The Diffusion of Innovations in Social Networks *

Bryony Reich, UCL

Job Market Paper

10 January 2016

Please download the latest version by clicking here:

https://sites.google.com/site/bryonyreich/documents-1/ReichJMP.pdf?attredirects=0&d=1

Abstract

This paper determines how different network structures influence the diffusion of innovations. We develop a model of diffusion where: 1. an individual's decision to adopt a new technology is influenced by his contacts; and 2. contacts can discuss, coordinate, and make adoption decisions together. A measure of connectedness, 'cohesion', determines diffusion. A cohesive community is defined as a group in which all members have a high proportion of their contacts within the group. We show a key trade-off: on one hand, a cohesive community can hinder diffusion by blocking the spread of a technology into the group; on the other hand, cohesive communities can be particularly effective at acting collectively to adopt an innovation. We find that for technologies with low externalities (that require few people to adopt before others are willing to adopt), social structures with loose ties, where people are not part of cohesive groups, enable greater diffusion. However, as externalities increase (technologies require more people to adopt before others are willing to adopt), social structures with increasingly cohesive groups enable greater diffusion. Given that societal structure is known to differ systematically along this dimension, our findings point to specialization in technological progress exhibiting these patterns.

*Bryony Reich, Faculty of Economics, University College London. Email: b.reich@ucl.ac.uk.
I would like to thank Alberto Alesina, Antonio Cabrales, Sanjeev Goyal, and Jorgen Weibull for their invaluable guidance and support. I benefited greatly from conversations with and comments of Marco Bassetto, Lars Nesheim and Imran Rasul. I am grateful to Jonathan Newton for numerous interactions at all stages of this project. For helpful comments I would also like to thank Lucie Gadenne, Terri Kneeland, and Sueyhun Kwon, as well as seminar participants at Alicante, Cambridge, INET Contagion Conference, Oxford, PET Luxembourg, and UCL. I gratefully acknowledge financial support from the UK Economic and Social Research Council (grant number ES/K001396/1).

1 Introduction

The spread of innovations is a 'social process'. For many new technologies, an individual's decision to adopt the technology depends on whether or not his contacts have adopted. For example, an individual's benefit from adopting a new communication technology depends on whether his contacts have also adopted that technology, and the benefit to his contacts of adopting in turn depends on who of their contacts have adopted, and so on.[1] A society's structure (the network of contacts that make up the society) is therefore important in determining whether or not an innovation spreads. Indeed, a large body of work examines the role of network structure in diffusion.[2] This work finds that diffusion of innovations occurs effectively through an inter-connected society, with insular communities considered anathema to diffusion. There is, however, a second social process that has been omitted in this literature. Humans communicate, coordinate and make decisions collectively. That is, family members, friends and neighbors do not just influence each other's adoption, they also discuss new technologies and jointly decide to adopt or not.

This paper develops a model that incorporates both social processes to understand why certain network structures diffuse innovations more effectively than others. We show that diffusion is determined by a measure of community, known as 'cohesion'. A cohesive community is a group in which all members have a high proportion of their contacts within the group. In the canonical diffusion setting, where individuals cannot make joint decisions, cohesion has a detrimental effect on diffusion. Preventing joint decision-making is not innocuous, since it changes key insights and results. Permitting agents to make joint decisions, our model generates a novel trade-off: on one hand, a cohesive community can hinder diffusion by blocking the spread of the innovation into the group; on the other hand, cohesive communities can be particularly effective at acting collectively to adopt the innovation.

Our framework thus predicts a pattern of specialization in technology adoption across different types of society. A particular network structure is not always 'best'. Instead, we show that for technologies with low externalities (that require few people to adopt before others are willing to adopt), social structures with loose ties, where people are not part of cohesive groups, enable greater diffusion. However, as externalities increase (as the number of people required to adopt to make the technology useful increases), social structures with increasingly cohesive groups enable greater diffusion.[3]

[1]Quotation from Acemoglu, Ozdaglar, and Yildiz (2011). For an overview see Rogers (2003) and Valente (1995). Goolsbee and Klenow (2002) and Björkegren (2015) show respectively that individuals are more likely to adopt a computer or mobile phone when their contacts adopt. A farmer's decision to adopt a new crop depends on whether neighboring farmers adopt (Bandiera and Rasul (2006)). A villager can be influenced to adopt malaria nets or safe sanitation when those living close by do so (Dupas (2014), Guiteras, Levinsohn, and Mobarak (2015)). Even the adoption of birth control or the form of contract used can be influenced by what others do (Munshi and Myaux (2006), Young and Burke (2001)).

