact that the receiver is using her inference on the types to rank the senders, and that therefore their rewards can vary discontinuously with her inference is crucial to our unraveling argument. If the senders' rewards varied continuously in the inference of the receiver (as they do in standard signaling models, in which the receiver's objective is to classify a single sender in the model), then the smaller success probability in more difficult tasks could outweigh the gains from the more beneficial inference in each event (success or failure).[16]

While we have not been explicit about the receiver's payoffs, we have assumed that she is attempting to rank the senders correctly. It turns out that if the receiver's utility is increasing in the probability of selecting the sender with the highest type, then pooling at the most difficult task is the least efficient task configuration from her perspective. When the two senders are of the same type, the receiver gets the same payoff from selecting either one of them. So what matters is her selection when one sender is a high type and the other is a low type. For concreteness assume the receiver gets 1 if she selects a high type, and 0 if she selects a low type. In this case, the receiver's expected payoff when both senders choose task m is

$$\frac{1}{2}\big(s(t_L, m)s(t_H, m) + f(t_L, m)f(t_H, m)\big) + f(t_L, m)s(t_H, m)$$

CLAIM **3** *If $\delta > 1$ both types of senders pooling at $m = m^z$ is the least efficient distribution of senders across tasks from the receiver's perspective.*

Proof of Claim 3: The receiver's payoff can be rewritten as

[16]The overall payoff to the sender is a weighted average of his expected payoff conditional on success and his expected payoff conditional on failure, so the higher weight on the expected payoff from failure stemming from a lower success probability can imply a lower overall success probability.

$$\frac{1}{2}(1 + s(s_H, m) - s(t_L, m))$$

This expression is increasing in the gap of the success probabilities between the two types. When $\delta > 1$ this gap is smallest at the most difficult task, and therefore having both senders choose this task is the least efficient distribution of senders across tasks.[17] ∎

The intuition for this result is that although the most difficult task has the greatest screening power conditional on one of the senders succeeding and the other one failing (due to the increasing success probability ratio property), the probability of the high type sender actually succeeding is too low. That is, the most likely outcome is that both senders fail, a case in which the selection becomes a coin toss. From the receiver's perspective it would be most efficient to observe both senders perform the easiest task. While the screening power of this task conditional on the outcomes of the senders being different is not very high, the event under which this screening power is actually used (when the performances of the senders differ) occurs with high probability. As discussed bove, $\delta > 1$, together with **(A1)** implies **(A2)**. This fact shows that all our assumptions can hold together for a wide variety of success probability functions.

On the other hand, if $\delta < 1$, pooling at $m = m^z$ is the most efficient outcome from the receiver's perspective.

We end this example, by considering a competition of n senders, and noting that when n is large then **(A1)** suffices to obtain the unraveling result discussed in Claims 1 and 2. That is, with sufficiently high competition, competitive unraveling towards the most difficult task will take place even in the absence of the increasing failure probability ratio property.

CLAIM **4** *Under **(A1)**, there exists some n^* such that when $n \geqslant n^*$, in all pure strategy pooling equilibria of the game described above, senders pool at the highest task m^z.*

Proof of Claim 4: Similarly as in the proof of Claim 2, we rule out pooling at any other task by contradiction. Suppose there exists a pooling equilibrium at $m < m^z$. We consider a deviation to a slightly higher task $m' \in (m, m + \gamma)$ $(m' < m^z)$ by a high type sender i. Let $\triangle = m' - m$.

Let $s_{-i}(m)$ be the probability that at least one sender succeeds other than sender i, and $f_{-i}(m) = 1 - s_{-i}(m)$. If sender i chooses task m, his payoff is at most

$$\frac{1}{2}s(t_H, m)s_{-i}(m) + s(t_H, m)f_{-i}(m) + \frac{1}{n}f(t_H, m)f_{-i}(m) \tag{4}$$

[17] Note that this includes all possible distributions, including probabilistic ones.

in which the payoff in the first term could be smaller than $1/2$ if more than two senders (including sender i) succeed.

Now consider the deviation to m'. By the trembles, the receiver holds the same interim beliefs for m and m'. Then by the increasing success probability ratio property, when seeing one success at m and the other success at m', the receiver will pick the one having succeeded at m'. So sender i's payoff at m' is at least:

$$(s(t_H, m) - \delta\triangle)s_{-i}(m) + s(t_H, m) - \delta\triangle)f_{-i}(m) \tag{5}$$

given that sender i is selected for sure when he succeeds. When $\triangle$ goes to zero, (5)−(4) equals

$$\frac{1}{2}s(t_H, m)s_{-i}(m) - \frac{1}{n}f(t_H, m)f_{-i}(m)$$

and this difference is bounded below by

$$\frac{1}{2}s(t_H, 1)(1 - (f(m^z)^n)) - \frac{1}{n}$$

Let n^* smallest integer exceeding $\max(-\frac{ln2}{lnf(m^z)}, \frac{4}{s(t_H,1)})$,[18] (5)−(4) is always positive. So the deviation is profitable. Thus pooling at $m < m^z$ cannot be an equilibrium. Finally, by relying on an analogous argument it can be shown that pooling in m^z is indeed an equilibrium. ∎

When there is sufficiently high competition, the receiver would observe at least one success with a probability close to 1, and thus selects only among the ones who succeed. By the increasing success probability ratio property, the agent succeeding in a harder task is more likely to be a high type, and it gives senders enough incentives to move to a slightly harder task. So the only pooling equilibrium must be pooling at the hardest task. We now generalize this set of results.

3 The General Model

In this section we describe our general model of competitive signaling, allowing for multiple types and general success probability functions (only retaining the idea that more difficult tasks are more conspicuous). As discussed in the arguments put forth in Section 2, the mechanism behind our results is a kind of unraveling: Given that the senders are being ranked, the increase in the their rewards from arbitrarily small changes in the re-

[18] The first term ensures $(f(m^z))^n < 1/2$ and the latter term ensures $s(t_H, 1)/4 > 1/n$. Combining both, the lower bound of (5)−(4) is positive.

ceiver's beliefs can be large, and therefore senders may be willing to face slightly higher costs (e.g. lower success probability) in order to induce such large increases in their rewards. Allowing them to alter their behavior continuously is sufficient for the existence of these kinds of profitable deviations, and thus enables a crisp presentation of the main idea. Baring in mind that the continuous model is just a convenient approximation to the possibly large, yet finite space of tasks available to senders in applications, we develop our generalization assuming that there are finitely many tasks, and that they are $\triangle$-dense on $[0, m^z]$, i.e. there is at least one task in any interval with length $\triangle$. $\triangle$ could be very small, such that the discrete model is close to the continuous model in the example. If the success probability functions are linear, or if there are three types or less then the main result discussed in the example continues to hold. However, when this is not the case the competitive unraveling only ensures that in equilibrium, senders must select messages towards the top of the range of messages available, but not necessarily all pooling in the most difficult one.

At stage 0, there are n senders $\{S_1, ..., S_n\}$, each one having some type t_i from k possible types, $T = \{t^1 < ... < t^k\} \subset [0, 1]$. The types are independently drawn and type t^l occurs with probability $p_\alpha(t^l)$, where $\sum_{l=1}^{k} p_\alpha(t^l) = 1$. To be clear, we reserve subindices to refer to the type of a specific sender or a given subset of senders. Specifically we let t_i denote the type of sender i and t_{-i} denote the vector of types of all senders other than i. $\overrightarrow{t}$ denotes the complete vector of types, $\overrightarrow{t} = (t_1, t_2, ..., t_n)$.

At stage 1 after observing his own type, S_i selects a task[19] m_i that he wants to perform from a finite set $M = \{m^0 < m^1 < m^2 < ... < m^z\}$. Let the gap $\triangle = \max(m^2 - m^1, ..., m^z - m^{z-1})$. We focus on the case in which there are many potential tasks and any two adjacent ones are very close to each other, e.g. $\triangle$ is very small. The sender can succeed or fail at the task. A sender of type t succeeds at task m with probability $\mathbf{s}(m, t) \in (0, 1)$. When tasks are close, the success rates are also close. To be precise, there exists some constant δ, such that $|\mathbf{s}(m, t) - \mathbf{s}(m', t)| \leqslant \delta |m - m'|$ for all $t \in T$ and all $m, m' \in M$.

