A Model of Confounded Entrepreneurial Choice

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Abstract

Should young people be encouraged to become entrepreneurs? The theory of occupational choice of Miller (1984) prescribes experimenting with more uncertain jobs at the early stage of one's career for the sake of fast identification of one's comparative advantages. According to the theory, there is value in learning even in the case of entrepreneurial failure. However, certain empirical findings put entrepreneurial learning in question. First, entrepreneurs keep on running under-performing businesses, not switching to payroll jobs even with long business tenure. Second, entrepreneurs are believed to be consistently over-optimistic to rationalise frequent entry in face of low business survival rates. I argue it is the more complex "jack of all trades" nature of entrepreneurship that is behind these facts. In my model, income of employees depends only on their ability while entrepreneurial income depends jointly on the ability and the business acumen. Both the factors are unknown and agents learn about them through noisy production. Entrepreneurial income only provides one signal on two unobservables. Entrepreneurs who over-estimate their acumen assign a fraction of disappointing performance to their ability which leads to a "lock-in" effect in entrepreneurship. Early quits and switching between the two types of employment, which produce high entry and exit rates, are required to identify one's business acumen.

JEL classification: E24, J24, J64, L26
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1 Introduction

I try to go against many of these bromides that people have in Silicon Valley and one of the ones that I really dislike is that somehow failure is this great learning experience or something like that

– Peter Thiel, co-founder of Pay Pal.

Should the society support entrepreneurial ventures? Without a doubt, we benefit from the innovations brought by super star entrepreneurs like Peter Thiel, Elon Musk or Steve Jobs. However, the experience of these superstars is markedly different from that of the average entrepreneur. It is the relative popularity of entrepreneurship in light of this harsher reality to which I turn my attention in this paper.

I focus on two groups of facts about entrepreneurship. First, most entrepreneurs in the cross-section earn less than what they could earn as employees. Furthermore, this difference persists even among entrepreneurs with long tenure in business (Hamilton, 2000). Second, many workers become entrepreneurs, quit relatively fast yet then re-enter entrepreneurship several times. Previous work aimed at exploring these facts in isolation, claiming existence of significant compensating non-pecuniary benefits in entrepreneurship or consistent initial over-optimism of entrepreneurs who quickly learn the true prospects of their businesses. These explanations, although plausible, are at odds with each other. In this paper I propose confounded entrepreneurial learning as a parsimonious explanation of all those facts jointly.

In my model, individuals endowed with general ability and business acumen choose between entrepreneurship, payroll employment and non-participation in the labor market. Ability and acumen are both unknown and can be learned about only via production that is subject to transitory idiosyncratic productivity shocks. Employees execute well-defined routine tasks so that payroll employment income depends only on the general ability. Entrepreneurs engage not only in repetitive but in creative activities as well. Thus, general ability and business acumen are jointly used, potentially to a different extent, in generating entrepreneurial income. Hence, income from entrepreneurship only provides one signal on two unobserved factors. This feature of entrepreneurial income leads to confounding of beliefs in Bayesian updating. The existence of non-participation generates additional incentives to learn the true values of acumen and ability. If business acumen is scarce, so that few agents improve in entrepreneurship on their payroll income, then more of individuals are likely to consider non-participation as an option superior to entrepreneuership which increases the fraction of short entrepreneurial spells.

Because of confounding, when confronted with worse than expected entrepreneurial income, a rational outcome of Bayesian updating is to decrease the belief not only on the business acumen, but also on the ability. This effect is strongest for individuals who initially overestimate their business acumen and underestimate their general ability. On top of that, these agents are the most likely to become entrepreneurs as their initial information points to strong comparative advantage in entrepreneurship. The strength of this effect is increasing in initial uncertainty on ability. Hence, the decision to switch back to payroll employment may be postponed for some time, or even forever. Based on a set of observable characteristics, such underperforming entrepreneurs may well look like enjoying additional non-pecuniary benefits of flexibility, not having a boss to report to etc.

The prevalence of short entrepreneurial episodes, usually interpreted as an evidence of high failure rate in entrepreneurship, is in fact required to refine individual information on business acumen. Because of confounding it is not possible to reduce uncertainty on acumen without gathering additional information on ability in payroll employment. This is why so many entrepreneurs instead of running their business for many years in a row rather prefer to switch back and forth. Only those who are sufficiently optimistic about their acumen don't switch.

I assume for tractability that ability and acumen are time invariant individual specific characteristics. This may seem restrictive, as at least some of entrepreneurs experiment with new business ideas, as did Peter Thiel and Elon Musk. However, in that case the effects of confounding would be magnified. If the potential of the first business idea was overestimated, the unsuccesful entrepreneur could be susceptible to trying out ideas of lower quality as her judgement on her own ability would be low. This story is in particular compelling for entrepreneurs who have their first start-up early if the young are more prone to over-optimism than the old. In this sense, the model developed in this paper provides the lower bound on the size of confounding effects in entrepreneurship. Apart from that, focusing on superstars may again be misleading. Several studies document (Hurst and Pugsley, 2011, Koellinger, 2008) that a majority of entrepreneurs run small scale businesses and don't innovate, yet, it is beyond argument that they face more complex challenges than employees.

This paper aims at making two contributions, one is theoretical and the other one is quantitative. As my theoretical contribution, I extend the model of learning developed in the seminal works of Jovanovic (1979, 1982) and Miller (1984). In those papers agents choose between independent alternatives with payoffs that depend on uncertain parameters. In particular, Jovanovic (1982) has entrepreneurial outside option (a payroll job) to be homogenous and certain across entrepreneurs. In my model one of the two outside options is uncertain and heterogeneous and the returns of payroll jobs and entrepreneurship are not independent. I also discuss the implications of the frequently made assumption on separate observation of all the relevant components of individual payoffs.

As a quantitative contribution, I test the relevance of learning for the type of employment choice using moments from a panel on US workers. The model can generate frequent short entrepreneurial spells and, due to the value of experimentation, most entrepreneurs earn less than employees. My model can generate about 40 percent of the earnings differential in the cross-section and also matches the fraction of underperforming entrepreneurs with longer business experience found in the data. The model predicts a 2-year exit rate from entrepreneurship at around 55 percent, in the ball park of the approximately 70 percent share of businesses that terminate early found in my sample.

There are two counterfactual predictions of the model. First, it generates self-employment rate that decreases with age. Second, it predicts the fraction of workers trying entrepreneurship to be more than two times the corresponding number in the data. This is due to the assumption on the "entrepreneurial factor" to be time-invariant business acumen. A reformulation of the model with business acumen replaced by business ideas arriving at a certain exogenous rate is bound to improve on the baseline specification. Part of the model overshooting the data in this respect may also be due to the annual frequency of measurement which leaves out, for example, entrepreneurial spells of quarterly length.

Then, I check the effects that reducing uncertainty about ability has on the productivity of entrepreneurs. This can be interpreted, for example, as an exogenous improvement in the quality of education by providing students with apprenticeships and alike. The background for the results is that in the model population of workers, the cost of information friction is moderate and amounts to about 4 percent of median annual earnings. However, this burden is distributed in a highly uneven way. In particular, without information friction the median entrepreneur would earn 24 percent more while for the median employee this gain is roughly equal to 2 percent.

I find that the reduction of uncertainty on individual ability by 10 percent increases the median of entrepreneurial income by about 1.1 percent. If the initial uncertainty on general ability was decreased by half, then the average entrepreneurial income would increase by about 4.8 percent only due to improved selection. In the absence of information friction, the entrepreneurship rate would be as low as 2.8 percent. Thus, the resource cost of information friction lies in experimentation of young agents and in inferior long-run selection in entrepreneurship.

At first glance, the improvement in the quality of entrepreneurs may seem rather small. However, entrepreneurs compete against each other for patents, financing and markets. Without a doubt they also affect the productivity of their employees. Thus, improving the selection and reducing the number of entrepreneurs can have potentially much greater implications for overall efficiency because of all the other effects my parsimonious model abstracts from.

Related Literature This paper is linked to two strands of literature. The first one is research on entrepreneurship. The contribution to this strand of the literature is threefold. First, it provides an alternative to the compensating differentials hypothesis in accounting for the differences between entrepreneurial and employee income distributions. Second, it proposes a mechanism behind relatively high exit and entry rates in entrepreneurship. Third, the confounded learning offers a way of bridging the learning and cognitive biases theories of entrepreneurship.

This paper is also related to the literature on occupational choices and learning which focus is mostly on employees choosing between different jobs and the costs of unemployment. The contribution of this paper is to propose a framework with some uncertain characteristics being transferable between activities which naturally corresponds to modeling jobs as compositions of tasks. Also, non-participation margin is introduced. This setting can be further expanded to account for interactions that go beyond simple correlations of returns. Below I explain those links in greater detail.

The model introduced in this paper is a partial equilibrium extension of the learning framework of Jovanovic (1982). In that paper the entrepreneurs enter with uncertainty about the quality of their business, there modeled as an unknown cost, and can switch at any point in time back to payroll employment that has a known value. In my model the value of payroll employment is also uncertain, the only choice of certain value is non-participation. A setting similar to the entrepreneurial learning component of my model is employed in recent work of Jovanovic (2015). He focuses on the dynamics of recombination of matches between agents of different, unknown abilities that are complementary in production. The key difference in my setup is the existence of an alternative source of information – payroll employment – and how its presence affects individual switching between the technologies of production.

The models of Roy (1951), Lucas (1978) and Jovanovic (1994) postulate that income in one activity (entrepreneurship versus payroll employment) is a one to one function of an ability dedicated solely to this particular activity. A notable exception is the "jack of all trades" theory by (Lazear, 2004, 2005). According to this paper entrepreneurs are generalists that have to do well in many different tasks, in contrast to specialists that make great narrowly-focused employees. My paper can thus be viewed as a dynamic learning version of Lazear (2004).

This paper is related to studies that explain the puzzle of low returns to entrepreneurship because of the option value of entrepreneurship. Vereshchagina and Hopenhayn (2009) show that the discrete occupational choice decision introduces non-concavities that can be dealt with by the choice of the size of investment and the option to switch back to payroll employment. A similar argument about a real option continuation value of start-ups has been made by Hintermaier and Steinberger (2005) and Campanale (2010). I argue that the assessment of the outside option value in entreprenuership may be subject to an information friction. The value of learning through entrepreneurship is also present in my model.

There are two recent papers that argue it is the learning mechanism that helps reproduce the observed cross-sectional distribution of earnings conditional on type of employment. Manso (2014) convincingly shows that the cross-sectional distribution of earnings may understate the premium to entrepreneurship when one does not account for the value of learning. His focus is on comparing life cycle earnings patterns as a function of entrepreneurial experience. He finds that short entrepreneurial spells have no negative effect on life cycle earnings profile and that omitting longitudinal dimension may lead to biased estimates of earnings differentials. Dilon and Stanton (2014) structurally estimate a complex dynamic model of type of employment choice. In their model the value of employment evolves stochastically subject to wage shocks but otherwise the learning is only on entrepreneurial ability, independent from the wage process. They argue that short entrepreneurial spells are a result of eirhet fast identification of the quality of business idea or a response to a wage shock. I complement those papers by providing an alternative specification of how entrepreneurs learn and introduce direct dependence between payroll and entrepreneurial incomes.

My paper is complementary to studies that argue in favor of the existence of significant non-pecuniary benefits (Hurst and Pugsley, 2011). The model developed here suggests that any empirical estimates of the size of non-pecuniary benefits may be biased if the nature of learning in entrepreneurship is not properly accounted for. However, as the results of my quantitative exercise suggest, introducing non-pecuniary benefits helps matching the negative difference between entrepreneur and employee incomes even in the presence of learning. This paper is also complementary to the literature that emphasizes the importance of financial constraints in entry into entrepreneurship (Buera, 2009, Evans and Jovanovic, 1989, Evans and Leighton, 1989), as financial constraints could be incorporated in the environment I build in this paper.

There is a vast body of research on cognitive biases and hubris in entrepreneurship, examples include Lowe and Ziedonis (2006), Hayward, Shepherd, and Griffin (2006) and Baron (1998) which are all put forth to explain either some form of lock-in effect in entrepreneurship or frequent business start ups despite seemingly high failure rates. In my paper this arises as a result of information updating by sophisticated agents and the link between the entrepreneurial performance and entrepreneur beliefs on the value of her outside options.

The type of employment decision is a version of occupational choice which has been studied extensively. The occupational mobility and learning models usualy assume that all factors that determine the income from a match (employee ability, employer characteristics, occupation) are learned about through observing separate signals and that those factors are either independent (McCall, 1990, Miller, 1984) or correlated (Moscarini, 2005, Papageorgiou, 2013). I contribute to this literature by lifting up the assumption of separate observation. Following up on the discussion on the confounding mechanism, my model nests (depending on the exact formulation of payoffs, random changes to the underlying heterogeneity etc.) the standard learning framework with or without correlation. Also, there is no unemployment in my model which is replaced with non-participation.

Gibbons, Katz, Lemieux, and Parent (2005) structurally estimate a model of learning in the labor market. They constrain the sample to include employees only. This is a standard practice as it permits to assume that a competitive market bids the employees wages on a particular job up to the publicly observable belief on worker's ability dedicated to that job. It is hard to argue the same would hold for entrepreneurs, thus, I rely to a different empirical strategy that relies on simulation based matching of moments in the data.

The signal extraction problem resembles the one found in Lucas (1972) and Li and Weinberg (2003). In those papers the learning friction is resolved over time as the observations arrive at different instants of time (one signal before agents make their choices and the other afterwards). In my model full identification of uncertain factors can only happen by switching between alternatives. Thus, the friction is not resolved by a late arrival of an additional signal.