[2]See Easley and Kleinberg (2010), Goyal (2011) and Jackson and Yariv (2011) and Jackson, Rogers, and Zenou (2015) for a selection of reviews of this literature.

[3]Societal structures are known to differ systematically along exactly this dimension, between more or less 'community-based' networks. For example Japan, Mexico and China are argued to consist of cohesive communities, while in the US and UK people have more disperse contacts. See Gorodnichenko and Roland (2011) and Fogli and Veldkamp (2013) for a review of work on this dimension of differences in network structure across different societies.

We study a model of diffusion in which an individual considers adopting an innovation, be it a new technology or behavior, and where there are complementarities in adoption. An underlying network captures the feature that an individual may care more about the adoption decision of some people than of others. For example, when considering a communication technology, an individual cares about the adoption decision of those with whom he communicates and this is represented by a link to those individuals (with links of possibly different weights depending on the importance of that communication). An individual adopts the innovation when a high enough proportion of those he is linked to also adopt. As well as making the decision to adopt or not independently, friends, family members or neighbors sometimes discuss new technologies and decide whether they would benefit by collectively adopting the innovation. A pair or group adopts collectively if each individual in that pair or group does better by adopting the innovation, given everyone else in the pair or group adopts. This framework nests the canonical model in which individuals make adoption decisions in isolation. Two examples demonstrate the relevance of the model:

- **Communication technologies:** WhatsApp is a cross-platform mobile messaging application that allows users to exchange instant messages, photos and current location with contacts and groups of contacts free of charge. It was sold to Facebook in 2014 for $21.8 billion, with 600 million users as of August 2014.[4] The adoption of WhatsApp illustrates the two key features of the model. First, the usefulness of such a messaging application depends completely on family, friends and other contacts using the same application. Second, individuals do not always act in isolation. There are many accounts of joint adoption of messaging applications: one user describes a coordinated change among his contacts from WhatsApp to another messaging application, Telegram: "there were a series of messages...that everyone should 'check out Telegram'. The debates on whether we should use it quickly shifted to becoming about how we should use it."[5] Indeed, it seems sensible for families or groups of friends to agree to adopt the same application.

  One puzzling feature of WhatsApp is that, despite being developed in the United States, it has been adopted much more widely elsewhere.[6] In fact, the US lags behind other countries in adopting mobile instant messaging generally, with 23% of smartphone users in the US and 30% in the UK using instant messaging in 2013, compared to 83% in Spain, 76% in Singapore and 78% in Mexico.[7] Our framework develops results to understand why messaging applications might diffuse widely in some societies, but not in others.

- **Health technologies:** The adoption of health technologies and safe health practices is a major

[4] 'Facebook's bill for WhatsApp climbs to $21.8bn', www.ft.com

[5] Mark Turrell, "Bye Whatsapp, Hello Telegram (And Later...Bye TG, Hello WhatsNext)", http://www.huffingtonpost.com/mark-turrell/

[6] In 2014, Spain had estimated 97% of smart phone users using WhatsApp each month compared to 10% in the US. Other messaging applications, for example Facebook Messenger, also have low usage in the US. http://www.mobidia.com/press-release/messaging-and-voip-apps-are-popular-among-mobile-users-but-the-global-market-is-very-fragmented and 'Social Media & Messaging Engagement: Chat, Social, Videoconferencing, Rich Media, and VoIP Apps' http://www.mobidia.com/

[7] 'What's Up with the US? (Not WhatsAppYet)' http://www.emarketer.com

global policy concern. One example is sanitation. One billion people worldwide practice open defecation rather than safe sanitation. In 2013, the Indian government pledged to eliminate open defecation - a practice followed by half the population.[8] While very different from communication technologies, health technologies can exhibit the same adoption behaviors. First, the benefit to a household of adopting improved sanitation can be increasing as neighboring households also adopt safe sanitation.[9] Second, collective adoption also plays a big role in the adoption of sanitation: many programmes to increase adoption are specifically designed to get neighbors and communities to overcome any collective action problem by providing workshops for discussion and the encouragement of joint adoption of sanitation.[10] Despite such efforts, getting people to adopt safe sanitation appears not only difficult, but produces varied results. The Total Sanitation Campaign introduced in India in 1999 markedly increased sanitation in some areas while in other areas it had little effect.[11] Our findings suggest some answers to why adoption of safe sanitation succeeds in some communities but not others, and whether it is worthwhile for such programmes to devote resources to promoting collective adoption.