[[Give a precise bound on $\triangle$?]]

At stage 2 after observing the tasks chosen by all senders and whether they succeed or fail, the receiver (R) takes an action $x \in X$ whereby

$$X = \{f : \{1, 2, 3, ..., n\} \to \{1, 2, 3, ..., n\}, f \text{ is one to one } \}$$

allocating the senders to $r \leqslant n$ jobs available, where $f(i) > r$ means that sender S_i remains unemployed.[20]

[19] The standard terminology involves *sending a message* rather than *performing a task*, but the fact that the sender can succeed or fail at it makes the second expression less awkward than the first.

[20] It is convenient to express the job assignment as a function onto $\{1, 2, 3, ..., n\}$ as it reflects the idea that

It is important to keep track of the beliefs of the receiver about the types of senders at three stages of the game.

1) τ_α denotes the random variable with *probability measure function* (pmf) $p_\alpha(t)$, representing any sender's type in view of the receiver before the sender chooses a task.

2) For each $m \in M$, $\tau_\mu(m)$ denotes the random variable with probability measure function $p_\mu(t|m)$, representing the type of a sender in view of the receiver after the sender chooses a task m.

3) For each $m \in M$ and $s \in \{0,1\}$, $\tau_\omega(m,s)$ denotes the random variable with probability measure function $p_\omega(t|m,s)$, representing the type of a sender in view of the receiver after the sender chooses a task m and yields performance $s \in \{0,1\}$ ($s = 1$ denotes success, and $s = 0$ denotes failure). [21]

Strategies

A (mixed) strategy for a sender S_i is a function $\sigma_i : T \to \Delta_M$, and we let $\sigma_i(m|t) \in [0,1]$ denote the probability that he chooses task m when his type is t, such that $\sum_{j=1}^{z} \sigma_i(m^j|t) = 1$. In the same fashion, a strategy for the receiver (R) is a function $\rho : (M \times \{0,1\})^n \to \Delta_X$ and we let $\rho(x|(m,s)^n) \in [0,1]$ denote the probability that R takes action x after observing the vector of task choices and successes/failures $(m,s)^n$. We denote the set of all strategies of the receiver by Σ_R.

Preferences

The preferences of the senders are represented by any collection of functions $\{u_i\}_{i=1}^n$ (Senders) and v (Receiver) with the following properties. We start with the senders.

- They strictly prefer being assigned to jobs with lower indexes:
  $u_i(x) > u_i(x') \Leftrightarrow x(i) < x'(i)$ and $x(i) < r$.

- They are only concerned with their own assignment:
  $u_i(x) = u_i(x')$ if $x(i) = x'(i)$.
  Thus, agent i's utility can be simplified to $u_i(y)$, $y \in \{1, ..., n\}$.

the receiver (R) is ranking the senders, which will become apparent once the utility functions are specified below.

[21] The only one of these random variables which is already fully specified is τ_α, distributed according to pmf p_α. The allowable distributions of the $\tau_\mu(m)$ and $\tau_\omega(m,s)$ will depend on the the strategies of the players and on the equilibrium concept. For simplicity the notation does not explicitly reflect these dependencies, but it is hopefully not at the expense of clarity as we only use it once the particular belief generation process in question is clear.

- There do not exist different kinds of unemployment:
  If $x(i) > r$ and $x'(i) > r$ then $u_i(x) = u_i(x')$.

In what follows we assume without loss of generality that all senders have the same utility function u.[22] Let $U(m_i, m_{-i}, \rho, t_i, t_{-i})$ denote the utility that sender i of type t_i would expect from choosing task m_i, knowing that the vector of types of all other senders was t_{-i}, and expecting them to choose tasks m_{-i} and the receiver to behave according to ρ.

$$\zeta(m_{-i}, t_{-i}, s_{-i}) = \prod_{k \neq i : s_k = 1} \mathbf{s}(m_k, t_k) \prod_{j \neq i : s_j = 0} (1 - \mathbf{s}(m_j, t_j))$$

which denotes the probability of performance vector s_{-i}, given that senders other than i having types t_{-i} choose tasks m_{-i}. Then,

$$U(m_i, m_{-i}, \rho, t_i, t_{-i}) =$$
$$\sum_{x \in X} u(x) \sum_{s_{-i}} \zeta(m_{-i}, t_{-i}, s_{-i}) \left[\mathbf{s}(m_i, t_i)\rho(x|m_{-i}, m_i, s_{-i}, 1) + (1 - \mathbf{s}(m_i, t_i))\rho(x|m_{-i}, m_i, s_{-i}, 0)\right]$$

If rather than expecting senders to choose tasks m_{-i}, i expects them to play according to strategy σ_{-i}, then based on the above description we have:

$$U(m_i, \sigma_{-i}, \rho, t_i, t_{-i}) = \sum_{m_{-i}} \left(\prod_{j \neq i} \sigma_j(m_j|t_j) \right) U(m_i, m_{-i}, \rho, t_i, t_{-i})$$

With some abuse of the notation, we denote by $U(m_i, \sigma_{-i}, \rho, t_i)$ the utility that a sender of type t_i expects from choosing task m_i, given that he expects the other senders to follow strategy σ_{-i} and the receiver to follow strategy ρ. Then:

$$U(m_i, \sigma_{-i}, \rho, t_i) = \sum_{t_{-i} \in T^{(n-1)}} \left(\prod_{k \neq i} p_\alpha(t_k) \right) U(m_i, \sigma_{-i}, \rho, t_i, t_{-i})$$

Finally, the expected payoff to player i when using strategy σ_i is:

$$U(\sigma_i, \sigma_{-i}, \rho, t_i) = \sum_{m_i} \sigma_i(m_i|t_i) U(m_i, \sigma_{-i}, \rho, t_i)$$

Next, we move on to the receiver. The receiver (R) in our game is a *representative agent* that stands for a matching process of the senders to r jobs, that works as follows. There are r principals, each seeking to hire one sender in order to maximize the

[22] As an example of a function satisfying these requirements, consider $u_i(x) = r + 1 - x(i)$ if $x(i) \leqslant r$ and $u_i(x) = 0$ if $x(i) > r$.

expected value $E[g(t)]$, where g is a strictly increasing function of t and which is identical for all principals. After observing the task and the performance of each sender (m_i, s_i), principals rank the senders by the expectation $E[g(t_i)|(m,s)^n]$ using the distribution $P(\overrightarrow{t}|(m,s)^n)$ and make job offers. One implication of the fact that g is strictly increasing and which we use repeatedly throughout what follows is:

(F1) If τ' and τ are random variables representing beliefs over of types, where τ' first order stochastically dominates τ (e.g. τ' FOSD τ), then $E_{\tau'}[g(t)] > E_{\tau}[g(t)]$.

We can think of this stage as a serial process whereby the principal of job 1 offers the job to her most preferred candidate, who automatically accepts the offer and leaves the pool of senders, then the principal of job 2 hires her most preferred candidate from the remaining senders, and so forth. We assume that principals cannot discriminate among senders by relying on labels, such that if a principal has identical beliefs over the types of several candidates (the most preferred), she must make offers to them uniformly at random.[23]

To simplify language we represent this kind of more elaborate matching process by a single receiver who makes an assignment of the senders ($x \in X$) with preferences over job assignments $v : X \times T^n \longrightarrow \mathbb{R}_+$ that are consistent with the underlying matching process. The receiver picks her strategy ρ to maximize:

$$V(\rho, P(\overrightarrow{t}|(m,s)^n), (m,s)^n) = \sum_{x \in X} \rho(x|(m,s)^n) \left(\sum_{\overrightarrow{t} \in T^n} P(\overrightarrow{t}|(m,s)^n) v(x, \overrightarrow{t}) \right)$$

We will make reference to the preferences of the principals at various points of our analysis in order to make use of the restrictions that they imply on the preferences of the receiver. Given two random variables τ and τ', taking values on T, we denote $E_{\tau'}[g(t)] \geqslant E_{\tau}[g(t)]$ by $\tau' \succeq_p \tau$.