The rest of the paper is organised as follows. In Section 2 I discuss the empirical regularities that my model seeks to explain. In section 3 I formulate the model. In section 4 the working of the learning process is derived analytically. Section 5 contains the description of the properties of the dynamic problem. Section 6 contains the results of simulation-based calibration of model parameters together with a study of the role of model assumptions, section 7 concludes.

2 Empirical Facts

This section documents key facts that will discipline the model using the data from the National Longitudinal Sample of Youth 1979 (NLSY79). The NLSY79 tracks a cohort of individuals across time, allowing for observation of pre- and post-entrepreneurship spells labor market outcomes. The key benefit of using NLSY 79 is that it allows to track the labor market outcomes of individuals starting at the very onset of their careers.

NLSY79 is a survey of 12686 individuals aged 14-22 when they were first surveyed in 1979. The survey continued on annual basis up to 1994, then the interviewees were contacted biannually and 2012 is the last available year. Hence, the sample in NLSY79 is relatively young. Each year a respondent has to answer a set of very detailed questions on, among other things, enrollment in education and labor market outcomes. The NLSY79 consists of three subsamples: an initial representative subsample of 6111 individuals and two additional subsamples introduced to track specifically individuals in the military forces and ethnic minorities.

Fairlie (2005) documents that individual characteristics including gender, education and race also influence the type of employment mobility for the reasons outside of the scope of this paper. Thus, I construct my sample using the representative subsample only and I focus on white men with at least some college experience. Furthermore, I also remove farmers and workers from disadvantaged families. These data adjustments serve the purpose of constructing a sample of ex-ante similar individuala, similar to the approach of Papageorgiou (2013) to avoid blurring the type of employment mobility patterns due to factors other than individual abilities. I consider an individual a worker (either an entrepreneur or an employee) for a particular year if they reported income and worked at least 400 hours in a given year and don't report more than 60 hours worked per week. Finally, I drop individuals with less than 3 observations on income as they are the most likely to be mismeasured. In my sample I have 1080 individuals and 15083 observations. Income is defined as annual earnings, assigned to the job the respondent reported working the most hours in a given year. An entrepreneurial spell (or spell in short) is defined as a set of consecutive observations (at least one) in entrepreneurship that ends with return to payroll employment or leaving the survey. Several stylised facts can be identified.

Many workers try entrepreneurship and switch between activities. I find that about 29 percent of workers have some entrepreneurial experience meaning they have at least one observation in entrepreneurship. The first entry into entrepreneurship usually happens in the early years of one's career. About 45 percent of individuals with some entrepreneurial experience have their first spell in the first five years after entering the labor market as depicted on Figure 4. First entrepreneurial entry after more than 10 years of labor market experience is very rare. Next, almost 40 percent of individuals who tried entrepreneurship are "serial" entrepreneurs, that is, they have more than one distinct spell of entrepreneurship in their career separated by periods of payroll employment. Thus, there is substantial type of employment mobility among the young workers. These facts are demonstrated on Figure 3. Re-entering entrepreneurship is most frequent among the earliest entrants. Interestingly, about two-third of entrepreneurial spells are very short and last up to two years. These spells are not limited to individuals who quit entrepreneurship and never return to it, they are also prevalent among the serial entrepreneurs. The average time spent in entrepreneurship, conditional on having at least one spell, is 2.8 years which is about a sixth of average individual history length.

Negative median earnings differential in entrepreneurship. The cross-sectional distribution of annual earnings conditional on type of employment features larger dispersion of entrepreneurial income. The fatter right tail of entrepreneurial income drives the mean enterpreneurial income up so that it exceeds, although only slightly, the mean of employee income. The median entrepreneur earns about 12 percent less than a median employee. The key characteristics of the pooled sample are described in Table 1 and on Figure 1 the histograms of these distributions are provided.

To some extent, the income differentials may come from the timing of entrepreneurial spells which for a large fraction of entrepreneurs happen at the early stage of their careers. Thus, the differential may be due, in particular, to human capital accumulation or aggregate time-varying conditions. The earnings may also depend on the pre-entry characteristics that determine the industry and occupation the individuals work at, including the university degree etc. To remove the effect of observable factors in shaping the income distribution I run a panel regression on log of annual earnings controlling for observables (labor market tenure, hours worked, years of education, degree obtained, year fixed effect, industry) with an interacted individual-type of employment fixed effect.

This approach, however, has some limitations given that for some workers with shortest entrepreneurial spells it's not possible to fully disentangle the fixed effect and the individual independent shock (residual in the earnings regression). The histogram of combined residual from the earnings equation (including the individual fixed effect) is provided on Figure 2. The residuals in entrepreneurship are more dispersed. Then, to obtain a measure of income conditional on observable characteristics I shift the residuals to have the mean that corresponds to the mean of the log-earnings. By converting back to levels it turns out that the median differential increases, the median entrepreneurial earnings are 84 percent of median employee earnings. The ratio of the means is roughly unchanged and stands at about mean entrepreneurial income being larger by 2 percent.

Most early entrepreneurs earn less than what they could earn as employees. The data on the subsample of workers with at least 3 years of experience in either of the types of employment show that about 52 percent of entrepreneurs would earn more upon return to payroll employment. For entrepreneurs with at least 5 years of experience this fraction drops to 36 percent. Thus, it seems that the entrepreneurial learning is slow. However, this is difficult to reconcile with large fraction of very short entrepreneurial spells. On the one hand, the first fact implies that it is necessary to spend a lot of time as entrepreneur to correctly asses the type of employment comparative advantage. On the other hand, the second fact implies, at least at face value, that entrepreneurs efficiently learn about abandoning entrepreneurship and quit fast.

Employee and entrepreneur earnings are positively correlated. After controlling for observable differences between agents the data show a positive correlation between earnings rank of employees and entrepreneurs. The location of an individual in the earnings distribution of employees is similar to her location among entrepreneurs when choosing that activity. The scatter plot of individual averages of income conditional on observable characteristics for workers with experience in entrepreneurship and payroll employment of at least 3 years is provided on Figure 5. The Spearman rank coefficient is 0.67 and is significant at 1-percent level.

A priori there is no clear argument for correlation of any particular sign. For example, it could be the case that business skills decline during formal education that increases general ability. Then, if business skills were more important in entrepreneurship and general ability more important in payroll employment, there should be a negative rank correlation.

The evidence presented in this section poses some targets for the theory of entrepreneurial learning. First, the theory should be able to address how can the frequent switches between entrepreneurship and payroll employment coexist with the large fraction of underperforming entrepreneurs. In the baseline learning by doing model the entrepreneurs run their businesses and return to payroll employment only if their business doesn't perform as well as expected. Entrepreneurial learning has no frequent switching back and forth between entrepreneurship and payroll employment. Second, the theory should be able to generate sufficient incentives for experimentation in entrepreneurship, reflected in the negative earnings differential, given the positive correlation of employee and entrepreneurial incomes. For example, the latter fact suggests that the baseline model should be extended with heterogeneity in the entrepreneurial outside option.

3 Basic Features of the Environment

In this section I introduce the key features of the entrepreneurial choice model which is further expanded in the next sections. I also provide a short discussion of the assumptions implicit in the model formulation.

3.1 Preferences and Technology.

The economy is inhabited by a unit mass of individuals who value their consumption c_t and have utility function $u(c_t)$. There is no storage of the consumption good so that the production within period y_t is consumed fully. All individuals live for T periods. The size of the population is kept constant as new generations enter the economy and the size of each population is constant and equal to $\frac{1}{T}$. The agents discount the future at a rate β.

There are two types of income generating activities: entrepreneurship and payroll jobs. Each individual i has some endowment of general ability a_i^* and business acumen m_i^*. The endowment of (a_i^*, m_i^*) is distributed in the economy according to a distribution function $F(a_i^*, m_i^*)$, bivariate normal with parameters $\left\{\mu_{a^*}, \mu_{m^*}, \sigma_{a^*}^2, \sigma_{m^*}^2, \rho^*\right\}$. The individuals may opt out from productive activities choosing not to participate in the labor market and enjoy a value of leisure of b which can be thought of as home production that does not require any specific skills.

The production function is type of employment specific and is subject to idiosyncratic productivity shocks which volatility can depend on the type of employment[1]. The agents supply their ability and acumen inelastcally. Most importantly, income in entrepreneurship can, in general, depend on *both* general ability and acumen while payroll income is postulated to be a function of a_i^*. Formally:

$$y_{i,t}^p = a_i^* + \varepsilon_{i,t}^p, \quad \text{where } \forall i \quad \varepsilon_{i,t}^p \sim \mathbb{N}\left(0, \sigma_p^2\right), \tag{1}$$

$$y_{i,t}^e = \gamma a_i^* + m_i^* + \varepsilon_{i,t}^e, \quad \text{where } \forall i \quad \varepsilon_{i,t}^e \sim \mathbb{N}\left(0, \sigma_e^2\right), \tag{2}$$

$$y_{i,t}^n = b. \tag{3}$$

The parametric specification of production in entrepreneurship features an additional parameter

[1] The equations (1)-(2) will describe either the level or log of product, depending on the context.

γ which scales the effect of general ability on entrepreneurial income. γ is assumed to be in the $[0,1]$ interval. This implies that there are some gains from higher ability in entrepreneurship on top of the business acumen and that for agents with low acumen the payroll employment dominates entrepreneurship.

3.2 Information friction

The individual ideas and abilities are not observable and can only be learned by observing production. Hence, the entrepreneurial income constitutes a *confounded* signal on the pair (a^*, m^*) whenever $\gamma > 0$. The entrepreneur faces an identification problem learning from one signal about two unobservables. All agents at time of birth receive an initial unbiased signal about true abilities (a^*, m^*). The signal produces individual agent's prior about her composition of abilities $\mathcal{I}_{i,0}$, bivariate normal with parameters $\left\{\hat{a}_{i,0}, \hat{m}_{i,0}, \hat{\sigma}^2_{a,i,0}, \hat{\sigma}^2_{m,i,0}, \hat{\rho}_{i,0}\right\}$.

Observe that the special case of $\gamma = 0$ and $\rho^* = 0$ implies that individual employee and entrepreneurial incomes are uncorrelated and each provides a signal on one dimension of unobservable heterogeneity. In that case the model developed here can be considered a version of the Jovanovic model of entrepreneurial learning, extended with learning about the outside option of payroll income.

3.3 The dynamic problem

The decision problem the worker is facing is to choose an *action plan* $\xi^T = (\xi_1, ..., \xi_T)$ that contains the decision to work in entrepreneurship, in a payroll or to remain out of the labor force. The actions ξ_t thus are in the discrete set $\Xi_t \in \{\xi^e, \xi^p, \xi^n\}$. Each action plan ξ^t generates a set of all possible income realization histories $\mathcal{S}^t$ with a typical element being the history of production $y^t = (y_1(\xi_1), ..., y_t(\xi_t))$. A priori, the plan is conditional on all future possible realizations of income. The optimization problem $\mathcal{P}$ that the households solve is given below, the dependence on all model-relevant parameters is omitted for convenience.

$$\mathcal{P}: \max_{\{\xi^{T-1}\}} V = \mathbb{E}_{\mathcal{I}_0} \sum_{t=0}^{T-1} \beta^t u\left(y_t\left(\xi_t\right)\right) \text{ subject to:}$$

$$\text{belief updating: } \mathcal{I}_t = f\left(\mathcal{I}_{t-1}, y^{t-1}\right), \text{ given } \mathcal{I}_0$$

$$\text{and the production function.}$$

Observe that $\mathcal{P}$ is an example of a multi armed bandit problem (MBP) with bandit arms corresponding to choices of activity. Unless $\gamma = 0$ and $\rho^* = 0$ the payoffs $y^p_{i,t}$ and $y^e_{i,t}$ are not independent. Without independence of arms the problem $\mathcal{P}$ does not meet the criteria for applicability of the standard approach of index policies proposed in Gittins (1979). It means that it is not possible to decompose the problem of choosing between M arms into M one-dimensional problems and solve them forward. The exact solution has to make use of M-dimensional dynamic programming techniques. Following the standard approach in the literature on MBPs, I work with the expected *Bayesian regret*. Intuitively, the regret is a measure of the cost of uncertainty about the optimal (in expectations) course of action.

Definition 1 (Bayesian regret) *Let ξ^*_t be the optimal arm in period t. The expected Bayesian regret R is:*

$$R = \mathbb{E}_{\mathcal{I}_0} \sum_{t=0}^{T-1} \beta^t \left[u\left(y_t\left(\xi^*_t\right)\right) - u\left(y_t\left(\xi_t\right)\right)\right]. \tag{4}$$

Observe that minimization of the discounted sum of expected Bayesian regret yields identical policy as the one that solves $\mathcal{P}$. As the optimal arm in each period is fixed and doesn't depend on the action plan ξ^T, we can decompose the sum in equation (4), as the returns to the best arm $\bar{V}$ and the target function V in problem $\mathcal{P}$ so that $V\left(\mathcal{I}_t\right) = \bar{V} - R\left(\mathcal{I}_t\right)$. From this relationship it follows that maximizing V is equivalent to minimizing R. Intuitively, the choices the agents make affect their future information set and the incoming information is used in the decision making. I start with a description of the learning process, the analytical characterization of the regret follows.