Innovations in this model are parameterized by a 'threshold of adoption'. An innovation has a low threshold of adoption when individuals are willing to adopt the innovation even when a low proportion of their contacts do so. An innovation has a high threshold of adoption when individuals are not willing to adopt the innovation unless a high proportion of their contacts do so. Messaging applications like WhatsApp are argued to have a high threshold of adoption. In contrast, while the utility of a home computer depends on an individual's contacts also having one (see Goolsbee and Klenow (2002)), it has independent value even when friends do not use one, and so has a lower threshold.[12]

We show a measure of connectedness referred to as 'cohesion' determines diffusion. The 'cohesion' of a group is the extent to which individuals in the group have their connections within that group rather than to those outside.[13] In the canonical diffusion setting without joint adoption, cohesion has a detrimental effect on diffusion. A key feature of our model is the emergence of a novel trade off: cohesion can hinder but can also help diffusion. Consider an individual who is aware of a new improved messaging application, but does not want to switch to this new application unless at least

[8]Guiteras, Levinsohn, and Mobarak (2015).

[9]There are two arguments: the first is spillovers, if neighboring households continue to practice open defecation then flies will continue to carry germs from neighboring household's waste; second, social norms might play a role so that as fewer people in a neighborhood defecate in the open it becomes more costly to do so.

[10]The "Community led total sanitation" program in Bangladesh is designed to get communities to adopt collectively by "investing in community mobilisation...shifting the focus from toilet construction for individual households to the creation of open defecation-free villages. By raising awareness that as long as even a minority continues to defecate in the open everyone is at risk of disease, CLTS triggers the communitys desire for collective change" http://www.communityledtotalsanitation.org/country/bangladesh

[11]Mission 2017: Global Water Security http://12.000.scripts.mit.edu/mission2017/implementation/

[12]Goolsbee and Klenow (2002) show individuals are more likely to adopt a home computer when their contacts adopt and that this is tied to the use of email.

[13]Cohesion is a measure of how insular a community is. A large body of work in psychology, sociology and recently in economics (see Gorodnichenko and Roland (2011) for a review), divides societies into those that are more individualist or collectivist. These terms are defined analogously to cohesion, whereby highly collectivist societies are those where people are typically part of highly cohesive groups. Whereas in highly individualist societies people have loser ties and are not part of such cohesive groups.

half of his friends also switch. The individual is part of a cohesive group, where most of his friends are also mostly friends with each other. Clearly, the individual will not switch to the new application until others in the group switch. But, since everyone in the group has most of their friends within the group, no one in the group will switch until others in the group switch. Cohesion thus makes it difficult for the new technology to penetrate the group. On the other hand, suppose this group of friends discuss whether or not to all adopt the new application. Each individual in the group has most of his friends within the group, therefore the group as a whole would do well by switching together.

Which effect dominates? Do more 'community based' societies, made up of highly cohesive groups, do better or worse at diffusing an innovation? The key result of the paper answers this question. We show that a given network structure can be effective at diffusing one type of innovation but ineffective when it comes to another. More precisely, for technologies with a low threshold of adoption, we show that an innovation diffuses further in societies with less cohesive groups, where individuals have more disperse contacts. For technologies with high thresholds of adoption the reverse is true, an innovation diffuses further in societies with more cohesive groups with fewer connections between groups. The intuition is as follows. For low thresholds of adoption, once one or two of an individual's contacts adopt the innovation, he will do so too, then so will his contacts, and so on. The innovation spreads largely without the need for joint decision making, and so cohesive groups hinder this spread. For higher thresholds of adoption, joint decision making becomes increasingly important in getting the innovation to diffuse (think of the extreme case where an individual needs nearly all of his contacts to adopt before he is willing to adopt). As joint decision making becomes more important, cohesive groups become increasingly advantageous to diffusion. This suggests that messaging applications should diffuse further in societies with more cohesive groups, as compared to computers, which should diffuse further in societies with less cohesive groups.