Our analysis focuses on what happens to equilibria when there exists some level of competition, e.g. $n \geqslant 2$, and when the amount of competition in the environment as given by n increases. Given a prior type distribution, sender and receiver preferences, and success function $\mathbf{s} : M \times T \longrightarrow (0,1)$, we denote the game described above on n players by $\mathcal{G}_n$.

[23] As the senders agree on their preferences regarding the various jobs, the unique Nash equilibrium of a variety of games whereby the principals make proposals and then the senders accept or reject, involves the principal of the preferred job getting her most preferred candidate, the principal of the 2^{nd} most preferred job getting her most preferred candidate after excluding the candidate chosen by principal 1 and so on.

Main Parametric Assumptions

Three assumptions about the success probabilities at the various tasks characterize our environment:

(A0) (Difficulty)

Higher types are more likely to succeed in all tasks, and higher tasks are less likely to succeed for all types:

$$\mathbf{s}(m,t') > \mathbf{s}(m,t), \quad \forall m, t' > t,$$
$$\mathbf{s}(m,t) > \mathbf{s}(m',t), \quad \forall t, m' > m.$$

(A1) (Increasing success probability ratio property)

The success likelihood ratio is increasing in m:

$$\frac{\mathbf{s}(m',t')}{\mathbf{s}(m',t)} > \frac{\mathbf{s}(m,t')}{\mathbf{s}(m,t)} \text{ whenever } m' > m \text{ and } t' > t$$

(A2) (Increasing failure probability ratio property)

The failure likelihood ratio is increasing in m:

$$\frac{1-\mathbf{s}(m',t')}{1-\mathbf{s}(m',t)} > \frac{1-\mathbf{s}(m,t')}{1-\mathbf{s}(m,t)} \text{ whenever } m' > m \text{ and } t' > t$$

(A0) and (A1) are critical assumptions and used in most of the results below. If (A2) holds, competitive unraveling happens even with a small groups of senders.

Sequential Equilibria with Trembles

We focus on strategies that are label independent, so despite having multiple senders we will always have a single sender strategy σ. We are interested in receiver behaviors that reflect the underlying principals' rankings of candidates given their inferences of their research abilities. We therefore restrict attention to receiver strategies which treat senders identically when the principals in the underlying matching process are indifferent.

A sequential equilibrium is a tuple $\langle \sigma, \rho, \{P(\overrightarrow{t}|(m,s)^n)\}\rangle$, where σ is a sender strategy, ρ is a receiver strategy, and for each $(m,s)^n \in (M \times \{0,1\})^n$, $P(\cdot|(m,s)^n) : T^n \to [0,1]$ is a *pmf* describing the beliefs of the receiver, with the following three properties:

(P1) **The receiver's strategy is sequentially rational given her beliefs:**

$$\forall (m,s)^n \;\; \rho \in argmax_{g \in \Delta_X} V(g, P(\overrightarrow{t}|(m,s)^n), (m,s)^n)$$

(P2) **The senders' strategy is sequentially rational for every sender type given the receiver's strategy:**

$$\forall t_i \in T, \;\; \sigma \in argmax_{\sigma_i \in \Delta_M} U(\sigma_i, \sigma, t_i, \rho)$$

(P3) **The beliefs of the receiver are consistent**

The belief consistency requirements of sequential equilibrium restrict the possible beliefs $\{P(\overrightarrow{t}|(m,s)^n)\}$ in three ways:[24]

(BC1) The beliefs of R regarding the type of a given sender must be independent of her beliefs regarding any other sender. This follows from the belief consistency requirement in the definition of sequential equilibrium, the assumption that senders' types are independently drawn and the assumption that a sender has no information about other senders' types when choosing his tasks. That is:

$$P(t_1, t_2, t_3, ..., t_n|(m,s)^n) = \prod_i p_\omega(t_i|m_i, s_i)$$

(BC2) On the equilibrium path the beliefs are fully determined by Bayes rule and the senders' strategy.

$$\forall m \text{ such that } \sigma(m|t) > 0 \text{ for some } t, \quad p_\mu(t|m) = \frac{p_\alpha(t)\sigma(m|t)}{\sum\limits_{t' \in T} p_\alpha(t')\sigma(m|t')}$$

(BC3) The beliefs of R after observing a sender's performance are completely determined by Bayes rule and her beliefs after observing the sender's task choice. In particular:

$$p_\omega(t|m,1) = \frac{p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t' \in T} p_\mu(t'|m)\mathbf{s}(m,t')} \quad \text{and} \quad p_\omega(t|m,0) = \frac{p_\mu(t|m)(1-\mathbf{s}(m,t))}{\sum\limits_{t' \in T} p_\mu(t'|m)(1-\mathbf{s}(m,t'))}$$

Note that given properties $(BC1)$ and $(BC3)$, we can describe a sequential equilibrium as a triple $\langle \sigma, \rho, \{p_\mu(t|m)\}\rangle$ in which ρ is sequentially rational given the terminal beliefs $\{p_\omega(t|m,s)\}$ induced by $\{p_\mu(t|m)\}$, σ is sequentially rational given ρ, and for all m such

[24] In fact, these are necessary and sufficient conditions for the beliefs to be consistent in the sense of Kreps and Wilson (1982) Kreps and Wilson (1982). More specifically by adjusting the rates of convergence of the strictly positive strategies in the modified game to the equilibrium strategies we can obtain any system of beliefs $\{p_\mu(t|m)\}$ as long as it satisfies these conditions (along with making the senders' strategies sequentially rational).

that $\sigma(m|t) > 0$ for some t, $p_\mu(t|m)$ is determined by Bayes rule, σ, and the prior type distribution $p_\alpha(t)$. As above we use τ_α, $\tau_\mu(m)$ and $\tau_\omega(m,s)$ to denote random variables distributed according to $p_\alpha(\cdot)$, $p_\mu(\cdot|m)$ and $p_\omega(\cdot|m,s)$. Note that in the matching process underlying the receiver's behavior, the beliefs about sender types induce a preference relation for the principals over the random variables induced by different task-performance profiles. In what follows $\tau_\omega(m,s) \geq_P \tau_\omega(m',s')$ ($\tau_\omega(m,s) >_P \tau_\omega(m',s')$) denotes that the principals weakly (strictly) prefer the random variable induced by $(m's')$ to the random variable induced by (m,s).

As in the example, we restrict off-equilibrium path belief by relying on the idea of trembles. Suppose actions are fuzzy, in the sense that when sender i intends to perform task m^l then he performs this task with probability $1-2\varepsilon$, task m^{l-1} with probability ε and task m^{l+1} with probability ε.[25] The receiver's off-equilibrium path belief has to be consistent with strategies with trembles.

4 Equilibrium Characterization

The following lemma summarizes some properties that are met by any belief system in a sequential equilibrium, and which we use repeatedly in the proofs of our main results. Intuitively, it shows that the receiver must believe that (1) a successful sender in any given task (on or off the equilibrium path), is more likely[26] to be a higher type than an unsuccessful sender with the same task; (2) a successful sender is also more likely to be a higher type than a sender with unknown performance when they choose the same task; (3) given two tasks, chosen by the same distribution of senders, a successful sender in the more difficult task is more likely to be a higher type than a successful sender in the less difficult one if (A1) holds; (4) given two tasks, chosen by the same distribution of senders, an unsuccessful sender in the more difficult task is more likely to be a higher type than an unsuccessful sender in the less difficult one if (A2) holds; (5) senders with a higher type have a higher utility in equilibrium. The proof can be found in the appendix.