4 Learning

The goal of this section is to build insight on the consequences of the structure of the signals and describe the dynamics of the evolution of the beliefs, $\mathcal{I}_t = f\left(\mathcal{I}_{t-1}, y^{t-1}\right)$. Without loss of generality let's consider an agent of age $t \leq T$. We consider an action plan ξ^T and some particular history of production $s^t = \left(y_1^p, ..., y_{t_p}^p, y_1^e, ..., y_{t_e}^e, b_1, ..., b_{t_n}\right)$ without taking stand about its optimality. A key feature of history s^t is how much time was spent not producing $t_n \geq 0$ or producing either in payroll employment $t_p \geq 0$ or in entrepreneurship $t_e \geq 0$ with $t_i = \sum \mathbb{1}_{\xi_t = \xi^i}$. Naturally, any such partition exhausts s^t in the following sense:

$$t - t_n = t_p + t_e. \tag{5}$$

The following two results can be shown with the proof relegated to Appendix B.

Proposition 1 (Sufficient statistic) *The sufficient statistic for $\mathcal{I}_t$ given $\mathcal{I}_0$ is $\{\bar{y_p}, \bar{y_e}, t_e, t_p\}$ with $\bar{y_k} = \frac{1}{t_k}\sum_{l=0}^{t_k} y_l^k$, the sample mean.*

Proposition 2 (No learning when not producing) *Let $\xi_t = \xi^n$, then $\mathcal{I}_t = \mathcal{I}_{t-1}$.*

The proposition 1 permits the use of standard dynamic programming techniques, in line with the results in chapter 10 of Bertsekas and Shreve (2007), including a recursive representation of the dynamic problem. Theorem 1 characterizes the belief $\mathcal{I}_t$ as a function of initial beliefs and history s^t, the dependence of the beliefs on agent's index i is omitted for convenience.

Theorem 1 *Consider an agent with an initial belief $\mathcal{I}_0$ with a type employment history s^t summarized by a sufficient statistic $(t_e, t_p, \bar{y}_p, \bar{y}_e)$, then the belief $\mathcal{I}_t$ has:*

- *posterior means:*

$$\hat{a}_t = \frac{\left(\frac{t_p \bar{y}_p}{\sigma_p^2} + \frac{\gamma t_e \bar{y}_e}{\sigma_e^2} + \frac{a_0}{(1-\rho_0^2)\sigma_{a,0}^2} - \frac{\rho_0 m_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}}\right)\left(\frac{t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{m,0}^2}\right)}{\left(\frac{t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{m,0}^2}\right)\left(\frac{t_p}{\sigma_p^2} + \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{a,0}^2}\right) - \left(\frac{\rho_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}\right)^2}$$
$$+ \frac{\left(\frac{\rho_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}\right)\left(\frac{t_e \bar{y}_e}{\sigma_e^2} + \frac{m_0}{(1-\rho_0^2)\sigma_{m,0}^2} - \frac{\rho_0 a_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}}\right)}{\left(\frac{t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{m,0}^2}\right)\left(\frac{t_p}{\sigma_p^2} + \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{a,0}^2}\right) - \left(\frac{\rho_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}\right)^2}$$

$$\hat{m}_t = \frac{\left(\frac{t_e \bar{y}_e}{\sigma_e^2} + \frac{m_0}{(1-\rho_0^2)\sigma_{m,0}^2} - \frac{\rho_0 a_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}}\right)\left(\frac{t_p}{\sigma_p^2} + \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{a,0}^2}\right)}{\left(\frac{t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{m,0}^2}\right)\left(\frac{t_p}{\sigma_p^2} + \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{a,0}^2}\right) - \left(\frac{\rho_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}\right)^2}$$
$$+ \frac{\left(\frac{\rho_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}\right)\left(\frac{t_p \bar{y}_p}{\sigma_p^2} + \frac{\gamma t_e \bar{y}_e}{\sigma_e^2} + \frac{a_0}{(1-\rho_0^2)\sigma_{a,0}^2} - \frac{\rho_0 m_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}}\right)}{\left(\frac{t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{m,0}^2}\right)\left(\frac{t_p}{\sigma_p^2} + \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{a,0}^2}\right) - \left(\frac{\rho_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}\right)^2}$$

- *posterior variances:*

$$\hat{\sigma}_{a,t}^2 = \frac{\frac{t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{m,0}^2}}{\left(\frac{t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{m,0}^2}\right)\left(\frac{t_p}{\sigma_p^2} + \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{a,0}^2}\right) - \left(\frac{\rho_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}\right)^2}$$
$$\hat{\sigma}_{m,t}^2 = \frac{\frac{t_p}{\sigma_p^2} + \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{a,0}^2}}{\left(\frac{t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{m,0}^2}\right)\left(\frac{t_p}{\sigma_p^2} + \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{a,0}^2}\right) - \left(\frac{\rho_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}\right)^2}$$

- *and the correlation coefficient:*

$$\hat{\rho}_t = \frac{\frac{\rho_0}{(1-\rho_0^2)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}}{\sqrt{\left(\frac{t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{m,0}^2}\right)\left(\frac{t_p}{\sigma_p^2} + \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{1}{(1-\rho_0^2)\sigma_{a,0}^2}\right)}}$$

Next, one can show that the standard property of rational updating holds, namely that the belief means $\hat{a}_t$ and $\hat{m}_t$ are martingales. Thus, the agents don't expect any confounding effects of entrepreneurial learning.

Theorem 2 (Martingale property of beliefs) *Take a belief $\mathcal{I}_t$ with some $\hat{a}_t$, $\hat{m}_t$ and history s^t*

with $t_e, t_p \neq 0$. It holds that

$$\forall \xi_{t+1} \quad \mathbb{E}_t \hat{a}_{t+1} | \mathcal{I}_t = \hat{a}_t \text{ and } \mathbb{E}_t \hat{m}_{t+1} | \mathcal{I}_t = \hat{m}_t.$$

4.1 Asymptotic beliefs

Now, for illustrative purposes let's consider an extreme situation of indefinitely long learning for an agent with life length $T \mapsto \infty$. To contrast the model developed here with the baseline learning model I start with an assumption that the agents choose their type of employment once-and-for-all at birth. For convenience let $\gamma = 1$.

Corollary 1 (Asymptotic beliefs, fixed type of employment.) *Let $\gamma = 1$. Consider the infinite horizon updating problem of an agent with true individual abilities bundle (a^*, m^*). Then the limiting belief distribution satisfies:*

- *in payroll employment:*

$$\lim_{t \mapsto \infty} \hat{a}_t = a^* \qquad \lim_{t \mapsto \infty} \hat{m}_t = \hat{m}_0 + \hat{\rho}_0 \frac{\sigma_{m,0}}{\sigma_{a,0}} (a^* - a_0)$$

$$\lim_{t \mapsto \infty} \hat{\sigma}^2_{a,t} = \lim_{t \mapsto \infty} \hat{\rho}_t = 0 \qquad \lim_{t \mapsto \infty} \hat{\sigma}^2_{m,t} = \left(1 - \rho_0^2\right) \hat{\sigma}^2_{m,0}$$

- *in entrepreneurship:*

$$\lim_{t \mapsto \infty} \hat{a}_t = \frac{(a^* + m^*) \left(\frac{1}{\sigma^2_{m,0}} + \frac{\rho_0}{\sigma_{a,0}\sigma_{m,0}} \right) + \frac{\hat{a}_0}{\sigma^2_{a,0}} - \frac{\hat{m}_0}{\sigma^2_{m,0}} + \frac{\rho_0}{\sigma_{a,0}\sigma_{m,0}} (\hat{a}_0 - \hat{m}_0)}{\frac{1}{\sigma^2_{a,0}} + \frac{1}{\sigma^2_{m,0}} + \frac{2\rho_0}{\sigma_{m,0}\sigma_{a,0}}},$$

$$\lim_{t \mapsto \infty} \hat{m}_t = \frac{(a^* + m^*) \left(\frac{1}{\sigma^2_{a,0}} + \frac{\rho_0}{\sigma_{a,0}\sigma_{m,0}} \right) - \frac{\hat{a}_0}{\sigma^2_{a,0}} + \frac{\hat{m}_0}{\sigma^2_{m,0}} + \frac{\rho_0}{\sigma_{a,0}\sigma_{m,0}} (\hat{m}_0 - \hat{a}_0)}{\frac{1}{\sigma^2_{a,0}} + \frac{1}{\sigma^2_{m,0}} + \frac{2\rho_0}{\sigma_{m,0}\sigma_{a,0}}},$$

$$\lim_{t \mapsto \infty} \hat{\sigma}^2_{a,t} = \lim_{t \mapsto \infty} \hat{\sigma}^2_t = \frac{1 - \rho_0^2}{\frac{1}{\sigma^2_{a,0}} + \frac{1}{\sigma^2_{m,0}} + \frac{2\rho_0}{\sigma_{m,0}\sigma_{a,0}}},$$

$$\lim_{t \mapsto \infty} \hat{\rho}_t = -1$$

Thus, an agent that spends the entire career as an employee can consistently estimate a^* and remove uncertainty about it completely. The correlation coefficient of beliefs on a^* and m^* goes to zero as the prior belief $\mathcal{I}_0$ is dominated by an infinite sequence of payroll income signals that have no correlation with m^*. If there's some correlation between (a^*, m^*) then the employee career provides additional insight about the business acumen. Otherwise the agent learns nothing about m^* on top of the information in the prior distribution. The correction of m_0 is the linear projection of the data on the initial beliefs.

The entrepreneur can identify her skill bundle up to a line on the (a^*, m^*) plane with -1 slope on which the limiting belief is bivariate normal distributed. The negative correlation in the limiting belief comes from a^* and m^* being complements to each other in entrepreneurship. The mass is symmetrically (as indicated by asymptotic m and a variances) centered around the *confusion-adjusted* point $(\lim_{t\mapsto\infty} \hat{a}_t, \lim_{t\mapsto\infty} \hat{m}_t)$. The *confusion adjustment* factor is a function of uncertainty-adjusted initial information about one's abilities. Whenever either $\hat{a}_0 \neq a^*$ or $\hat{m}_0 \neq m^*$ the entrepreneur can not consistently estimate her endowment. This is contrary to the payroll employment case where the limiting belief on a^* is consistent irrespectively of prior information.

Observe, however, that the entrepreneur updating problem allows for consistent estimation of the individual entrepreneurial return, exactly as in the Jovanovic model, as by taking the sum of the limits we get:

$$\lim_{t\mapsto\infty} \hat{a}_t + \lim_{t\mapsto\infty} \hat{m}_t = a^* + m^*.$$

Because of the confounding nature of the signal the prior information constantly weighs in on the estimates of the means of both unobservable factors. If an agent believes that her acumen/ability is higher than its true value, then this decreases the mean of the ability/acumen, respectively, in the updated belief. Thus, it may not be possible to infer the value of the outside option as individual ability and the business acumen jointly determine entrepreneurial income. The magnitude of the confusion effect hinges on the quality of initial information. To see why this is the case consider the rewritten formulation of the asymptotic belief about the business acumen $\lim_{t\mapsto\infty} \hat{m}_t$ under an additional simplification of $\rho_0 = 0$:

$$\lim_{t\mapsto\infty} \hat{m}_t = \frac{\sigma^2_{m,0}}{\sigma^2_{m,0} + \sigma^2_{a,0}} (a^* - \hat{a}_0) + \frac{\sigma^2_{m,0}}{\sigma^2_{m,0} + \sigma^2_{a,0}} m^* + \frac{\sigma^2_{a,0}}{\sigma^2_{m,0} + \sigma^2_{a,0}} \hat{m}_0. \tag{6}$$

Equation (6) is vital to understanding the implications of the model. The belief is a weighted sum of the prior information $\hat{m}_0$ and the true value m^* shifted by the rescaled error on the general ability a^*. Thus, there are two factors that make agents value entrepreneurship above payroll employment even if the evidence points to $a^* + m^* < a^*$. First, agents that have initial belief on their general ability below its true value will tend to have better opinion about their business acumen. Second, a similar effect has an initial overestimation of m^*.

Next, the more precise the estimate of the general ability, the smaller the interference of ability on the belief on acumen due to the confounded nature of the entrepreneurial learning. In particular, it follows that:

$$\lim_{\hat{\sigma}^2_{a,0} \mapsto 0} \lim_{t \mapsto \infty} \hat{m}_t = m^*,$$

as in the baseline model. However, this discussion depends to some extent on the assumption of fixed careers, as the following corollary demonstrates.

Corollary 2 *If $t_e = t_p$ and $t_p + t_e = t \mapsto \infty$, then the asymptotic beliefs have:*

$$\lim_{t \mapsto \infty} \hat{a}_t = a^* \lim_{t \mapsto \infty} \hat{m}_t = m^*$$
$$\lim_{t \mapsto \infty} \hat{\sigma}^2_{a,t} = \lim_{t \mapsto \infty} \hat{\rho}_t = \lim_{t \mapsto \infty} \hat{\sigma}^2_t = 0$$

Hence, the agents can refine their belief on the business acumen by switching between the two types of employment. Payroll employment experience is necessary to recover the true value of business acumen. Now we are in position to characterize the regret function and to solve the dynamic problem $\mathcal{P}$.

5 Illustration of the working of the model

In this section the analytical properties of the regret function are derived. Then, the dependence of the dynamics of type of employment choice on model parameters is discusssed in a 3-period setting. Unless stated otherwise, the earnings process is considered to hold in levels of income and the utility is linear. To begin with, I provide the solution to the dynamic problem $\mathcal{P}$ sans information friction.