The degree to which an innovation will diffuse in a given society depends systematically on the extent to which a society is made up of cohesive groups. In fact, societies are known to vary significantly along this dimension. A large body of work in psychology, sociology, and recently in economics, divides societies into those that are more or less 'community-based', and refers to this dimension as individualism versus collectivism (see Gorodnichenko and Roland (2011) and Fogli and Veldkamp (2013) for a review, also Greif (1993), Greif and Tabellini (2010) Gorodnichenko and Roland (forthcoming), Greif and Tabellini (2015)). This literature has extensively documented such differences across societies and has even ranked countries along this dimension. The best known ranking was originally compiled by Hofstede (2003) who defined the different societies analogously to the definition of cohesion used in network theory: 'Individualism on the one side versus its opposite, collectivism, is the degree to which individuals are integrated into groups. On the individualist side we find societies in which the ties between individuals are loose[...] On the collectivist side, we find societies in which people from birth onwards are integrated into strong, cohesive in-groups'. Existing theories of diffusion predict (using the terminology above) that societies ranked as more individualist should be closer to the technology frontier than those ranked as more collectivist, for all types of technology. Our results suggest that this is true for technologies with low thresholds of adoption.

However, for technologies with higher thresholds of adoption more collectivist societies enable greater diffusion compared to more individualist societies.

In Section 2 and 3 we present the model and main results. The first is a general result which characterizes where an innovation will diffuse (who will adopt and who will not) in any given network. The outcome is unique. This characterization produces two conditions that identify the trade-off described above. This is an important first step, but the main question remains: which networks are better at diffusing an innovation? To answer this question, we consider how far an innovation spreads in a given network, and how this changes when we increase the weight of a link (in the weighted network). This is analogous to adding a link in a binary network and asking whether the addition of a link helps or hinders diffusion. We show that for any possible network the set of all pairs of individuals (all potential links) can be completely partitioned into two types, where links of one type have the opposite effect on diffusion to links of the other type. We refer to the two types of link as 'strong ties' and 'weak ties'. The way links are partitioned means that links categorized as strong ties are those which connect between two individuals within a group which is already 'highly cohesive'.[14] Weak ties are all remaining ties and so include those that connect between two different groups, and so make those groups less cohesive. We show that increasing the weight of a strong tie reduces total adoption in the network for innovations with a low threshold of adoption, however it increases total adoption for innovations with a high threshold of adoption. Increasing the weight of a weak tie increases the diffusion of innovations with a low threshold of adoption, but decreases diffusion of innovations with a high threshold of adoption. Thus for innovations with low thresholds of adoption this suggests structures with more weak ties and fewer strong ties (i.e. more disperse ties and less cohesion) are better for diffusion. For innovations with high thresholds of adoption, societies with more strong ties and fewer weak ties (more cohesion) are better for diffusion.

Section 4 extends the model to incorporate heterogeneous preferences over technology and minimal restrictions on which groups can and which groups cannot make joint decisions. Section 5 examines how far we can extend the results when we do not know who is able to make a joint decision with whom. In Section 5 we also test the model using simulations. We generate a series of societies moving from less cohesive to more cohesive societies. We examine the extent to which an innovation diffuses in the different societies when who can make joint decisions is selected randomly. We show that for different technologies with increasing thresholds of adoption, the optimal network for diffusion changes from one made up of more disperse connections to networks made up of increasingly cohesive groups. In Section 6 we review the importance of this dimension of social structure across different populations. If this is an important feature of different societies then our findings have consequences not only for patterns of technological progress and specialization, but also for policy and marketing strategies of firms (which we also consider in this section). Alongside this, we discuss the empirical literature on diffusion and use this to micro-found the 'threshold of adoption'.

[14]We make this statement precise in the paper.

1.1 Related Literature

The diffusion of innovations, being central to growth, is studied by an extensive literature. We consider the role of network structure in diffusion, a large literature in itself. Easley and Kleinberg (2010), Goyal (2011), Jackson and Yariv (2011), and Jackson, Rogers, and Zenou (2015) provide excellent reviews.[15] This paper makes two key novel contributions. This is the first paper to consider how different network structures facilitate diffusion in the presence of joint decision-making, and to demonstrate that allowing for joint decision-making substantially changes our understanding of which societies are effective at diffusing innovations.