Lemma **1** *Let $\langle \sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ denote a sequential equilibrium.*

1. *If $(A0)$ holds, $\tau_\omega(m,1)$ FOSD $\tau_\omega(m,0)$ for all tasks m such that $\tau_\mu(m)$ is non-degenerate.*

2. *If $(A0)$ holds, $\tau_\omega(m,1)$ FOSD $\tau_\mu(m)$ for all tasks m such that $\tau_\mu(m)$ is non-degenerate.*

[25] If the intended task is at one of the two boundaries then, the "tremble" concentrates entirely on the only neighboring task, with probability 2ε. The task performed is the intended one with probability $1-2\varepsilon$.

[26] In a stochastic sense, the type distribution of the succeeding sender FOSD the one of the unsuccessful one.

3. *If* $(A1)$ *holds,* $m' > m$ *and* $\tau_\mu(m') = \tau_\mu(m)$ *then* $\tau_\omega(m', 1)$ *FOSD* $\tau_\omega(m, 1)$.

4. *If* $(A2)$ *holds,* $m' > m$ *and* $\tau_\mu(m') = \tau_\mu(m)$ *then* $\tau_\omega(m', 0)$ *FOSD* $\tau_\omega(m, 0)$.

5. *If* $(A0)$ *holds,* $t' > t$ *then* $U(\sigma^*, \rho^*, t') > U(\sigma^*, \rho^*, t)$.

First we show that in our game there do not exist sequential equilibria in which a type gets fully separated from all others. Suppose some type t is fully separated from all others in some equilibrium. If it is the lowest type, this full separation would fully reveal the lowest type and the sender would be given the lowest priority in the matching. If it is not the lowest type, the lowest type could choose some task to mimic the behavior of type t and earn a higher utility by the last point of Lemma 1. So full separation is not a possible equilibrium outcome.

PROPOSITION **1 (No Separation)** *If (A0) holds and* $\langle \sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ *is a sequential equilibrium, then there do not exist* m *and* t*, such that* $\sigma(m|t) > 0$ *and* $\sigma(m|t') = 0\ \forall t' \neq t$.

Proof of Proposition 1: Suppose to the contrary that there exist m and t such that $\sigma(m|t) > 0$ and $\sigma(m|t') = 0\ \forall t' \neq t$.

Case (I) $\exists\, t'' < t$. Consider some $t'' < t$. By Lemma 1 $U(\sigma^*, \rho^*, t) > U(\sigma^*, \rho^*, t'')$. Moreover as t is the only type for which $\sigma(m|t) > 0$, the payoff $\tau_\omega(m, 0) = \tau_\omega(m, 1)$ and therefore the payoffs under success or failure are the same. Type t'' can therefore do strictly better by playing m, and this contradicts the sequential rationality of σ^* for t''.

Case (II) $\nexists\, t'' < t$. Then $\tau_\omega(m', 0)$ FOSD $\tau_\omega(m, 1)$ for any m' such that $\sigma(m'|t'') > 0$ for some $t'' \neq t$. Let σ' be the same as σ^* except $\sigma'(t) = m'$. By optimality of ρ^* we must therefore have $U(\sigma', \rho^*, t) > U(\sigma^*, \rho^*, t)$, contradicting the sequential rationality of σ^* for t. ∎

Proposition 1 shows that in our setting any task on the equilibrium path is always populated by at least two types. The next proposition shows that when (A0), (A1) and (A2) hold then competition among senders leads to a kind of unravelling. Specifically, all the pure strategy sequential equilibria with trembles must involve all sender types selecting tasks towards the top of the range of tasks $\{m^0, m^1, m^2, ..., m^z\}$.

PROPOSITION **2 (Weak Competitive unraveling)** *When (A0)-(A2) hold and* $\triangle$ *is small enough, then in any pure strategy sequential equilibrium with trembles,* $\langle \sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$,

- *Some senders must pool at* m^z*, i.e.* $\sigma^*(m^z|t) = 1$ *for at least two types.*

- *If* $\sigma^*(m|t) > 0$ *for some* t*,* $m \geqslant m^{z-k+2}$ *if* k *is even and* $m \geqslant m^{z-k+3}$ *if* k *is odd. (Recall* k *is the total number of different types.)*

Furthermore, there always exists a pure strategy sequential equilibrium with trembles in which all senders pool at m^z.

Proof of Proposition 2: Suppose $\langle \sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ is a sequential equilibrium with trembles. Let m^h be the highest task that senders choose with a positive probability in this equilibrium. We first show that m^h must be m^z.

Suppose instead $m^h < m^z$, and t^h is one of the types who choose m^h. We bound the gain and loss for a sender (sender i) with type t^h from deviating to m^{h+1}. m^{h+1} is an off-equilibrium task, and by trembles, the receiver holds the same interim belief $p_\mu(t|m^h) = p_\mu(t|m^{h+1})$. Then by Lemma 1, $\tau_\omega(m^{h+1}, 1)$ FOSD $\tau_\omega(m^h, 1)$ and $\tau_\omega(m^{h+1}, 0)$ FOSD $\tau_\omega(m^h, 0)$. The only loss occurs because of the decrease in success probability, which is at most $\delta\triangle$. The most severe case is getting the best job ($u_i(1)$) with a success and being unemployed ($u_i(n)$) with a failure. Let $U = u_i(1) - u_i(n)$. The loss is bounded above by

$$loss \leqslant \delta\triangle U$$

The gain could happen in many realized outcomes, we just consider a simple one to provide a lower bound. With probability $p_\alpha(t^h)^{n-1}s(t^h, m^h)^n$, all other senders' type is t^h and all senders succeed in m^h. The receiver holds the same posterior belief of all senders, and thus sender i's utility is $u_{avg} = \frac{1}{n}\sum_{y=1}^{n} u_i(y)$. If sender i deviates to m^h and succeeds, his utility is $u_i(1)$ instead. So the gain from the deviation is bounded below by

$$gain > p_\alpha(t^h)^{n-1}s(t^h, m^h)^n(u_i(1) - u_{avg})$$

Note that the decrease in success probability is captured in the loss analysis, and it doesn't show up in the gain analysis. So by deviating to task m^{h+1} from m^h, sender i can get a non-trivial increase in the probability of being selected for better jobs, while as $\triangle$ is small enough, the change in success probability is negligible. So deviation to m^{h+1} is profitable, which is a contradiction. Then in a pure strategy sequential equilibrium, $m^h = m^z$.

We now show that whenever $m^* > m'$, if $\sigma^*(m^*|t^*) = 1$ for some t^*, $\sigma^*(m'|t') = 1$ for some t' and $\sigma^*(m|t) = 0$ for all t and m such that $m' < m < m^*$, then $m^* - m' < 2\triangle$; that is, there can be a gap of at most one task between any two succeeding tasks in the sequence of tasks chosen by senders in equilibrium. Suppose that there are at least two other tasks between m^* and m', and let m'' be the one right above m'. By trembles, $p_\mu(t|m'') = p_\mu(t|m')$, and by the same argument above, it is profitable for senders to deviate from m' to m'', which contradicts the fact that m' is chosen in equilibrium.

Finally, pooling at the most difficult task m^z can always be supported as a sequential equilibrium with trembles. Due to an argument analogous to the one at the first part of the proof, under such a strategy profile a sender, regardless of his type, would never find it profitable to deviate to task m^{z-1}. Deviations to tasks $m' < m^{z-1}$ (outside of the trembling region) can be precluded in any sequential equilibrium by assuming that the receiver's interim beliefs upon observing a sender choose such a task place probability 1 on him being of the lowest type. ∎

Remark 1: If there are only two or three types, then it follows from Proposition 2 that in all pure strategy sequential equilibria with trembles all types must pool at the most difficult task m^z.

An alternative way to view the proposition is that if m is chosen in equilibrium, then $|m - m^z| < k\triangle$. As $\triangle$ get very small, senders pool around the very top of the tasks.