5.1 Choice in the absence of information friction

As we have three choices and the composition of the ability-acumen endowment is fixed, one choice is optimal in all periods and the optimal action plan sequence $\{\xi_1^*, \xi_2^*, \xi_3^*\}$ satisfies:

$$\forall t \in \{1,2,3\} : \xi_t^* = \begin{cases} \xi^p \text{ if } \max\{b, \gamma a^* + m^*\} \leq a^* \\ \xi^e \text{ if } \max\{b, a^*\} \leq \gamma a^* + m^* \\ \xi^n \text{ otherwise.} \end{cases}$$

When the endowments are unobservable and can only be learned by producing, the individual outcomes will differ from the perfect information case.

5.2 Baseline regret function: no opportunity cost of ability

For ease of exposition I start with a somewhat special parametric case, focusing first on the choice between the two types of employment by putting $b = -\infty$. This rules out non-participation as a valid choice for the agents. I also fix $\gamma = 1$ so that the returns to general ability are identical in the two activities. The opportunity cost of switching between the activities is the value of agent's business acumen, m_i^*. It is also assumed that $\rho^* = 0$ so that refining the precision of information on a_i^* does not provide additional information on m_i^*. Then, the definition of the regret within a particular period t is:

$$R(\mathcal{I}_t, \xi_t) = \begin{cases} \mathbb{P}_{\mathcal{I}_t}(m^* > 0)\, \mathbb{E}_{\mathcal{I}_t}(m^* | m^* > 0) \text{ if } \xi_t = \xi^p \\ \mathbb{P}_{\mathcal{I}_t}(m^* < 0)\, \mathbb{E}_{\mathcal{I}_t}(-m^* | m^* < 0) \text{ if } \xi_t = \xi^e \end{cases} \tag{7}$$

Observe that the regret is an expectation of a truncated normal variable with parameters $\hat{m}$, $\hat{\sigma}_m^2$ weighted by the probability of a tail event of m^* being either below or above zero. Thus, the analytical representation of the regret in period t is:

$$\mathbb{E}_t\left[y(\xi^*) - y(\xi_t)|\mathcal{I}_t\right] = \begin{cases} \left(1 - \Phi\left(-\frac{\hat{m}_t}{\hat{\sigma}_{m,t}}\right)\right)\hat{m}_t + \hat{\sigma}_{m,t}\phi\left(-\frac{\hat{m}_t}{\hat{\sigma}_{m,t}}\right) \text{ if } \xi_t = \xi^p \\ -\Phi\left(-\frac{\hat{m}_t}{\hat{\sigma}_{m,t}}\right)\hat{m}_t + \hat{\sigma}_{m,t}\phi\left(-\frac{\hat{m}_t}{\hat{\sigma}_{m,t}}\right) \text{ if } \xi_t = \xi^e \end{cases}$$

Proposition 3 (Regret dependence on $\mathcal{I}_t$.) *Let $b = -\infty$, $\rho = 0$, $\gamma = 1$, then the regret is increasing in $\hat{\sigma}_m^2$. The minimum of the regret is decreasing in $|\hat{m}|$. The regret does not directly depend on $\hat{a}$, $\hat{\sigma}_a^2$.*

The regret of each arm is a convex function of the mean of individual belief on m^*, as demonstrated on Figure 6. The minimum envelope of the regret is always non-negative, has a maximum at $\hat{m}_i = 0$ which is the threshold for the change of arm, and decreases to zero the more extreme the values of the mean of the beliefs on m_i^*. In other words, the more convinced agents are about their advantage in one of the types of employment, the less likely they expect themselves to choose the wrong arm ex ante. This is the *exploitation* motive in the agent's optimization problem. The arm that seems to generate a higher reward is more desirable to choose.

Regret is also increasing in uncertainty on m^* which reflects the *experimentation* motive. Getting rid of uncertainty is valued because it decreases the probability of choosing the wrong arm given all other available information. The impact of decreasing uncertainty diminishes the more extreme means of the beliefs are. Hence, the more extreme the mean of the belief about the acumen, the smaller is the expected gain from learning. Intuitively, the more extreme the belief on m^* is, the less the agents expect to be wrong about m^* which makes the gains from reducing uncertainty about it smaller than for the beliefs near the $\hat{m} = 0$ switching threshold.

Knowledge of a^* plays no role in this formulation as the choice of $\xi_t = \xi^e$ does not entail any losses of the true ability a^* which only holds when $\gamma = 1$. In this case the uncertainty on a^* can only create additional (on top of initial uncertainty about the acumen) confusion about m^* and interact with the pace of uncertainty reduction on m^*.

5.3 The role of ρ, γ and non-participation.

Now, let's allow $0 \leq \gamma < 1$ and $-1 < \rho < 1$ keeping $b = -\infty$. Observe that in this case the regret definition is:

$$R(\mathcal{I}_t, \xi_t) = \begin{cases} (1 - \mathbb{P}_{\mathcal{I}_t}(m^* < (1-\gamma) a^*)) \, \mathbb{E}_{\mathcal{I}_t}(m^* + (\gamma - 1) a^* | m^* > (1-\gamma) a^*) & \text{if } \xi_t = \xi^p \\ (1 - \mathbb{P}_{\mathcal{I}_t}(m^* > (1-\gamma) a^*) \, \mathbb{E}_{\mathcal{I}_t}((1-\gamma) a^* - m^* | m^* < (1-\gamma) a^*)) & \text{if } \xi_t = \xi^e \end{cases}$$

This can be rewritten introducing a new variable $z^* = m^* - (1-\gamma) a^*$ which is distributed as univariate normal with mean $\hat{z} = \hat{m} - (1-\gamma)\,\hat{a}$ and variance $\hat{\sigma}_z^2 = \hat{\sigma}_m^2 + (1-\gamma)^2\,\hat{\sigma}_a^2 + 2\hat{\rho}|\gamma - 1|\hat{\sigma}_a\hat{\sigma}_m$. Then, one can rewrite the regret equation (7) using $\hat{z}$ and $\hat{\sigma}_z$ instead of $\hat{m}$ and $\hat{\sigma}_m$. In that case the proposition 3 extends naturally to z^*. The minimum of the regret is decreasing in the belief on the ability-adjusted individual advantage in entrepreneurship $|\hat{z}|$. The regret increases with $\hat{\sigma}_z$. This fact is further illustrated on Figure 7 with changes in $\hat{\sigma}_a$ now translating in different values of regret, contrary to Figure 6.

Thus, having $\gamma \neq 1$ introduces a motive for learning a^*. This comes from the fact that now the agents advantage in entrepreneurship is a function of ability as well and the agents may make mistakes due to imperfect identification of the switching threshold $m^* = (1-\gamma)\,a^*$.

Finally, non-participation is an absorbing state. In other words, when $b > -\infty$, if an agent chooses $\xi_t = \xi^n$ the subsequent periods will have $\xi_{t+k} = \xi^n, k > 0$. The intuition behind this result is as follows - agents with beliefs that imply their value of home production exceeds their value of productive activities can still decide to begin their career producing solely for the sake of learning. The benefits of learning are decreasing with agent's experience thus if they don't justify undertaking production in period t, they will also not be big enough to make the agent produce in all future periods. However, because of this property of non-participation the agents have strong incentives to improve the finesse of their estimates of ability and acumen before leaving the labor market. The introduction of non-participation also implies that the optimal choice is not only driven by the comparative advantage between the two productive abilities, z^*. Because of non-participation, location of agent's beliefs in $(\hat{a}, \hat{m})$ space is also relevant for the dynamics.

5.4 Dynamics of Type of Employment Choice

Let's again assume that $b = -\infty$ so that the choice is between two productive arms only. Deriving the optimal decision rule is easiest for the last period. Observe that for $t = 0, 1, 2$ the following identity holds:

$$R(\mathcal{I}_t, \xi_t = \xi^p) - R(\mathcal{I}_t, \xi_t = \xi^e) = \hat{z}.$$

Hence, in the final period the optimal decision is to choose the e-arm when $\hat{m}_2 > (1-\gamma)\,\hat{a}_2$ and p-arm when $\hat{m}_2 < (1-\gamma)\,\hat{a}_2$, breaking the tie arbitrarily. This reflects the fact that in a one-period setting there is no value from experimentation, solely the exploitation motive is present. To put it simply, the benefits of reducing uncertainty today are only available in the next period which is absent when $t = 2$. In the first two periods wtih $t = 0, 1$, however, there is the experimentation gain, namely the decrease of regret coming from the reduction in $\hat{\sigma}_z^2$ which may dominate the current-period increase of regret from choosing a suboptimal arm. From the fact that the decrease of the regret function due to reducing $\hat{\sigma}_z$ diminishes with $|\hat{z}|$ the following result naturally follows.

Observation 1 (Reservation property) *Consider an agent with belief $\mathcal{I}_t$ with some $\underline{\hat{z}_t}$. If it is optimal for this agent to choose $\xi_t = \xi^e$ then it is also optimal to choose $\xi_t = \xi^e$ for all agents with posterior $\tilde{\mathcal{I}}_t$ such that $\tilde{z}_t \geq \underline{\hat{z}_t}$ other things equal.*

5.4.1 Informational gains

In the first and second period the agents may opt for choosing an inferior arm to decrease their uncertainty about the (a^*, m^*) pair. The size of those informational benefits depends, however, on the signal to noise ratio in both types of employment and initial uncertainty. Intuitively, the more uncertain agents are about one of their endowments, the greater the incentive to experiment. The more noisy an activity is, the less desirable a choice for learning it represents. The more correlated are the individual beliefs, the smaller the relative difference in informational gains of the two choices shall be.

An additional source of complexity is that the use of ability and acumen in entrepreneurship yields *some* information on *both* endowments, yet in a confounded way. On the one hand, entrepreneurship brings informational gains on both components of $\hat{z}_t$. On the other hand, informational gains of choosing entrepreneurship in consecutive periods decrease faster than in payroll employment as there are limits to learning in entrepreneurship.

For the magnitude of the informational gains the relative initial uncertainty and signal to noise ratios in the two activities are key. Thus, let's have all the variances $\sigma^2_{m,0}, \sigma^2_{a,0}, \sigma^2_p, \sigma^2_e$ normalised to one to investigate the role of γ and ρ^*. On Figure 8 the reduction of uncertainty about z^* is presented, measured as the ratio of period-1 variance of the $\hat{z}$ estimate to its initial variance. On Figure 9 the same measure after two periods is plotted. In each case, the informational gains depend on the choices the agents made.

In the noisy learning case the initial period informational gains are identical, as the uncertainty about both unobserved endowments and the noise-to-signal ratios are the same. This is not the case in the initial period in the confounded learning case. There, for larger values of γ entrepreneurship dominates informationally payroll employment. This is because the uncertainty about m^* weighs more in the overall uncertainty about z^*. Only when γ goes to zero, the relative informational advantage of entrepreneurship vanishes.

In the second period, using each arm once produces uniformly largest informational gase n the noisy learning case. If there is confounding, however, this is not always the case. In particular, for large values of γ choosing entrepreneurship twice may informationally dominate other choices. What is more, choosing entrepreneurship twice and in particular, switching between occupations dominates the choice of payroll employment in the first two periods to a significantly larger extent when $\gamma > 0$ than in the noisy learning case.

The magnitudes of the effects described above depend to some extent on the desire to experiment about entrepreneurship, expressed as a ratio of initial beliefs $\frac{\sigma^2_{m,0}}{\sigma^2_{a,0}}$ and the speed of learning in each activity measured by the signal to noise ratio. To fix ideas, let's keep $\sigma^2_{a,0} = \sigma^2_p = 1$. Then, by increasing $\sigma^2_{m,0}$ the incentives to learn about m^* increase. These incentives are diminished when variance of shocks to entrepreneurial income increases. However, the interaction of the two effects can be non-trivial. As demonstrated on Figures 10 and 11, with $\hat{\sigma}^2_{m,0}$ increased twofold and σ^2_e increased threefold, the informational gains of entrepreneurship in the confounded learning case increase in the first two periods while they decrease in the noisy learning case.

5.4.2 Reservation thresholds

For illustration of trade-offs in the dynamic problem, two cases are considered, strong correlation ($\rho = 0.8$, $\gamma = 0$) in a noisy, confounding-free setting and strong ($\gamma = 1$, $\rho^* = 0$) confounding, labelled NL and SC respectively. The assumption of all relevant variances normalised to one made at the beginning of the previous section is maintained. In line with lack of the experimentation motive, in the last period the switching threshold in all cases is at $\underline{\hat{z}_2} = 0$.

Let's start with the noisy learning case. As in the first period both arms offer identical learning benefits, the agents have no experimentation motive to prefer one arm over the other. Thus, the switching threshold is $\underline{\hat{z}_0^{NL}} = 0$. In the second period agents have an experimentation incentive to switch. This incentive is identical in the case of initial period entrepreneurs and employees given the symmetry of all relevant variances. The individuals who started as employees are now willing to try entrepreneurship provided their estimate of $\hat{z}_1$ is not too low. The same holds for first period entrepreneurs, who are willing to give away some benefits of entrepreneurship. Thus, we obtain the following set of conditions:

$$\underline{\hat{z}_0^{NL}} = \underline{\hat{z}_2^{NL}} = 0 \quad \wedge \quad 0 > \underline{\hat{z}_1^{NL}} \left(\xi_0 = \xi^p\right) = -\underline{\hat{z}_1^{NL}} \left(\xi_0 = \xi^e\right).$$

Now, in the confounding case there are strong incentives to identify m^* which dominate analogous incentives to identify a^*. In line with the results on informational gains, it holds that $\underline{z_0}^{SC} < 0$. Then, the first-period employees will have an incentive to switch into entrepreneurship for some $\underline{z_1} \left(\xi_0 = \xi^p\right) < 0$. The first period entrepreneurs have no incentives to switch as the learning via switching and continuation provide identical relative information gains. Thus, in the strong confounding setup we have:

$$\underline{\hat{z}_1^{SC}} \left(\xi_0 = \xi^e\right) = \underline{\hat{z}_2^{SC}} = 0 \quad \wedge \quad 0 > \underline{\hat{z}_1^{SC}} \left(\xi_0 = \xi^p\right) > \underline{\hat{z}_0^{SC}}.$$

Thus, keeping initial uncertainty constant, the increases in γ yield larger experimentation incentives via entrepreneurship and increase the fraction of shorter entrepreneurial spells compared to changes in the correlation of beliefs. This comes from the difference between $\underline{\hat{z}_1^{SC}} \left(\xi_0 = \xi^e\right) - \underline{\hat{z}_0^{SC}}$. The switching tresholds are depicted on Figure 12.