In early work, Morris (2000) presents a deterministic model of diffusion on a network and provides general results for any network structure.[16] Morris' main finding is an elegant result showing that cohesive groups block diffusion. Easley and Kleinberg (2010) describe this as a 'general principle' of such diffusion on networks, and show the result is robust to changes in the framework. Acemoglu, Ozdaglar, and Yildiz (2011) extend the model and develop new findings, again highlighting a negative effect of cohesion on diffusion.[17] Thus, without the possibility of joint decision-making, existing work finds that cohesive groups will be less effective at diffusing innovations.[18] Our paper takes the canonical diffusion framework of Morris (2000), and incorporates collective decision-making between contacts.[19] We show that, when joint decision-making occurs, very different societal structures will facilitate diffusion.

This paper also contributes to a literature which considers how social structure influences cooperation of various sorts. However, no work in this literature has looked at the interaction between different social structures and cooperation in diffusing innovations. Karlan, Mobius, Rosenblat, and Szeidl (2009) show how different network structures support the exchange of favors and valuable assets. Jackson, Rodriguez-Barraquer, and Tan (2012) consider which networks can sustain favor exchange and, importantly, are robust to concerns such as ripple effects that break down cooperation. Kets, Iyengar, Sethi, and Bowles (2011) suppose that cooperation depends on the network structure (such that some connected individuals can form coalitions), and consider how social structure influ-

[15] Rather than the diffusion of innovations (our focus), one branch of the literature studies the diffusion of information. See Goyal (2011) for a review of this literature. See Bala and Goyal (1998) for early work and Banerjee, Chandrasekhar, Duflo, and Jackson (2013), Banerjee, Chandrasekhar, Duflo, and Jackson (2014) for recent work on information diffusion.

[16] Morris (2000) is the first paper to provide results for any network structure. Previous work using a similar framework includes Blume (1993), Blume (1995), Young (1996), Anderlini and Ianni (1996), Goyal (1996) and Ellison (2000).

[17] Work by Jackson and Yariv (2007), López-Pintado (2008), Galeotti and Goyal (2009), Galeotti, Goyal, Jackson, Vega-Redondo, and Yariv (2010), Jackson and López-Pintado (2013) amongst others, focuses on the role of neighbor degree (how many contacts do individuals have) in diffusion. This work has a similar underlying framework, but takes an approach that is tractable for understanding the role of neighbor degree, and less so for understanding general features of network structure that we consider here. See Jackson and Yariv (2011) for a review and discussion.

[18] In a framework with perturbed dynamics, Young (2011) considers the waiting time before an innovation is widely adopted. Waiting time is not linked to the extent of diffusion, which is our interest. Young (2011) shows that cohesive groups can reduce the waiting time by enabling an innovation to gain a foothold in a population. Young (2011) does not allow joint decision-making. The reason for the reduction in waiting times is very different: when multiple agents within a cohesive group make a mistake and adopt an innovation before it is in their interest to do so, they are less likely to want to switch back.

[19] Our framework nests the model in Morris (2000), which corresponds to the case in our framework when no agents can make joint adoption decisions.

ences inequality. Newton and Angus (2015) do not compare the role of different network structures but, similarly to Kets, Iyengar, Sethi, and Bowles (2011), they suppose that cooperation depends on the network structure and look at the effect of coalitional behavior on the waiting time to converge to a norm in a coordination game.

2 A Model of Diffusion with Joint Decision Making

In our model, a new technology is invented. Agents, who are all using some old technology, choose whether or not to adopt the new one. We consider technologies with complementarities, so the usefulness of the new technology depends on who else is using it. For example, the usefulness of a communication technology is dependent on an individual's friends, family and other contacts using it. Agents choose to adopt the new technology only when a high enough proportion of their social contacts also adopt it. Agents can choose to adopt or not independently. They may also take joint decisions with others to adopt the new technology together. Once some agents adopt the technology, their contacts may adopt, then their contacts may adopt, and so on. We examine how far the technology diffuses in the population.

We first illustrate the diffusion process with an example with no formal notation, and then present the formal model.