In addition, if the success probabilities are linear in types, i.e. there exists some constant $\beta > 0$. s.t. $s(m,t) - s(m,t') = \beta(t - t')$, then the results discussed in Proposition 2 can be strengthened considerably. In fact in this case all pure strategy sequential equilibria with trembles involve all agents pooling at the most difficult task. Intuitively, with linearity, we can show that between $m > m'$, if a lower type prefers a higher task m then a higher type would also prefers m. So it must be higher types pooling in m and lower types pooling in m', and if so the receiver would prefer failure in m to success in m'. Then it is not optimal for senders to choose m'. We will prove the proposition under the linearity assumption first, and then discuss its necessarity in the remark.

PROPOSITION **3 (Strong Competitive unraveling)** *When (A0)-(A2) hold, $s(m,t)$ is linear in t and $\triangle$ is small enough, $\mathcal{G}_n$ has a unique pure strategy sequential equilibrium with trembles, pooling in m^z, e.g. $\langle \sigma^*, \rho^*, \{p_\mu(t|m)\}\rangle$ such that $\sigma^*(m^z|t) = 1\ \forall t$.*

Proof of Proposition 3: Suppose m^h and m' are the highest and second highest task that senders choose, $m^h > m'$. As shown in the proof pf Proposition 2 , when $\triangle$ is small, the probabilities of success of any given type at m^h and at the following task m' which some senders choose are very close. The receiver has preferences over the agents achieving the different possible outcomes at these tasks determined by his posterior beliefs $p_\omega(t|m^h, 1)$, $p_\omega(t|m^h, 0)$, $p_\omega(t|m', 1)$ and $p_\omega(t|m', 0)$. The receiver's preference must satisfy the following statements:

- Since both tasks are pooled by more than one type, by Lemma 1, success must be preferred to failure in any given task.
- Success in one task must be preferred to failure in the other task, because otherwise the other task is dominated and no one would want to choose it.
- If success in one task is preferred to success in the other one, then failure in that task must be less preferred to failure in the other one.

The last statement can be proved by an argument similar to the one proving $m^h = m^z$. Suppose both success and failure in m^h are preferred to the corresponding outcomes from m'. Recall that the difference in success probabilities in m^h and m' is at most $2\delta\triangle$. When $\triangle$ is small enough, it is profitable to deviate from m' to m^h, which contradicts the fact that m' is chosen in equilibrium. So the last statement must hold. Without loss of generality, suppose $p_\omega(t|m^h, 1)$ is preferred to $p_\omega(t|m', 1)$ and $p_\omega(t|m', 0)$ is preferred to $p_\omega(t|m^h, 0)$. Let $U(\sigma^*_{-i}, \rho^*_{-i}, 1_m)$ (or $U(\sigma^*_{-i}, \rho^*_{-i}, 0_m)$) be sender i's expected utility with a success (or failure) in a task m, $U(1_m)$ (or $U(0_m)$) for short. Then the receiver's preference implies

$$U(1_{m^h}) \geqslant U(1_{m'}) > U(0_{m'}) \geqslant U(0_{m^h}) \tag{6}$$

in which the first and third relationships cannot hold with equality at the same time.[27] Let $t = \underline{t}(m^h)$ be the lowest type who chooses m^h, and let $t' = \bar{t}(m')$ be highest type who chooses m'. To rationalize the receiver's preference, t must be lower than t', since otherwise the receiver should prefer those choosing m^h regardless of the performance. A type-t sender finds it profitable to choose m^h,

$$s(t, m^h)U(1_{m^h}) + (1 - s(t, m^h))U(0_{m^h}) \geqslant s(t, m')U(1_{m'}) + (1 - s(t, m'))U(0_{m'})$$

It can be rewritten as

$$s(t, m^h)(U(1_{m^h}) - U(0_{m^h})) + U(0_{m^h}) \geqslant s(t, m')(U(1_{m'}) - U(0_{m'})) + U(0_{m'})$$

By linearity, $s(t', m) = s(t, m) + \beta(t' - t)$ and
note that $U(1_{m^h}) - U(0_{m^h}) > U(1_{m'}) - U(0_{m'})$, So

$$s(t', m^h)(U(1_{m^h}) - U(0_{m^h})) + U(0_{m^h}) > s(t', m')(U(1_{m'}) - U(0_{m'})) + U(0_{m'})$$

which contradicts the fact that a type-t' sender prefers m'. Thus, pooling at multiple different tasks cannot happen. In equilibrium, all senders pool at m^z. ∎

Remark 2. It is possible to have senders pool at multiple tasks without the linearity assumption. Here is an illustrative example. Take (6) from the proof and four types $t^1 < t^2 < t^3 < t^4$. Construct the success probability as follows. Suppose $s(t^1, m^h) < s(t^1, m')$ and $\xi > 0$ is small, such that

$$s(t^1, m^h)(U(1_{m^h}) - U(0_{m^h})) + U(0_{m^h}) = s(t^1, m')(U(1_{m'}) - U(0_{m'})) + U(0_{m'}) + \xi$$

[27] If both of them hold with equality, all senders would strictly prefer m' since it gives a higher success probability.

so t^1 prefers m^h. Then let

$$s(t^2, m^h) = s(t^1, m^h) + \frac{\xi}{U(1_{m^h}) - U(0_{m^h})}, \ \ s(t^2, m') = s(t^1, m') + \frac{3\xi}{U(1_{m'}) - U(0_{m'})}$$

$$s(t^3, m^h) = s(t^2, m^h) + \frac{\xi}{U(1_{m^h}) - U(0_{m^h})}, \ \ s(t^3, m') = s(t^2, m') + \frac{\xi}{U(1_{m'}) - U(0_{m'})}$$

$$s(t^4, m^h) = s(t^3, m^h) + \frac{3\xi}{U(1_{m^h}) - U(0_{m^h})}, \ \ s(t^4, m') = s(t^3, m') + \frac{\xi}{U(1_{m'}) - U(0_{m'})}$$

It is easy to check that $s(t^i, m^h) < s(t^i, m')$ for $i \in \{2, 3, 4\}$. Type t^2 and t^3 prefer m' and type t^1 and t^4 prefer m^h. So it is possible to have t^1 and t^4 pooling at m^h and have t^2 and t^3 pooling at m' (the construction of the rest of the example is omitted).

Lack of Robustness to Competition

As in the case of the example in Section 2, we are also interested in characterizing the collection of sender behaviors that are robust to competition when dropping (A2).

DEFINITION **1 (Robustness to Competition)** *A sender strategy σ^* is robust to competition if for each game $\mathcal{G}_n$ it is part of some sequential equilibrium $\langle \sigma^*, \rho, \{p_\mu(t|m)\}\rangle$.*

This refinement selects equilibria which are preserved as the number of senders changes. The main motivation for this refinement is that in many settings senders may only have a rough idea of the number of their competitors, and this number may vary over time frequently. On the other hand we are attempting to understand patterns of behavior, which characterize institutions to the point that they become observable regularities.[28]

We denote *simple pooling equilibrium* if all types pool at the same task. If we restrict our attention to equilibria which are robust to competition, then the remaining equilibria are simple pooling equilibria and all other possible equilibria will disappear. For example, suppose given some n, there is one equilibrium where senders pool over several tasks. If the receiver favors successes in a unique task, say m^*, as the number of senders goes to infinity, the probability that as least r senders succeed in the m^* goes to 1. Senders choosing other tasks almost have no probability of getting the job. Thus senders will pool only in m^*. Otherwise if the receiver favors successes in several tasks equally, senders would pool only in the easiest task among the most favorable tasks. In addition, if we consider trembles, senders want to deviate to a slightly higher task, so the only possible equilibrium is pooling at m^z. We states the equilibrium characterization without trembles first.

[28]See the application of the model to theoretical particle physics or to conspicuous consumption.