6 Quantitative Analysis

This section starts with the description of the numerical simulation procedure that pins down the parameters of the model. Then, I investigate the implications of introducing non-pecuniary benefits of entrepreneurship and the role of non-participation margin. The learning model has $\gamma > 0$ which makes entrepreneurial learning confounded, the ability and acumen are independently distributed. Finally, I use the parameter values for the confounded entrepreneurial learning specification and conduct an experiment on the impact of the reduction of uncertainty of the initial prior on the general ability.

6.1 Simulation based estimation setup

I employ Simulated Method of Moments (SMM) to pin the parameters of the model. The preferences are logarithmic, $u(c) = \log(c)$ and the production equations are assumed to hold in logs of product. Observe that this yields the regret function analogous to the level-level case in *expected utilities*. Thus, agents seek to minimize the distance to the arm offering highest expected utility.

Because there is substantial evidence that pre-retirement considerations, health and technological changes interact with type of employment choice of older workers, I assume that agents in my model have in total 30 periods/years to choose their activity and then stay on this activity until retirement after 40 years since entering the labor market. This assumption is due to that in later age the learning motive may be dominated by aforementioned factors.

The sole externally calibrated parameter is the discount factor β set to 0.96. The parameters of interest include the parametrization of $F(a, m)$, the precision of initial beliefs, the variance of idiosyncratic shocks σ_p^2 and σ_e^2 and the value of non-participation b. Then, I assume that acumen and ability are independent, $\rho^* = 0$ and allow γ to vary. Below I provide a discussion of identification of each of the parameters.

The primary source of targets for the simulations are the moments of the log-residuals from the earnings equation. In particular, the targets include the standard deviations of those residuals and standard deviations of individual averages of the residuals, all conditional on type of employment. These four distributions provide natural targets for $\sigma_{a^*}^2$, $\sigma_{m^*}^2$ and σ_p^2, σ_e^2.

As the choice of productive activities works is driven by relative returns, I normalise the mean of the general ability in the population μ_{a^*} to zero. To compare the model with the data I then shift the log-residuals to produce the same mean log-residual earnings in payroll employment. Then, conditionally on the standard deviations of ability, acumen and individual shocks the mean of the business acumen distribution μ_{m^*} is matched by a restriction of the ratio of the means of the two income distributions. The level of initial uncertainty about ability is matched exogenously to the noise variance in payroll employment so that an agent who never tries entrepreneurship learns the ability at the end of the 30-year period[2]. The identification of uncertainty on individual business acumen relies on the desire to experiment between the productive activities as relative informational

[2] An alternative calibration strategy would have the value of non-participation calibrated from the literature as a function of average employee income which would allow the uncertainty on the general ability to vary.

gains are a function of relative uncertainty on m^* and a^*. Thus, I target the entrepreneurship rate in the cross section which is decreasing in initial uncertainty, given all other parameters of the model.

The untargeted moments consist of the features of the data that will be driven by the learning process. Those include the distribution of spells and switches, the differential of median earnings of entrepreneurs and employees in the cross-section and the fraction of underperforming entrepreneurs among the individuals with 10 years of labor market experience, frequency of returns to entrepreneurship after the first spell. The model is solved in the space of beliefs on a grid that approximates the $F(\hat{a}, \hat{m})$ distribution. The description of the numerical algorithm is relegated to Appendix C.

6.2 Learning, non-pecuniary benefits and non-participation.

The model is able to match most of the targeted moments relatively well, as demonstrated in the Table 2. Due to strong experimentation motives, both against non-participation and payroll employment, the cross-sectional entrepreneurship rate is slightly above and non-participation rate is below those in the data. For the untargeted moments (Table 3), the model is able to generate negative median earnings differential, although a bit less than a half (6 percent) of the magnitude of the differential (16 percent) in the data. The model replicates about 90 percent of the magnitude of short spells pointing to high exit rates from entrepreneurship. The estimated parameters are presented in Table 4.

Let's start with the analysis of the true skills space which are depicted on Figure 13. There are three blue lines which demark six regions in the parameter space. First, there is the $a^* = b$ line which separates the beliefs that point to payroll employment being better/worse than non-participation. Second, there is a line $m^* = (1-\gamma)\, a^*$ that separates the beliefs that imply entrepreneurship dominating/being dominated by payroll employment. Finally, the lne $m^* + \gamma a^* = b$ separates the regions of non-participation being better/worse than entrepreneurship.

Then, the implied distribution of initial begins is presented on Figure 14. In the regions $\mathcal{R}_1$ and $\mathcal{R}_6$ the incentives to choose entrepreneurship are the strongest. These beliefs are likely to have the acumen overestimated, as indicated by the comparison of the corresponding regions on the true types distribution plot. The regions $\mathcal{R}_4$ and $\mathcal{R}_5$ imply that the agents should leave the labor market. In those two regions experimentation determines the behavior of agents until they learn better their underlying type or indeed leave the labor market. Those two regions generate additional entrepreneurial spells. In the last two regions $\mathcal{R}_2$ and $\mathcal{R}_3$ can only suffer from some of the high-quality entrepreneurs choosing payroll employment.

The dynamics of the model are such that at early age the experimentation thresholds shift in favor of entrepreneurship which provides information on a^* and m^*. This is responsible for a counterfactual prediction of the model on the share of entrepreneurs among the agents with tenure in the labor market no longer than 5 years which well exceeds the low entrepreneurship rate in the data. However, this is a mechanical feature of the model as the young workers have the strongest incentives to experiment and that there are no entry restrictions in entrepreneurship. Imposing a random idea arrival rate would solve this problem but it would also require tracking the full history of ideas agents have tried leading to an increase of the state space size. On the other hand, that also means the potential of confounding is diminished and, in line with the discussion of this issue in the introduction, the median differential reported here constitutes a lower bound of the difference if one was to consider mulitple ideas etc. Thus, the entry rate into entrepreneurship is too high in the model, as reflected by the share of agents who ever tried entrepeneurship. However, the size of this discrepancy is most likely overstated in the model, as the shortest entrepreneurial spells are likely to be underrepresented in the sample.

Then, I reestimate the parameters considering the role of several features of the model. To this end, I test the importance of the lack of separate observation of ability and acumen in entrepreneurship and the role of non-participation. The results of this exercise are represented in Tables 2 - 3.

The assumption of confounded signal is an important one. The model with separate observation of the two underlying dimensions of individual heterogeneityalthough also able to generate the negative median income differential, predicts an even larger entrepreneurship rate and completely misses the features of the entrepreneurial spells distribution. For example, the exit rate in the first two years is by a factor of ten smaller than the corresponding moment in the data. There are fewer serial entrepreneurs. Those predictions of the model are a consequence of too great an informational advantage of entrepreneurship. Essentially all agents try entrepreneurship at their early age and stay for several years before they efficiently learn what type of employment suits them best.

The role of non-participation is less trivial. Certainly, it decreases the exit rate from entrepreneurship by one-fifth. The cross-sectional cost of experimentation decreases because there is one less activity available for the agents, now relatively fewer highly-skilled agents select into entrepreneurship. The non-participation in the baseline model was removing mostly least skilled employees so that the median negative earnings differential is larger than for the baseline model and gets closer to unity as the distribution of employee earnings has more mass to the left.

Then, using the estimated values of the model parameters I introduce non-pecuniary compensation in entrepreneurship to match exactly the median differential between entrepreneurial and employee incomes. As expected and demonstrated in Table 6, the frequency of switches between the two activities decreases and the entrepreneurship rate goes up.

6.3 Cost of uncertainty and the effects of reducing uncertainty.

Based on the estimated values of the parameters I check the effects of an exogenous decrease in the dispersion of the prior beliefs. The results of this exercise are reported in Table 7. A real world counterpart of the exogenous decrease in prior uncertainty is easier to find for the case of general ability. For example, it can be thought of as an exogenous improvement in the quality of teaching and its alignment to the needs of the labor market. The reduction in business acumen uncertainty is presented solely for comparison. It seems plausible to say that the only way of learning about entrepreneurial abilities is to try entrepreneurship.

In the absence of information friction the median entrepreneur earns about 17 percent more than the median employee. In the cross-section the cost mostly comes from excessive selection into entrepreneurship, as there is almost four times as many entrepreneurs as in the case without the information friction. By removing the uncertainty and improving selection the productivity increase in entrepreneurship can go up as much as by 24 percent. The intuition behind this result is simple. First, there are many well-skilled agents trying entrepreneurship who are, however, below the optimal threshold of $m^* \geq (1-\gamma)\, a^*$. The high ability agents who select intro entrepreneurship incur larger costs of scaling of the ability. Furthermore, low ability agents face lower opportunity cost of trying entrepreneurship and drag the entrepreneurial income distribution further to the left. However, exactly for that reason, the effect of information friction on the median employee earnings is relatively small and equal to 2 percent. Intuitively, it's easier to identify the non-participation selection margin from the entrepreneurship selection margin.

Now, let's consider a change in the precision of initial information about the general ability. Such change can be thought of as promoting apprenticeships and student internships. I assume that through providing the students with additional working experience the reduction in initial uncertainty reaches either 10 or 50 percent of its initial value and investigate its effects on productivity and selection into entrepreneurship. Two key findings emerge.

First, as there is now relatively more uncertainty about business acumen, the fraction of young individuals that wish to learn through entrepreneurship goes up. Thus, fewer talented entrepreneurs are left behind because of their initial pessimism about their acumen. Second, long-run selection improves as fewer agents become entrepreneurs or employees by mistake due to uncertainty about their ability. Hence, the productivity of an average entrepreneur, measured as a mean in the cross-section distribution of income increases by about 1.1/4.8 percent, respectively. For comparison, reducing uncertainty on business acumen has stronger effects on the productivity of entrepreneurs. Providing a reduction in $\sigma^2_{m,0}$ by 10/50 percent increases the median entrepreneur earnings by 3.3/10.6 percent, respectively.

7 Conclusions

I show that incorporating learning on ability and business acumen into a model of entrepreneurial choice helps meeting certain puzzling facts about entrepreneurship. The value of experimentation yields negative earnings differential between entrepreneurs and employees because there are stronger incentives to learn about business acumen than there are to learn about general ability. Unlike the theory of non-pecuniary benefits and heterogeneous tastes, my model can explain the large frequency of short entrepreneurial spells and frequent termination and re-entering by many entrepreneurs.

Future work should investigate the dynamics of entrepreneurial outside options. An important question is on the form of the returns to ability in entrepreneurship and its variability across industries. For example, assuming decreasing or increasing returns to scale will lead to change in the magnitude of the confounding effects. It is also interesting to consider a case of workers with deteriorating skills and business ideas. In the framework I propose, entrepreneurship offers a clear advantage over payroll employment which comes from the usage of ability in entrepreneurship. Next, a natural extension is to think of a life cycle of an enterprise. At its nascent stage, the ability of an entrepreneur should shape the value of a business to a larger extent. More mature businesses, that employ many workers, have their value driven to a greater extent by the quality of the initial business ida or the acumen of the owner. Finally, linked employer-employee-tax register data can offer ultimate answers to the questions on the dynamics of entrepreneurial income and provide a more accurate benchmark for the theory.

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A Plots and Tables

Figure 1: The distribution of annual earnings, source: NLSY79.