A Simple Example of Diffusion

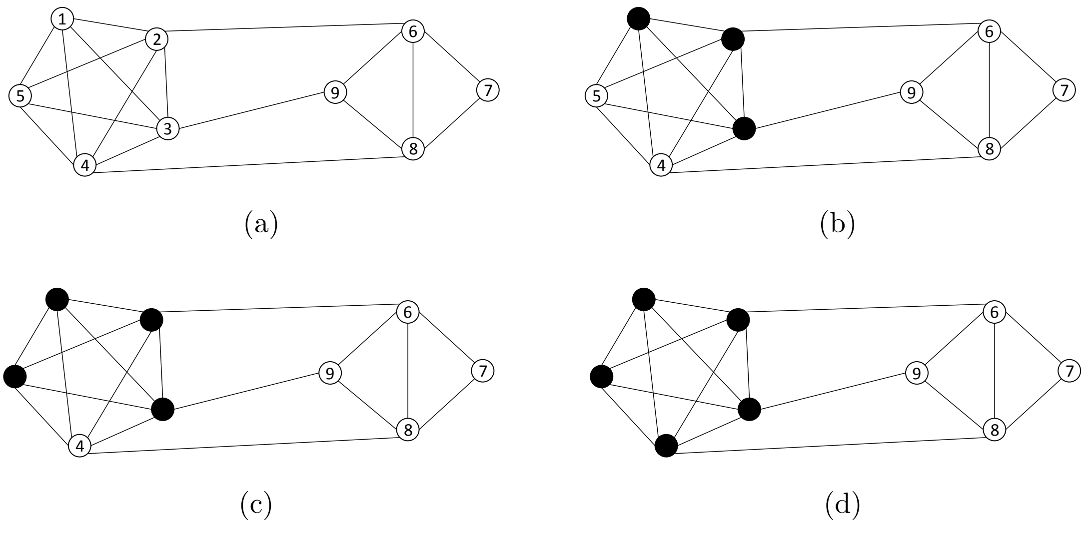


Figure 1: An example of diffusion without joint decision-making

Figure 1a illustrates a network of 9 individuals. Initially all use some current technology. A new technology is invented and individuals may adopt the new technology. A link exists between two individuals in the network if the adoption decision of one individual influences the adoption decision of the other. To give a further example (in addition to those in the introduction), consider the much studied topic of learning in agriculture. Suppose individuals are farmers who initially all grow the same crop. Growing crops can involve continual learning to improve processes and outcomes. Farmers share information on inputs, for example whether and how to use fertilizer, or how much of a crop to plant and how best to plant it. Farmers learn from neighboring farmers or those with similar soil conditions (Foster and Rosenzweig (1995), Munshi (2004), Bandiera and Rasul (2006), Conley and Udry (2010)). Therefore, for crops where learning is involved, farmers want to adopt a similar crop to their neighbors in order to benefit from their experience (Bandiera and Rasul (2006)). Suppose a new crop is introduced and a farmer will adopt the new crop only if strictly more than 2/3 of those he is linked to adopt the new crop.