PROPOSITION **4** *The only sender strategies that are robust to competition are given by* $\sigma(m|t) = 1\ \forall t \in T$*, for some* $m \in M$*.*

The proof of Proposition 4 relies on the following lemma. For clarity we introduce some notation:

• Tasks on the equilibrium path: $e(\sigma) = \{m : \sigma(m|t) > 0$ for some $t \in T\}$

• Set of tasks such that a success in any one of them is weakly preferred by the principals to success in any other task: $\phi(\sigma) = \{m : \tau_\omega(m, 1) \succeq_p \tau_\omega(m', 1)$ for all $m'\}$.[29]

The Lemma says that when the number of senders in the market is large enough, the tasks chosen on the equilibrium path must be among the receiver's most preferred (as defined above). That is, given the receiver's preferences over task-performance profiles induced by her beliefs, for sufficiently high n the difference in success probabilities across different tasks becomes a second order consideration for all sender types.

LEMMA **2** *For sufficiently large* n*, if* $\langle \sigma, \rho, \{p_\mu(t|m)\} \rangle$ *is a sequential equilibrium of* $\mathcal{G}_n$ *it must be the case that* $e(\sigma) \subseteq \phi(\sigma)$*.*

The idea of the proof is as follows. When n is large enough, there are many senders with each type t_i. If some most preferred task ($m \in \phi(\sigma)$) occurs on the equilibrium path, it is almost certain that there are more senders succeeding in this task than the number of jobs such that each job almost certainly gets assigned to senders who choose from the set of most preferred tasks. So it cannot be optimal for any sender type to choose a task m' such that $(m', 1)$ is not among the preferred tasks.[30] On the other hand, if none of the most preferred tasks can occur on the equilibrium path, then by deviating to one of the most preferred tasks a sender would for sure obtain the best job upon success. And regardless of how low this success probability is, the expected payoff from deviation would eventually exceed the payoff on the equilibrium path which converges to 0 when increasing the number of senders.

Proof of Lemma 2: Suppose it is not the case that $e(\sigma) \subseteq \phi(\sigma)$. We split the argument into two cases: (I) $e(\sigma) \cap \phi(\sigma) \neq \emptyset$ and (II) $e(\sigma) \cap \phi(\sigma) = \emptyset$.

(I) $e(\sigma) \cap \phi(\sigma) \neq \emptyset$. Consider any $m' \in e(\sigma)$ such that $m' \notin Pf(\sigma)$. A necessary condition for a sender to get a job by choosing $m' \in e(\sigma)$ is that one of the r principals is forced to resort to his less preferred options (call this event E_1). E_1 the complement of the

[29] This set depends on the equilibrium sender strategy σ through the beliefs $\tau_\omega(m, 1)$.

[30] For low values of n this is not the case as when the principal gets to choose, it is possible that there is no sender left with the principal's most preferred tasks so it is likely that he needs to continue going down the list to his second most preferred tasks or even further.

event (E_1^c) that each principal is able to assign her respective job to some candidate with one of her most preferred tasks. A sufficient condition for E_1^c is that at least r senders are successful in some task $m \in e(\sigma) \cap \phi(\sigma)$. Let $q = min\{\sigma(m|t) : t \in T\}$. We can bound the probability of E_1^c with the probability of the less likely event that r senders succeed in m with the lowest success probability.[31] So we have that:

$$\begin{aligned}
P(E_1) &= 1 - P(E_1^c) \\
&\leqslant 1 - \sum_{i=r}^{n} \binom{n}{i} q\mathbf{s}(m,t^0)^i (1 - q\mathbf{s}(m,t^0))^{n-i} \\
&= \sum_{i=0}^{r-1} \binom{n}{i} q\mathbf{s}(m,t^0)^i (1 - q\mathbf{s}(m,t^0))^{n-i} \\
&\leqslant \frac{1}{2} e^{-2\frac{(nq\mathbf{s}(m,t^0)-(r-1))^2}{n}}
\end{aligned}$$

where the last inequality is Hoeffding's bound for the cumulative distribution function of binomially distributed random variables. So the payoff to a sender from choosing m' can be bounded above for large enough n by $u(1)\frac{1}{2}e^{-2\frac{(nq\mathbf{s}(m,t^0)-(r-1))^2}{n}}$

On the other hand, the probability that a sender gets a job from choosing $m \in e(\sigma) \cap \phi(\sigma)$ is bounded below by $\frac{\mathbf{s}(m,t^0)}{n}$, which is the probability that the sender succeeds at m and is then chosen uniformly at random from among all other succeeding senders, which are less than n. Thus the payoff can be bounded from below by $u(r)\frac{\mathbf{s}(m,t^0)}{n}$. We therefore have that:

$$\begin{aligned}
\frac{U(m',\sigma_{-i},\rho,t)}{U(m,\sigma_{-i},\rho,t)} &\leqslant \frac{u(1)\frac{1}{2}e^{-2\frac{(nq\mathbf{s}(m,t^0)-(r-1))^2}{n}}}{u(r)\frac{\mathbf{s}(m,t^0)}{n}} \\
&= \frac{u(1)}{2u(r)\mathbf{s}(m,t^0)} n e^{-2(nq^2\mathbf{s}(m,t^0)^2 - 2(r-1)q\mathbf{s}(m,t^0) + \frac{(r-1)^2}{n})} \\
&\longrightarrow 0 \text{ as } n \text{ goes to } \infty
\end{aligned}$$

The convergence to 0 holds because ne^{-cn} converges to 0 as n goes to infinity for any constant $c > 0$. Thus it is not optimal to choosing m', which contradicts the assumption $m' \in e(\sigma)$. We can conclude that $e(\sigma) = \phi(\sigma)$.

[31] For the purpose of establishing this bound we think of the event of succeeding in a specific task as two independent events in a sequence: (1) choosing the task,which happens with probability at least q and (2) succeeding at it, which happens with probability at least $\mathbf{s}(m,t^0)$.

(II) $e(\sigma) \cap \phi(\sigma) = \emptyset$. The payoff that the sender would get from choosing some task $m \in \phi(\sigma)$ is bounded from below by $u(r)\mathbf{s}(m, t^0)$, whereas his payoff from choosing any task $m \in e(\sigma)$ converges to 0 as $n \longrightarrow \infty$. This is because for any large enough $w > r$, the probability that at least w other senders choose m and succeed converges to 1 as n goes to infinity. Upon such a tie, the r jobs are assigned uniformly at random among these $w + 1$ senders, so the probability that any given sender gets a job converges to 0. ∎

Proof of Proposition 4: By Lemma 2 for sufficiently large n it is true that for all tasks m chosen on the equilibrium path $\tau_\omega(m, 1) \succeq_p \tau_\omega(m', 1)$ where m' is any other task. The proposition now follows from noting that for sufficiently large n if $e(\sigma) \subseteq \phi(\sigma)$ then $e(\sigma)$ must be a singleton. The reason is that for sufficiently large n all r jobs almost certainly get assigned uniformly at random among the succeeding senders in the tasks in $e(\sigma)$[32] and as a result the part of any sender's expected payoff that corresponds to the event of being assigned a job after failing converges exponentially fast to 0, as in the proof of Lemma 2. As a result it becomes strictly better for each sender to pick the task in $e(\sigma)$ at which he is most likely to succeed, which by assumption (A1) is the one in $e(\sigma)$ with the lowest index, for all sender types. So unless this is the unique element of $e(\sigma)$, σ can't possibly be part of a sequential equilibrium $\langle \sigma^*, \rho, \{p_\mu(t|m)\} \rangle$. ∎

The argument in the proof of Proposition 4 is similar to the one behind the competitive unraveling in our example in Section 2. Underneath any equilibrium there are some tasks which are most preferred by the principals (inside or outside the equilibrium path) ($\phi(\sigma)$ in our notation). Although at low levels of competition, senders may trade off attempting one of the principal-preferred tasks $m \in \phi(\sigma)$ for a higher chance of success at some task $m' \notin \phi(\sigma)$ which is still likely to be rewarded with a job, as n rises, the cost of not competing for one of the employer-preferred tasks comes to exceed the benefits of the higher success probability at some other task.