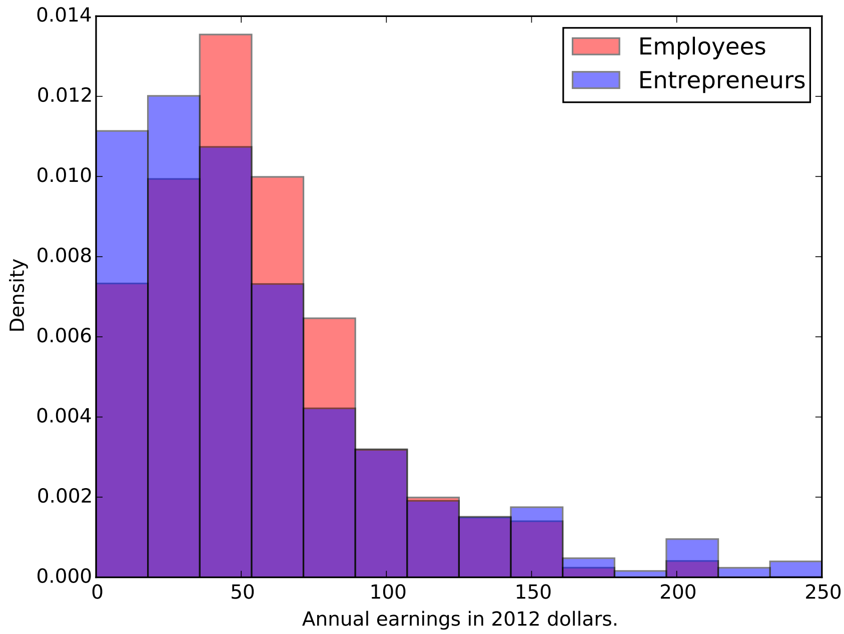


Figure 2: The residuals from log-earnings regression

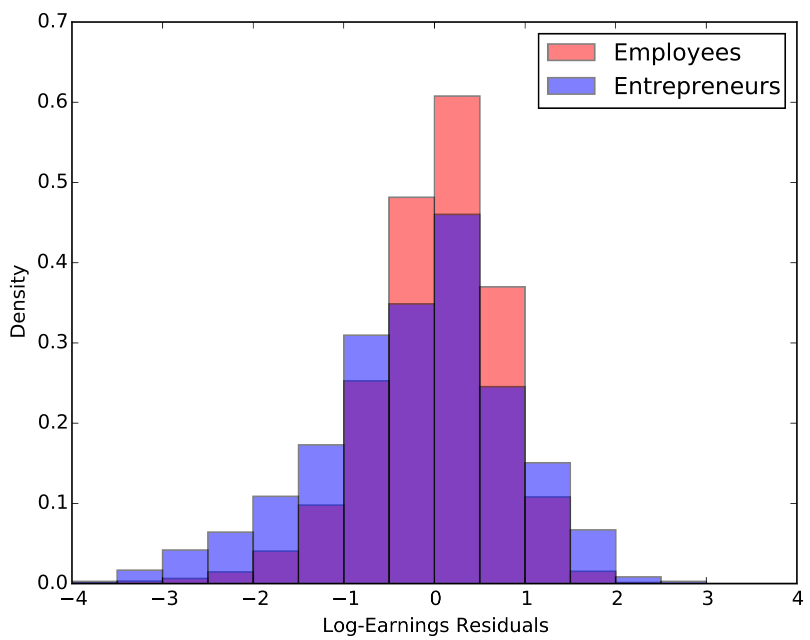


Figure 3: Distributions of the number of entrepreneurial spells and their length for agents with at least one spell.

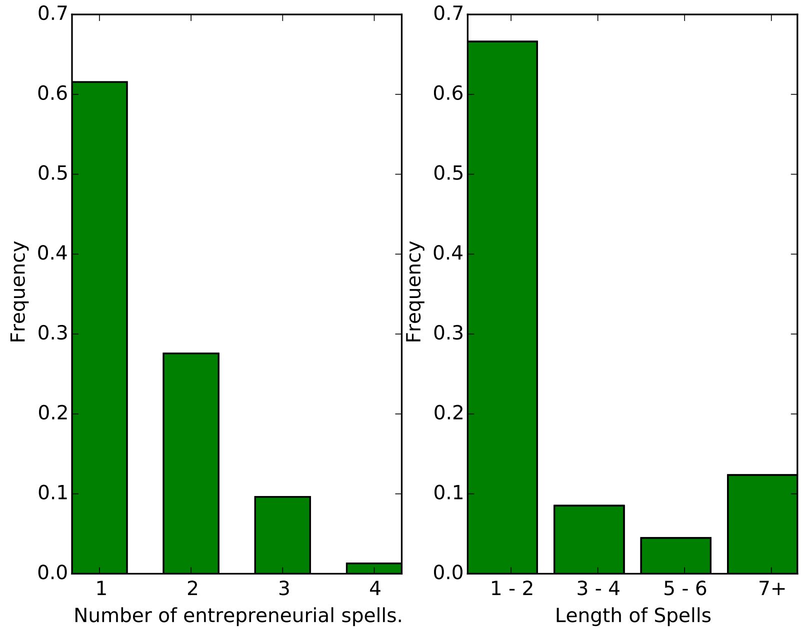


Figure 4: Distribution of years of labor market experience at first entry into entrepreneurship.

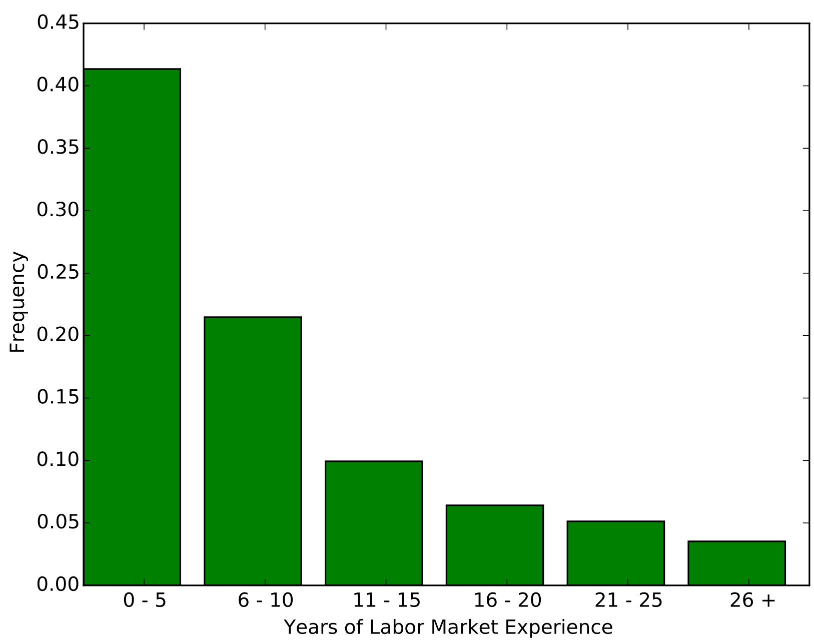


Figure 5: Individual averages of earnings condtional on observables in each type of employment for agents with at least 3 observations in each of the activities.

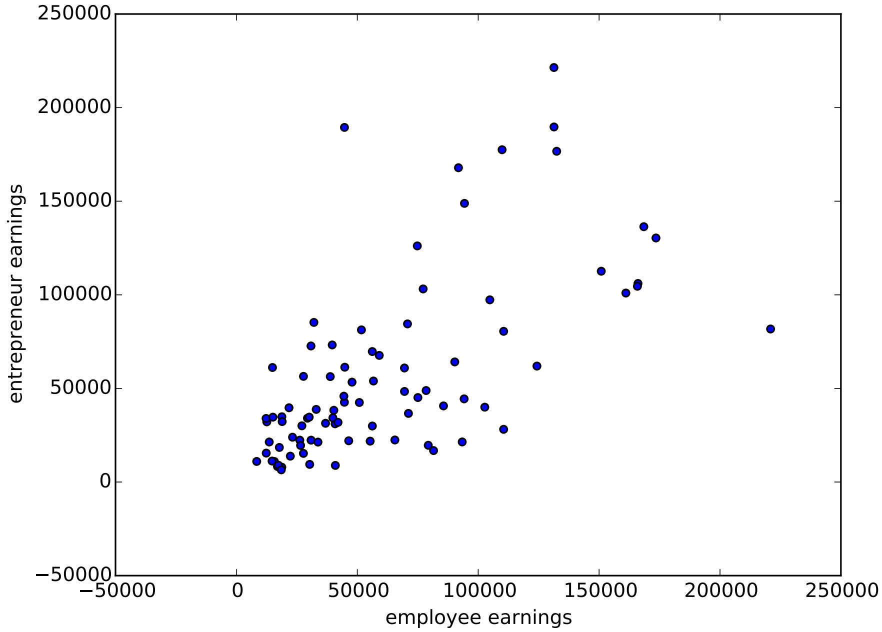


Figure 6: Regret function in the full confounding case.

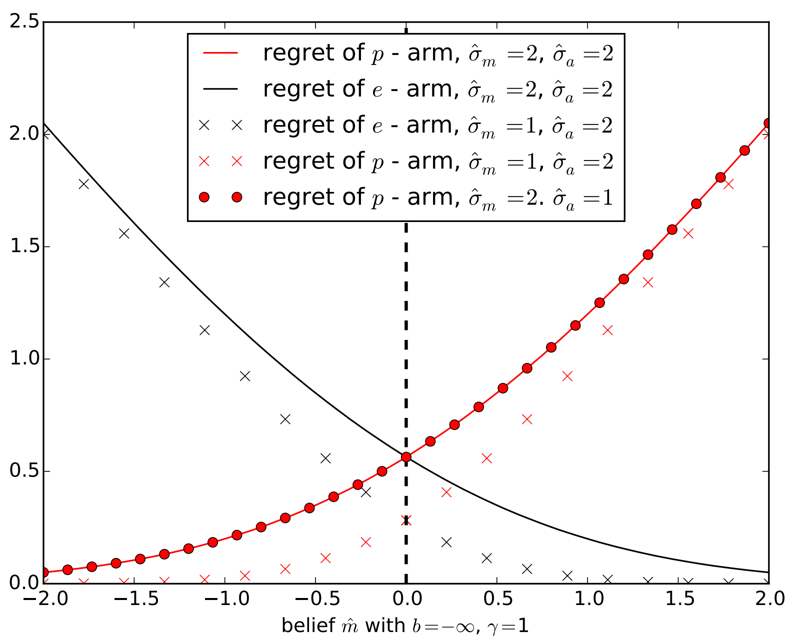


Figure 7: Regret function in the incomplete confounding case.

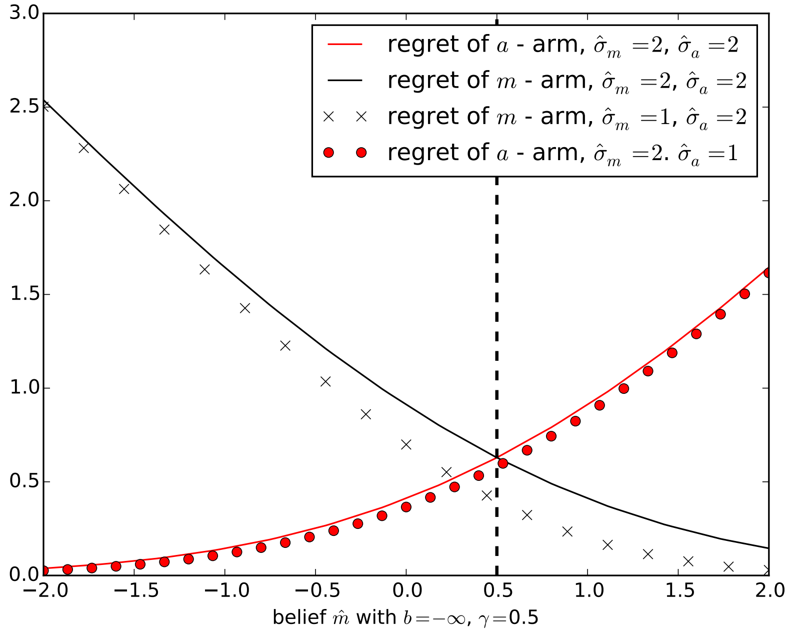


Figure 8: Informational benefits after the first period, homogeneous initial uncertainty and noise.

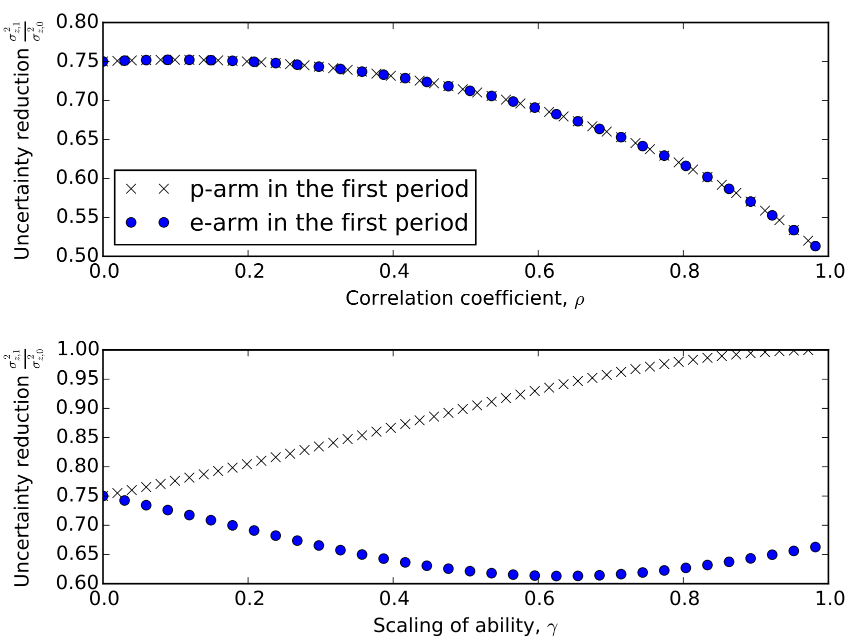


Figure 9: Informational benefits after the second period, homogeneous initial uncertainty and noise.

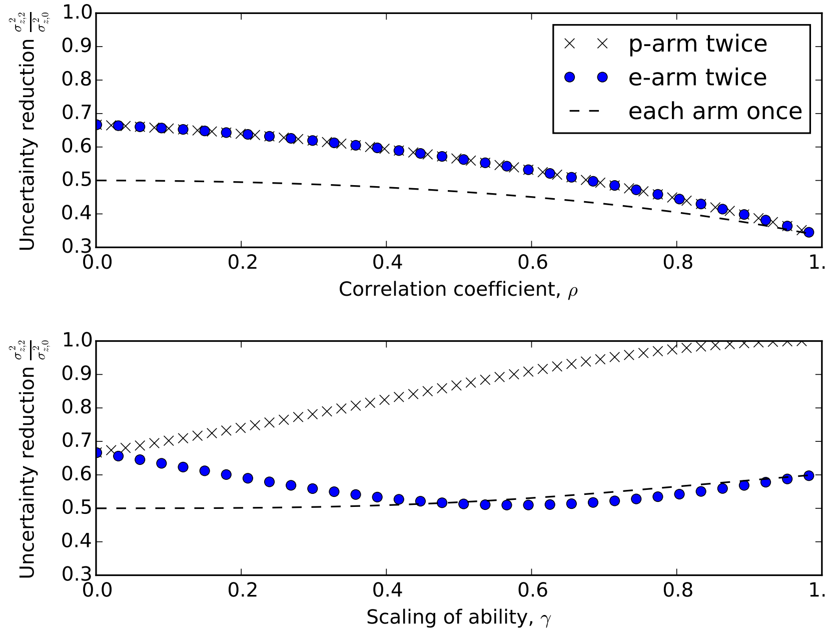


Figure 10: Informational benefits after the first period, heterogeneous initial uncertainty and noise.