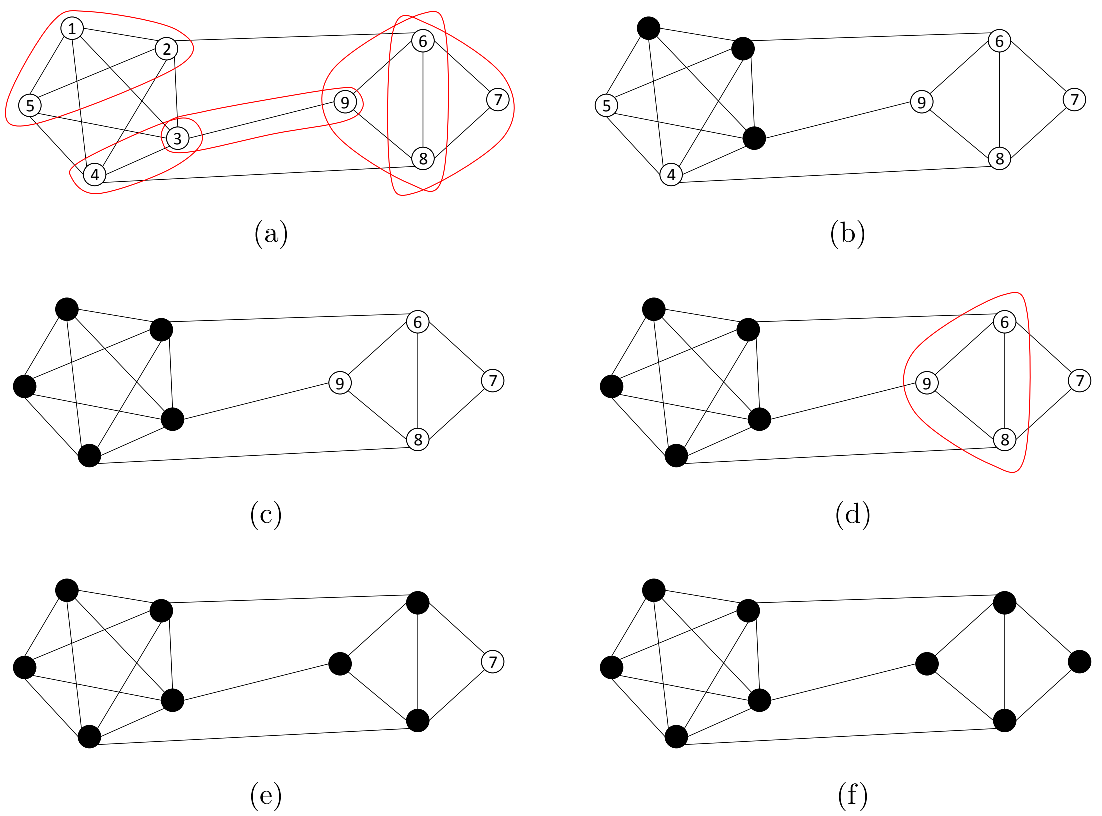


Figure 2: An example of diffusion with joint decision-making

Figure 1 illustrates diffusion when agents make decisions in isolation. Figure 2 illustrates the diffusion process with joint decision-making. We start by seeding the network in Figure 1b: individuals 1, 2 and 3 automatically adopt the innovation. These farmers might be given the seeds and some land, or paid to try out the new crop. Seeding is not a central part of the model, but it simplifies this example. For the remainder of the population, an individual will adopt the innovation if strictly more than 2/3 of those he is linked to adopt. For example, individual 9 has 1/3 of those he is linked to adopting the innovation and will not adopt. Individual 4 has 3/5 of those he is linked to adopting the innovation and will also not adopt. Individual 5 however has 3/4 of those he is linked to adopting the innovation and will adopt (Figure 1c). This then changes individual 4's decision since he now has 4/5 of those he is linked to adopting the innovation and will also adopt (Figure 1d). The innovation will spread no further. Individuals 6, 7, 8, and 9 all have at least 2/3 of their links to one another and will not adopt. The remaining group of farmers 6, 7, 8, and 9 continue to grow the old crop since they predominantly learn from each other and are less connected to the other group of farmers who have adopted the new crop.

Suppose however, that farmers 6, 8, and 9, since they discuss and learn from each other about growing crops, get together and discuss the new crop and whether or not to adopt it. If all three adopt together, then each of them has strictly more than 2/3 of those he is linked to growing the new crop. Thus they will adopt. This is how we think about joint decision-making.

In Figure 2 we run the diffusion model allowing for joint decision-making. Assume any two individuals who are linked can make a joint decision, and any group of individuals where each individual in the group has a link to all others can make a joint decision. That is, any farmers who learn from each other also have the opportunity to discuss adoption and coordinate on adopting the new crop. They decide to adopt jointly if, given each individual in the group adopts the new crop, each individual in the group then has strictly more than 2/3 of those he is linked to adopting the new crop. Figure 2a illustrates some of these groups (circled) which can make a joint decision. Seed individuals 1, 2, and 3 as before (Figure 2b). We know that individuals 4 and 5 will then also adopt the innovation (Figure 2c). Allowing agents to make a joint decision does not change this. Now, individuals 6, 8 and 9, circled in Figure 2d, can make a joint decision. They all adopt the innovation if, given each of them adopts, each has more than 2/3 of those he is linked to adopting. This is satisfied and 6, 8 and 9 will adopt (Figure 2e). Individual 7 then adopts (Figure 2f).

2.1 A Formal Model

A population consists of a finite set of individuals $N = \{1, 2, \ldots, n\}$. Time is indexed by $t \in \mathbb{N}_+$. At time $t = 0$, all individuals use an old technology. A new technology is invented. At each time $t > 0$, each individual i takes action $a_i^t \in \{0, 1\}$, he either keeps the old technology (action 0) or adopts the innovation (action 1). The vector $a^t = (a_1^t, ..., a_N^t)$ describes who adopts the old or new technology at time t.