Formally speaking equilibria of $\mathcal{G}_n$ and $\mathcal{G}_{n'}$ when $n \neq n'$ are necessarily different as the number of players is a primitive of the game and thus to begin with the spaces of receiver strategies are formally different in both games. However, as can be seen throughout the discussion, the only components of the receiver strategy that play a role in satisfying the definition of sequential equilibrium in our model are her strategies on the equilibrium path, and on information sets that are reached by deviations by a single sender from the equilibrium path. Given a pooling strategy of the senders on any given m, there are equivalent equilibria sustaining it in the corresponding games $\mathcal{G}_n$ for all values of n in the sense that they involve exactly the same receiver beliefs about each individual sender under any possible behavior on and off the equilibrium path.

[32] Keeping in mind that the principals choose uniformly at random among the senders among which they are indifferent.

Proposition 4 established that the only sender strategies that are *robust to competition* are those which prescribe that every type chooses a given task with probability 1. Then if we add the restriction on off-equilibrium path belief introduced by trembles, the only surviving equilibrium is pooling at m^z.

PROPOSITION **5** *When (A0)-(A1) hold and n is large enough, in any sequential equilibrium with trembles, the only sender behavior that is robust to competition is $\sigma(m^z|t) = 1\ \forall t$.*

Proof of Proposition 5: By Proposition 4, we can focus on simple pooling equilibria, i.e. all senders pool at the same task. Suppose all senders pool at $m^h \neq m^z$. We consider if the lowest type t^0 (say sender i) wants to deviate to m^{h+1}. If sender i stays with m^h, the expected utility is at most $\frac{1}{n}\sum_{j=1}^{n} u_i(x(j))$, because the lowest type has the lowest probability of success. If sender i chooses m^{h+1}, he succeeds with probability $s(t^0, m^{h+1})$ and gets $u_i(x(1))$. When n is large enough,

$$s(t^0, m^{h+1})u_i(x(1)) > \frac{1}{n}\sum_{j=1}^{n} u_i(x(j))$$

So pooling at m^h cannot be an equilibrium, and the only equilibrium must be pooling at m^z. ∎

We remark that Proposition 3 and Proposition 5 both justify that competition could lead to all senders pooling at the most difficult task. The difference is that Proposition 3 holds for any level of competition ($n \geqslant 2$) but needs stronger restrictions, e.g. (A2) and pure strategies. While Proposition 5 holds when the competition is severe (large n), but it can also rule out mixed strategies.

Lastly, we discuss efficiency, i.e. the receiver's utility. In Claim 3 of the example, it is possible that pooling in m^z is the least efficient outcome for the receiver. The problem is that the probability of success in m^z could be too low to be useful. While when n is large enough, especially to ensure enough success in m^z, pooling in m^z is the most efficient outcome since it has the strongest screening power by $(A1)$.

PROPOSITION **6** *When (A0)-(A1) hold and n is large enough, pooling in m^z is the most efficient outcome among all simple pooling equilibria.*

Proof of Proposition 6: Let $\underline{s} = \min_{t,m} s(t, m)$ be the lowest probability of success. We first claim that when all senders pool at m ($n_s(n, m)$ is the number of senders who succeed),

$$\lim_{n\to\infty} prop(n_s(n, m) \geqslant r) = 1 \tag{7}$$

Let $n = lr + z$, in which l and z are nonnegative integers and $z < r$. The probability of at least one success among l senders is higher than $1 - (1 - \underline{s})^l$, then

$$prop(n_s(n, m) \geqslant r) \geqslant (1 - (1 - \underline{s})^l)^r$$

As n goes to infinity, so does l. The RHS goes to 1. Also $prop(n_s(n, m) \geqslant r) = 1 - \eta$ increases in n.

By Lemma 1, if $(A1)$ holds, $m' > m$ and $\tau_\mu(m') = \tau_\mu(m)$, then $\tau_\omega(m', 1)$ FOSD $\tau_\omega(m, 1)$. So in the case with at least r senders who succeed, m^z must be the most efficient outcome. Suppose $prop(n_s(m) \geqslant r) = 1 - \eta$ and the principle's utility $g(t) \in [g_L, g_H]$. With senders pooling in m^{h+1}, the principle's utility is at least

$$(1 - \eta)E_{\tau_\omega(m^{h+1},1)}g(t) + \eta g_L$$

While with senders pooling in m^h, the principle's utility is at most

$$(1 - \eta)E_{\tau_\omega(m^h,1)}g(t) + \eta g_H$$

The difference is at least

$$(1 - \eta)(E_{\tau_\omega(m^{h+1},1)} - E_{\tau_\omega(m^h,1)}g(t)) - \eta(g_H - g_L)$$

Let $e = \min_h E_{\tau_\omega(m^{h+1},1)} - E_{\tau_\omega(m^h,1)}$. By (7), there exists some n^* such that when $n \geqslant n^*$, $\eta < \frac{e}{g_H - g_L + e}$. So the difference above is alway positive, implying pooling in m^z gives the principle the highest utility. ▮

5 Conclusion

We study in detail a signaling model that captures scholars' choice of research tasks as they seek to make progress in the early stages of their academic careers. The model fits other settings of on-the-job screening, and more generally to situations in which senders are competing for prizes by sending noisy messages about unobservable characteristics. The key feature of the setting[33] is that the more difficult tasks are more conspicuous in the sense that they distinguish more clearly among different underlying research abilities. As it is often the case in settings with asymmetric information, in our model there are multiple sequential equilibria, reflecting a variety of conventions which society can uphold by relying on appropriate off-the-equilibrium-path beliefs. We focus on the sub-

[33] Formally represented by assumptions (A1) and (A2).

set of sequential equilibria in which belief about the underlying research ability in an off-the-equilibrium-path task must be consistent with that in an on-the-equilibrium-path task if there is one very close. It can be induced by the idea of trembles.

We show that under this refinement on the beliefs, the only sequential equilibrium is pooling in the most difficult task in two popular scenarios. First, if the more difficult tasks are more conspicuous in the case of both success and failure, then pooling in the most difficult task is the only pure strategy sequential equilibrium as long as competition exists ($n \geqslant 2$). Second, if the more difficult tasks are more conspicuous in the case of success and the competition is sufficiently high, then pooling in the most difficult task is the only sequential equilibrium. However, pooling in the most difficult task may not be efficient for the society since it could have a very low success rate. While this tendency towards *conspicuousness* may be seen as detrimental in certain contexts, it can also serve the crucial role of providing the kind of continuity and persistence required for certain scientific breakthroughs. In fact, as can be seen throughout the history of science, the apparent fertility or barrenness of a scientific agenda at a given moment in time is often a bad predictor of its future.