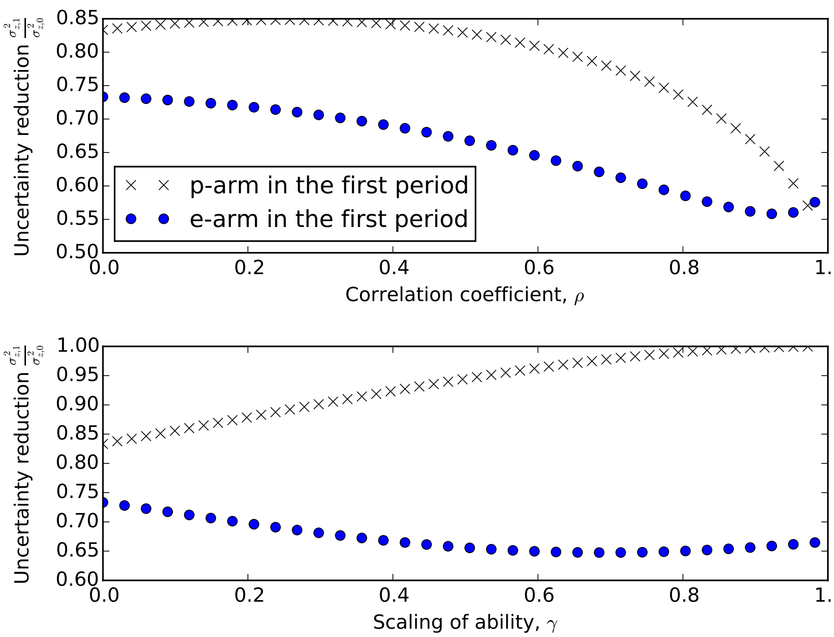


Figure 11: Informational benefits after the second period, heterogeneous initial uncertainty and noise.

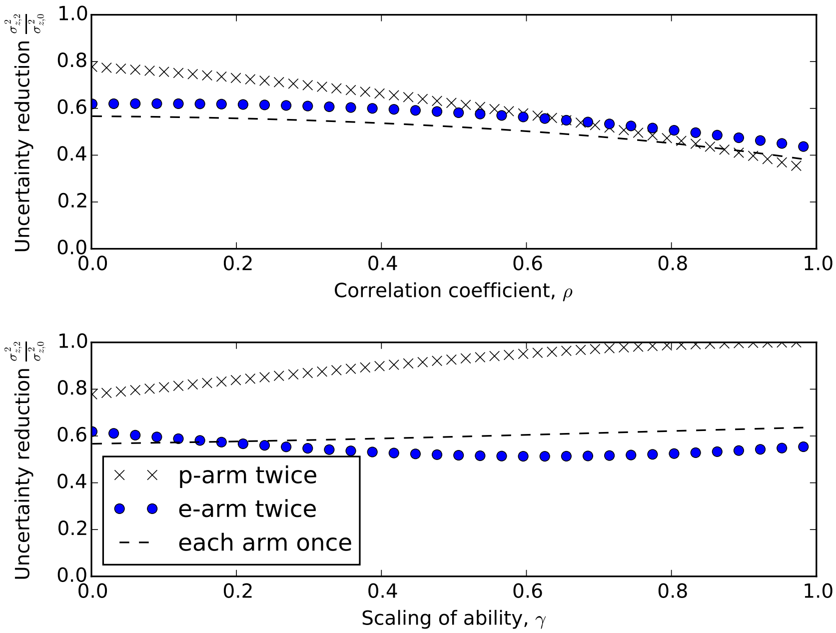


Figure 12: Illustration of reservation thresholds as a function of ρ and γ.

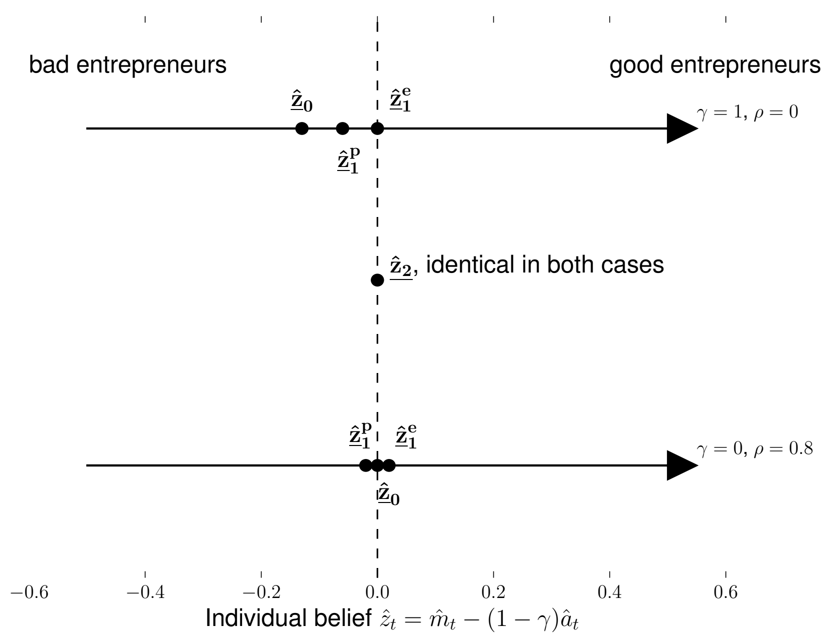


Figure 13: Frictionless assignment.

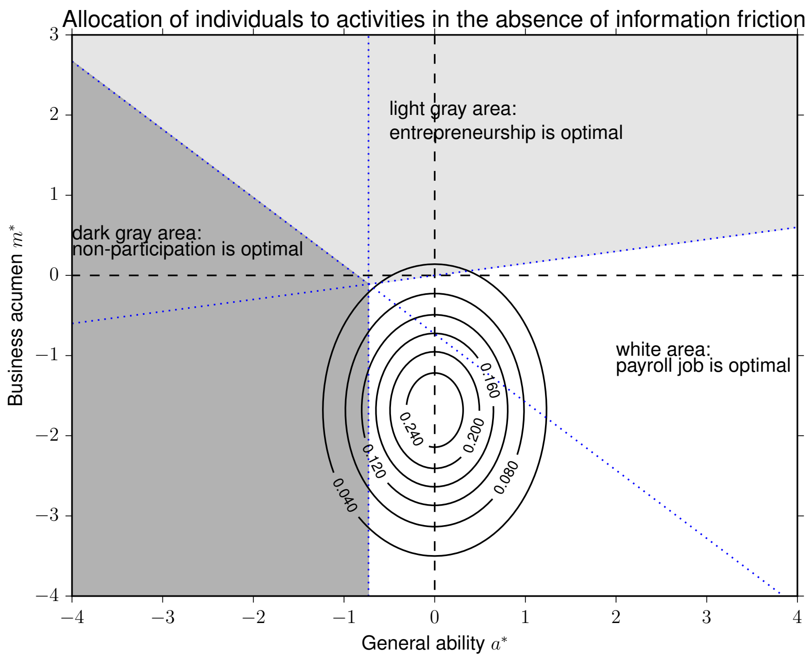
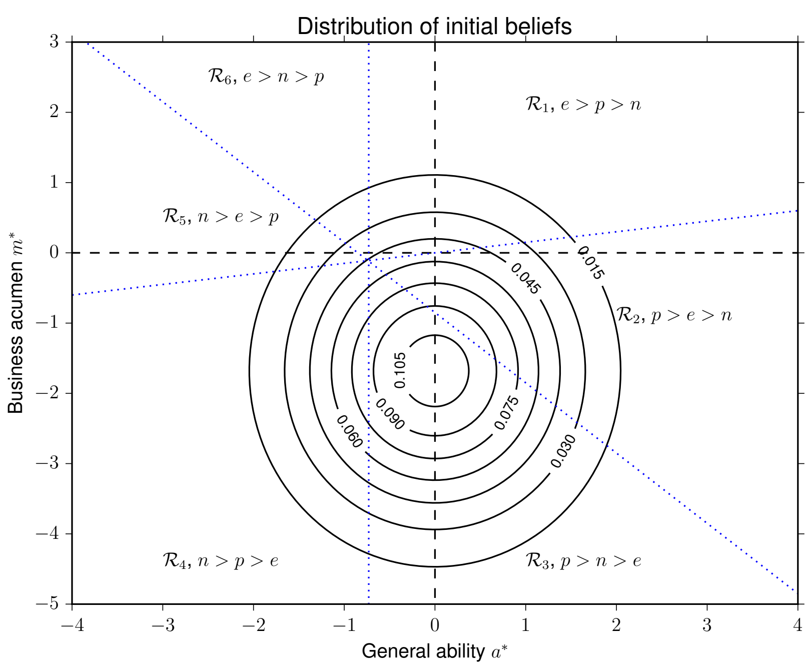


Figure 14: Initial Beliefs.



	Entrepreneurs	Employees
mean annual earnings	66.5	64.3
median annual earnings	44.9	51.0
std of annual earnings	72.2	57.8

Table 1: NLSY 1979-2012, pooled data on white men, farmers, members of military and disadvantaged families excluded. Income in thousands of 2012 dollars.

Targeted Moments	data	model	no confounding	no non-partic.
$\sigma\left(\log\left(y^p_{i,t}\right)\right)$	0.73	0.71	0.73	0.73
$\sigma\left(\overline{\log\left(y^p_i\right)}\right)$	0.60	0.59	0.60	0.61
$\sigma\left(\log\left(y^e_{i,t}\right)\right)$	1.08	1.07	1.16	1.14
$\sigma\left(\overline{\log\left(y^e_i\right)}\right)$	0.83	0.86	0.92	0.64
$\text{corr}\left(\log\left(y^p_{i,t}\right), \log\left(y^e_{i,t}\right)\right)$	0.68	0.65	0.29	0.60
ratio of mean incomes $\frac{\text{entr.}}{\text{emp.}}$	1.02	1.04	1.03	1.05
non-participation rate	0.10	0.08	0.07	–
cross-sec. entrepreneurship rate	0.06	0.10	0.19	0.10

Table 2: SMM implied targeted moments - consequences of confounding and non-participation (reestimated for each).

Untargeted Moments	data	model	no confounding	no non-partic.
ratio of medians $\frac{entr.}{emp.}$	0.84	0.94	0.86	0.96
frac. of underpeforming entr.	0.36	0.32	0.20	0.22
share of serial entrepreneurs	0.39	0.51	0.06	0.76
ever trying entrepreneurship	0.29	0.71	1.00	0.88
share of 1-2 p. spells in entr.	0.68	0.54	0.04	0.42
share of entries with 0-5 experience	0.41	0.65	1.00	0.86
entr. rate up to 5 yrs. of activity	0.03	0.45	0.86	0.49

Table 3: SMM implied targeted moments - consequences of confounding and non-participation (reestimated for each).

Parameter	estimate
mean population ability μ_{a^*}	0(norm.)
mean population acumen μ_{m^*}	-1.68
std population ability σ_{a^*}	0.64
std population acumen σ_{m^*}	0.91
scaling of ability in entrepreneurship γ	0.85
initial uncertainty about ability $\sigma_{a,0}$	1.02
initial uncertainty about acumen $\sigma_{m,0}$	1.39
std of individual shocks in entr. σ_e	0.87
std of individual shocks in emp. σ_p	0.51
value of home production b	-0.72

Table 4: Estimated parameters of the baseline model. The variance of initial estimates $\sigma^2_{a,0}, \sigma^2_{m,0}$ are a sum of population uncertainty $\sigma^2_{a^*}, \sigma^2_{m^*}$ and the variance of initial signals $\sigma_{\varepsilon_{a,0}} = 0.81, \sigma_{\varepsilon_{m,0}} = 1.05$. Value of initial uncertaint on ability pinned down by an exogenous requirement of full identification of a^* at the end of the learning period.

Targeted Moments	data	model	+ non-pecuniary
$\sigma\left(\log\left(y^p_{i,t}\right)\right)$	0.73	0.71	0.74
$\sigma\left(\overline{\log\left(y^p_i\right)}\right)$	0.60	0.59	0.63
$\sigma\left(\log\left(y^e_{i,t}\right)\right)$	1.08	1.07	1.13
$\sigma\left(\overline{\log\left(y^e_i\right)}\right)$	0.83	0.86	0.84
corr $\left(\log\left(y^p_{i,t}\right), \log\left(y^e_{i,t}\right)\right)$	0.68	0.65	0.60
ratio of mean incomes $\frac{\text{entr.}}{\text{emp.}}$	1.02	1.04	0.97
non-participation rate	0.10	0.08	0.06
cross-sec. entrepreneurship rate	0.06	0.10	0.20

Table 5: SMM implied untargeted moments based on the main specification of the model estimated parameters - consequences of non-pecuniary benefits in entrepreneurship.

Untargeted Moments	data	model	+ non-pecuniary
ratio of medians $\frac{\textit{entr.}}{\textit{emp.}}$	0.84	0.94	0.84(target)
frac. of underpeforming entr.	0.36	0.32	0.47
share of serial entrepreneurs	0.39	0.51	0.36
ever trying entrepreneurship	0.29	0.71	0.92
share of 1-2 p. spells in entr.	0.68	0.54	0.22
share of entries with 0-5 experience	0.41	0.65	0.77
entr. rate up to 5 yrs. of activity	0.03	0.45	0.53

Table 6: SMM implied untargeted moments based on the main specification of the model estimated parameters - consequences of non-pecuniary benefits in entrepreneurship.

	Δ entr	Δ emp.	median $\frac{\text{entr.}}{\text{emp.}}$
decrease in $\sigma^2_{a,0}$ by 10%	1.011	1.000	0.95
decrease in $\sigma^2_{a,0}$ by 50%	1.048	1.006	0.99
decrease in $\sigma^2_{a,0}$ by 100%	1.207	1.011	1.14
decrease in $\sigma^2_{m,0}$ by 10%	1.033	1.005	0.96
decrease in $\sigma^2_{m,0}$ by 50%	1.106	1.010	1.05
decrease in $\sigma^2_{m,0}$ by 100%	1.238	1.016	1.16

Table 7: Relative productivity change Δ in response to reduction of initial uncertainty measured as a ratio of median incomes before and after the change in prior uncertainty and the accompanying ratio of median incomes in the two types of employment.

B Proofs

In this section the proofs for analytical results are provided.

Proof of Proposition 1 and 2. Let the data be $y^t = \left(y^p_1, ..., y^p_{t_p}, y^e_1, ..., y^e_{t_e}\right)$. The likelihood of the data can be written as:

$$L\left(y^t | a^*, m^*\right) \propto \exp\left\{-\left(\frac{1}{2\sigma_e^2}\sum_{j=1}^{t_e}\left(y^e_j - a^* - m^*\right)^2 + \frac{1}{2\sigma_p^2}\sum_{j=1}^{t_p}\left(y^p_j - a^*\right)^2\right)\right\} =$$

$$\exp\left\{\frac{-t_e}{2\sigma_e^2}\left(\frac{1}{t_e}\sum_{j=1}^{t_e}(y^e_i)^2 - 2\left(a^* + m^*\right)\bar{y^e} + \left(a^* + m^*\right)^2\right)\frac{-t_e}{2\sigma_e^2}\left(\frac{1}{t_e}\sum_{j=1}^{t_e}(y^e_i)^2 - 2\left(a^*\right)\bar{y^e} + \left(a^*\right)^2\right)\right\}.$$

So that $\bar{y}_e$ is a sufficient statistic for $a^* + m^*$ with variance $\frac{\sigma_e^2}{t_e}$ and $\bar{y}_p$ is a sufficient statistic for a^* with variance $\frac{\sigma_p^2}{t_p}$ which proves Corollary 1. Observe that the difference $\bar{y}_e - \bar{y}_p$ is a sufficient statistic for m^*. Corollary 2 follows as the value of home production is independent of $\mathcal{I}_t$.

The ommission of γ is without loss of generality, as an alternative is to formulate the model with ability gains $\frac{1}{\gamma}, 0 < \gamma \leq 1$ in payroll employment as follows:

$$y_p = \frac{1}{\gamma}a^* + \varepsilon_p$$

$$y_e = a^* + m^* + \varepsilon_e$$

so that it's possible to introduce a new signal $\tilde{y}_p = \gamma y_p$ with variance $\tilde{\sigma}^2_p = \gamma^2\sigma^2_p$. ■

Proof of Theorem 1. Let's assume that the prior on (a^*, m^*) is a bivariate normal with mean vector μ and variance-covariance matrix Σ as follows:

$$\mu = \begin{pmatrix} a_0 \\ m_0 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \sigma_{a,0}^2 & \rho\sigma_{a,0}\sigma_{m,0} \\ \rho\sigma_{a,0}\sigma_{m,0} & \sigma_{m,0}^2 \end{pmatrix}.$$

Then, the posterior $H\left(a^*, m^* | y = \left\{y_1^e, y_{t_e}^e, y_1^p, ..., y_{t_p}^p\right\}\right)$ is:

$$H \propto \exp -\left\{ \left(\frac{1}{2\sigma_e^2} \sum_{j=1}^{t_e} \left(y_j^e - \gamma a^* - m^*\right)^2 + \frac{1}{2\sigma_p^2} \sum_{j=1}^{t_p} \left(y_j^p - a^*\right)^2 \right) + \right.$$
$$\left. \frac{1}{1-\rho^2} \left(\frac{(a^* - a_0)^2}{\sigma_{a,0}^2} + \frac{(m^* - m_0)^2}{\sigma_{m,0}^2} - \frac{2\rho\,(a^* - a_0)\,(m^* - m_0)}{\sigma_{a,0}\sigma_{m,0}} \right) \right\}.$$

The posterior can be collapsed to the following representation:

$$\begin{aligned} H \propto \exp -\frac{1}{2}\Bigg\{ & (a^*)^2 \left(\frac{\gamma^2 t_e}{\sigma_e^2} + \frac{t_p}{\sigma_p^2} + \frac{1}{\left(1-\rho_0^2\right)\sigma_{a,0}^2} \right) + (m^*)^2 \left(\frac{t_e}{\sigma_e^2} + \frac{1}{\left(1-\rho_0^2\right)\sigma_{m,0}^2} \right) \\ & + a^* \left(\frac{\gamma t_e \bar{y}_e}{\sigma_e^2} + \frac{t_p \bar{y}_p}{\sigma_p^2} + \frac{a_0}{\left(1-\rho_0^2\right)\sigma_{a,0}^2} - \frac{\rho_0 m_0}{\left(1-\rho_0^2\right)\sigma_{a,0}\sigma_{m,0}} \right) \\ & + m^* \left(\frac{t_e \bar{y}_e}{\sigma_e^2} + \frac{m_0}{\left(1-\rho_0^2\right)\sigma_{m,0}^2} - \frac{\rho_0 a_0}{\left(1-\rho_0^2\right)\sigma_{a,0}\sigma_{m,0}} \right) \\ & + a^* m^* \left(\frac{\rho_0}{\left(1-\rho_0^2\right)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2} \right) \Bigg\} \end{aligned}$$

Lemma 1 (Parametrization of the posterior) *The distribution of (x, y) on the plane that is proportional to*

$$\exp -\frac{1}{2}\left[ax^2 - 2bx + cy^2 - 2dy - 2exy\right]$$

is a bivariate normal with means μ_x, μ_y:

$$\mu_x = \frac{bc + ed}{ac - e^2} \quad \mu_y = \frac{da + eb}{ac - e^2},$$

variances σ_x^2, σ_y^2*:*

$$\sigma_x^2 = \frac{c}{ac - e^2} \quad \sigma_y^2 = \frac{a}{ac - e^2},$$

and correlation ρ*:*

$$\rho = \frac{e}{\sqrt{ac}}$$

The proof follows from algebraic expansion of the distribution formula and can be found in Farzinnia and McCardle (2010)]. By applying this approach we can identify the posterior parameters as a, b, c, d, e naturally follow[3]:

$$\begin{aligned}
a &= \frac{\gamma^2 t_e}{\sigma_e^2} + \frac{t_p}{\sigma_p^2} + \frac{1}{\left(1-\rho_0^2\right)\sigma_{a,0}^2} \\
b &= \frac{\gamma t_e \bar{y}_e}{\sigma_e^2} + \frac{t_p \bar{y}_p}{\sigma_p^2} + \frac{a_0}{\left(1-\rho_0^2\right)\sigma_{a,0}^2} - \frac{\rho_0 m_0}{\left(1-\rho_0^2\right)\sigma_{a,0}\sigma_{m,0}} \\
c &= \frac{t_e}{\sigma_e^2} + \frac{1}{\left(1-\rho_0^2\right)\sigma_{m,0}^2} \\
d &= \frac{t_e \bar{y}_e}{\sigma_e^2} + \frac{m_0}{\left(1-\rho_0^2\right)\sigma_{m,0}^2} - \frac{\rho_0 a_0}{\left(1-\rho_0^2\right)\sigma_{a,0}\sigma_{m,0}} \\
e &= \frac{\rho_0}{\left(1-\rho_0^2\right)\sigma_{a,0}\sigma_{m,0}} - \frac{\gamma t_e}{\sigma_e^2}.
\end{aligned}$$

What is left is to plug these results back to Lemma 1 equations. ■

Martingale property of beliefs. The proof involves tedious and cumbersome algebraic manipulations. Mathematica codes deriving the martingale property of beliefs are available on request from the author. ■

Regret Properties. First, let's show that the regret is increasing in uncertainty. To do that let's consider within-period regret. It has the following form:

$$\frac{\partial R\left(\mathcal{I}_t, \xi\right)}{\partial \hat{\sigma}_{m,t}} = \phi\left(-\frac{\hat{m}_t}{\hat{\sigma}_{m,t}}\right)\left[1 + \left(\frac{\hat{m}_t}{\hat{\sigma}_{m,t}}\right)^2 + \frac{\hat{m}_t}{\hat{\sigma}_{m,t}}\right] > 0 \quad \forall \hat{m}_t, \hat{\sigma}_{m,t}$$

Observe that when $\hat{m} > 0$ all the factors of the sum in the square bracket are positive. When

[3] Observe that the formula for the e-term in (Farzinnia and McCardle, 2010) contains an error.

$-1 < \frac{\hat{m}}{\hat{\sigma}_{m,t}} < 0$ the last two factors are jointly negative but less than 1 in absolute value and the sum remains positive. Finally, when $\frac{\hat{m}}{\hat{\sigma}_{m,t}} < -1$ the squared term dominates the last negative factor and the sum again is positive. For the case of $0 < \gamma < 1$, let's introduce a variable $z^* = m^* + (\gamma - 1) a^*$ and then the regret can be rewritten as:

$$\mathbb{E}_t \left[y \left(\xi^* \right) - y \left(\xi_t \right) | \mathcal{I}_t \right] = \begin{cases} \left(1 - \Phi_{\mathcal{I}_t} \left(-\frac{\hat{z}_t}{\hat{\sigma}_{z,t}} \right) \right) \hat{z}_t + \hat{\sigma}_{z,t} \phi \left(-\frac{\hat{z}_t}{\hat{\sigma}_{z,t}} \right) & \text{if } \xi_t = p \\ -\Phi \left(-\frac{\hat{z}_t}{\hat{\sigma}_{z,t}} \right) \hat{z}_t + \hat{\sigma}_{z,t} \phi \left(-\frac{\hat{z}_t}{\hat{\sigma}_{z,t}} \right) & \text{if } \xi_t = s. \end{cases}$$

■

C Numerical algorithm

The problem of each agent is uniquely identified by initial beliefs $\hat{m}_0$, $\hat{a}_0$, $t = t_e + t_p$ and $\bar{y}_e$, $\bar{y}_p$. At step t the belief distribution is given accordingly to the Theorem 1 in the paper. The algorithm proceeds as follows:

1. Given the distribution $F(\hat{a}, \hat{m})$ construct a grid of pairs $(\hat{a}_i, \hat{m}_j)_{i,j \in \{1,\ldots,N_g\}}$ that approximates F.

2. For each initial belief means $(\hat{a}_i, \hat{m}_j)$ construct a grid on sufficient statistics space $\left(\bar{y_e}(i,j)_k, \bar{y_p}(i,j)_l \right)_{i,j \in \{1,\ldots,N_m\}} \times (t_e, t_p)$. The set of productive times (t_e, t_p) has the power equal to the number of integer solutions to the diophantine equation $t = t_e + t_p$ which equals $\binom{t+2}{2}$.

3. Solve the $\mathcal{P}$ problem by backward induction, approximating the within-period regret with a product Gauss-Hermite quadrature and using two-dimensional linear interpolation for the next period value function to obtain the switching thresholds for each sufficient statistic. Observe that choosing $\xi_t = \xi^n$ implies ξ^n is chosen for all future periods.

4. Simulate N_a agents indexed by k with initial belief $(\hat{a}_i, \hat{m}_j)$ and true endowment (a_k^*, m_k^*), interpolating the switching thresholds implied by the solution to $\mathcal{P}$.

5. Store the status in employment for each agent k in time t and the earnings $y_{k,t}$ weighted by the probability mass at $(\hat{a}_i, \hat{m}_j)$, discard the value function and policy functions, move to the

next $(\hat{a}, \hat{m})$ pair.

The main advantage of the last two steps is that they save on backward induction step which is the most costly part of the algorithm. It does introduce, however, some inaccuracy in the first period as all $N_a\,(i, j)$ agents make identical first period choice, their trajectories begin to differ with the first period different income realisations. The SMM exercise target function is a weighted sum of squared deviations from the targeted moments. The key source of possible inaccuracy is the location of first period switching threshold in the $\hat{z}$ space and the rate at which it goes to $\hat{z}_t = 0$ boundary. Thus, I use a finer grid for the approximation of F at the expense of the grid on average returns. The policy functions tend to differ only near the switching thresholds, thus the policy on sufficient statistics which are off the grid are usually well approximated by the policy at the boundary. The grid on average returns is chosen endogenously, conditionally on the initial belief means and uncertainty. Observe that the number of agents N_a has virtually no bearing on the computational cost, as it relies on the same value function at a given $(\hat{a}_i, \hat{m}_j)$ pair which is collapsed to two reservation thresholds at any of the histories t_e, t_p. Thus, I have $N_g = 20$, $N_m = 15$, $N_a = 500$.