Appendix: Omitted Proofs

Proof of Lemma 1:

Part 1. By $(BC3)$ we have that:

$$P(t \geqslant t^l|m,1) = \sum_{t \geqslant t^l} p_\omega(t|m,1) = \sum_{t \geqslant t^l} \frac{p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t \in T} p_\mu(t|m)\mathbf{s}(m,t)} \quad \text{and}$$

$$P(t \geqslant t^l|m,0) = \sum_{t \geqslant t^l} p_\omega(t|m,0) = \sum_{t \geqslant t^l} \frac{p_\mu(t|m)(1-\mathbf{s}(m,t))}{\sum\limits_{t \in T} p_\mu(t|m)(1-\mathbf{s}(m,t))}$$

So we have that:

$P(t \geqslant t^l|m,1) \geqslant P(t \geqslant t^l|m,0)$

$$\iff \sum_{t \geqslant t^l} \frac{p_\mu(t|m)\mathbf{s}(m,t)}{\sum\limits_{t \in T} p_\mu(t|m)\mathbf{s}(m,t)} \geqslant \sum_{t \geqslant t^l} \frac{p_\mu(t|m)(1-\mathbf{s}(m,t))}{\sum\limits_{t \in T} p_\mu(t|m)(1-\mathbf{s}(m,t))}$$

$$\iff \sum_{t \in T} p_\mu(t|m) \sum_{t \geqslant t^l} p_\mu(t|m)\mathbf{s}(m,t) \geqslant \sum_{t \geqslant t^l} p_\mu(t|m) \sum_{t \in T} p_\mu(t|m)\mathbf{s}(m,t)$$

$$\iff \left(\sum_{t<t^l} p_\mu(t|m) + \sum_{t\geqslant t^l} p_\mu(t|m)\right) \sum_{t\geqslant t^l} p_\mu(t|m)\mathbf{s}(m,t) \geqslant$$
$$\sum_{t\geqslant t^l} p_\mu(t|m) \left(\sum_{t<t^l} p_\mu(t|m)\mathbf{s}(m,t) + \sum_{t\geqslant t^l} p_\mu(t|m)\mathbf{s}(m,t)\right)$$
$$\iff \sum_{t<t^l} p_\mu(t|m) \sum_{t\geqslant t^l} p_\mu(t|m)\mathbf{s}(m,t) \geqslant \sum_{t\geqslant t^l} p_\mu(t|m) \sum_{t<t^l} p_\mu(t|m)\mathbf{s}(m,t)$$
$$\iff \frac{\sum_{t\geqslant t^l} p_\mu(t|m)\mathbf{s}(m,t)}{\sum_{t\geqslant t^l} p_\mu(t|m)} \geqslant \frac{\sum_{t<t^l} p_\mu(t|m)\mathbf{s}(m,t)}{\sum_{t<t^l} p_\mu(t|m)}$$

which is necessarily true since $\mathbf{s}(m,t)$ is strictly increasing in t for all tasks m (A1):

$$\frac{\sum_{t\geqslant t^l} p_\mu(t|m)\mathbf{s}(m,t)}{\sum_{t\geqslant t^l} p_\mu(t|m)} \geqslant \mathbf{s}(m,t^l) \geqslant \frac{\sum_{t<t^l} p_\mu(t|m)\mathbf{s}(m,t)}{\sum_{t<t^l} p_\mu(t|m)}$$

Note that if $\tau_\mu(m)$ is non-degenerate then at least one of these two inequalities becomes strict under $(A0)$. ▮

Part 2. From Part 1, we know $\tau_\omega(m,1)$ FOSD $\tau_\omega(m,0)$. As $P(t \geqslant t^l|m) = P(s_i = 1|m)P(t \geqslant t^l|m,1) + P(s_i = 0|m)P(t \geqslant t^l|m,0)$ this implies $\tau_\omega(m,1)$ FOSD $\tau_\mu(m)$. ▮

Part 3. $P(t \geqslant t^l|m',1) \geqslant P(t \geqslant t^l|m,1)$

$$\iff \sum_{t\geqslant t^l} \frac{p_\mu(t|m')\mathbf{s}(m',t)}{\sum_{t\in T} p_\mu(t|m')\mathbf{s}(m',t)} \geqslant \sum_{t\geqslant t^l} \frac{p_\mu(t|m)\mathbf{s}(m,t)}{\sum_{t\in T} p_\mu(t|m)\mathbf{s}(m,t)}$$

$$\iff \sum_{t\geqslant t^l} \frac{p_\mu(t|m')\mathbf{s}(m',t)}{\sum_{t\in T} p_\mu(t|m')\mathbf{s}(m',t)} \geqslant \sum_{t\geqslant t^l} \frac{p_\mu(t|m')\mathbf{s}(m,t)}{\sum_{t\in T} p_\mu(t|m')\mathbf{s}(m,t)}$$

$$\iff \sum_{t\in T} p_\mu(t|m')\mathbf{s}(m,t) \sum_{t\geqslant t^l} p_\mu(t|m')\mathbf{s}(m',t) \geqslant \sum_{t\in T} p_\mu(t|m')\mathbf{s}(m',t) \sum_{t\geqslant t^l} p_\mu(t|m')\mathbf{s}(m,t)$$

$$\iff \sum_{t<t^l} p_\mu(t|m')\mathbf{s}(m,t) \sum_{t\geqslant t^l} p_\mu(t|m')\mathbf{s}(m',t) \geqslant \sum_{t<t^l} p_\mu(t|m')\mathbf{s}(m',t) \sum_{t\geqslant t^l} p_\mu(t|m')\mathbf{s}(m,t)$$

$$\iff \frac{\sum_{t\geqslant t^l} p_\mu(t|m')\mathbf{s}(m',t)}{\sum_{t<t^l} p_\mu(t|m')\mathbf{s}(m',t)} \geqslant \frac{\sum_{t\geqslant t^l} p_\mu(t|m')\mathbf{s}(m,t)}{\sum_{t<t^l} p_\mu(t|m')\mathbf{s}(m,t)}$$

Note that for each $t^* \geqslant t^l$ such that $p_\mu(t^*|m') > 0$, we have that:

$$
\begin{aligned}
&\frac{p_\mu(t^*|m')\mathbf{s}(m',t^*)}{\sum\limits_{t<t^l} p_\mu(t|m')\mathbf{s}(m',t)} > \frac{p_\mu(t^*|m')\mathbf{s}(m,t^*)}{\sum\limits_{t<t^l} p_\mu(t|m')\mathbf{s}(m,t)} \\
\iff &\frac{\sum\limits_{t<t^l} p_\mu(t|m')\mathbf{s}(m,t)}{p_\mu(t^*|m')\mathbf{s}(m,t^*)} > \frac{\sum\limits_{t<t^l} p_\mu(t|m')\mathbf{s}(m',t)}{p_\mu(t^*|m')\mathbf{s}(m',t^*)}
\end{aligned}
$$

which is true due to $(A1)$. Note that if $p_\mu(t^*|m') = 0$ then the two expressions are equal. Note that in particular when $t^l = t^0$ and provided that $\tau_\mu(m) = \tau_\mu(m')$ is non-degenerate the inequality involving the cumulative distributions is strict. We therefore have the first order stochastic dominance relationship stated in the Lemma. ∎

Part 4:. The proof is identical to that in Part 3 except replacing the success probability $\mathbf{s}(m,t)$ by the failure probability $1 - \mathbf{s}(m',t)$. ∎

Part 5:. First assume that t does not fully separate itself from other types. That is, there exists m such that $\sigma(m|t) > 0$ and $\sigma(m|t'') > 0$ for some $t'' \neq t$. From part 1 we have that $\tau_\omega(m,1)$ FOSD $\tau_\omega(m,0)$ and by (F1) the receiver strictly prefers success to failure. Moreover since $t' > t$ and $\mathbf{s}(m,t') > \mathbf{s}(m,t)$, type t' is more likely to succeed in all tasks. So if type t' mimics types t it would get a strictly higher payoff.

So we just need to rule out that t fully separates itself from all other types, which is when performance does not provide the receiver with additional information. Suppose that t fully separates itself. We claim that this implies that type t^0 is not acting optimally . If $t = t^0$ then due to full separation the receiver knows for sure that it is t^0 and she strictly prefers any sender choosing any task different from those chosen by t on the equilibrium path regardless of success or failure. So a type $t = t^0$ sender is not acting optimally as he would do strictly better by imitating any other type. Now suppose that $t > t^0$ and notice that by the discussion above t^0 does not use a fully separating strategy, which in turn implies that $U(\sigma^*,\rho^*,t) > U(\sigma^*,\rho^*,t^0)$. Moreover, if t^0 mimics t, then t^0 would earn utility $U(\sigma^*,\rho^*,t)$ since t fully separates itself and therefore performance is irrelevant.

It is therefore never the case that a sender type ever fully separates itself, and we can conclude from above that $U(\sigma^*,\rho^*,t') > U(\sigma^*,\rho^*,t)$. ∎