A network consists of the population N and set of links $\mathcal{L}$. A link is an unordered pair of distinct individuals $(i, j) \in \mathcal{L}$ who influence each other's adoption. We consider varying degrees of influence, to allow for one contact to be more important than another.[20]

Definition 1 *The influence of individual j on individual i is given by the value $w_{ij} \in [0, 1]$, where $w_{ij} > 0$ if $(i, j) \in \mathcal{L}$, $w_{ij} = 0$ if $(i, j) \notin \mathcal{L}$ and $w_{ij} = w_{ji}$.*

For example, in the case of a new communication technology, w_{ij} will be higher for two family members who communicate frequently, than for two friends who speak only every few months, and will be zero for two individuals who never communicate. In a slight abuse of notation, we refer to the weighted network by $\mathcal{L}$.

Individual i's per period utility from keeping the old technology, $a_i^t = 0$, is normalized to zero, $u(a^t) = 0$. Individual i's per period utility from adopting the innovation, $a_i^t = 1$, is $u(a^t) = v(Q_i(a^t))$

[20]Symmetric influence is not necessary for the results but simplifies the exposition. We follow the notation in Ambrus, Mobius, and Szeidl (2014) for a weighted network.

which is continuous and strictly increasing in the (weighted) proportion of those i is linked to who also take action 1 at time t

$$Q_i(a^t) = \frac{\sum_{j\in N} w_{ij} a_j^t}{\sum_{j\in N} w_{ij}}.$$

As more of i's contacts adopt the innovation, his utility from adopting the innovation increases.

Individuals do not always make adoption decisions in isolation. Family or friends may discuss a new communication technology and together agree to adopt it if it improves their ability to communicate. Two individuals, i and j, if they have a strong enough relationship, can *make a joint decision.* That is, if $w_{ij} \geq \alpha$, where $\alpha \in (0, 1]$ then we say i and j can *make a joint decision.* Any group of individuals, where $w_{ij} \geq \alpha$ for all individuals i and j in the group, can *make a joint decision.* Denote by Ω the set which includes (as its elements) all individuals, as well as all groups of individuals that can make a joint decision.

At each point in time, individuals play a best response. At time zero, all individuals adopt the old technology, $a_i^0 = 0$ for all $i \in N$. At each time $t > 0$, an individual, or group that can make a joint decision, is chosen uniformly at random from Ω to update their action by best response dynamics.[21] The individual or group will switch to action 1 if all members do strictly better by taking action 1 given all other members take action 1, and given the actions of the rest of the population in period $t - 1$. Formally:

$$\text{For } i \in T, a_i^t = \begin{cases} 1 & \text{if } u_i(a_T = \mathbf{1}, a_{N\setminus T}^{t-1}) > u_i(a^{t-1}) \text{ for all } i \in T \\ a_i^{t-1} & \text{otherwise} \end{cases}$$

$$\text{For } i \notin T, a_i^t = a_i^{t-1}$$

This determines a^t and defines a Markov process.

A key parameter here is the value $\underline{Q}$ that satisfies $v(\underline{Q}) = 0$. When the proportion of an individual's contacts who adopt the new technology is above $\underline{Q}$, then his per period utility from the new technology is higher than his utility from the old technology. Thus an individual's best response is to adopt the new technology if the proportion of those he is linked to who have adopted the new technology is above $\underline{Q}$. The best response of a group of agents that can make a joint decision is to all switch to the new technology if, given everyone in the group adopts the new technology, each individual in the group has strictly more than $\underline{Q}$ of those he is linked to adopting. An innovation is completely summarized by its threshold $\underline{Q}$.

Our interest is in the set of final adopters. We are interested in the penetration rate of the technology: how many individuals adopt in an absorbing state of the Markov process, when no further agents will adopt.[22]

Definition 2 *For population N, network $\mathcal{L}$ and innovation characterized by threshold $\underline{Q}$, 'total adoption' is the number of individuals who adopt the innovation in an absorbing state.*

[21] Alternatively let some element $T \in \Omega$ be chosen from a distribution $\Xi(.)$ which has full support on Ω.

[22] We show in Proposition 1 that the absorbing state is unique.

Discussion of the Modeling Assumptions

The network. In the decision to adopt a new technology, an individual may not care as much about the decisions of the population as a whole as about the decisions of family, friends, neighbors or colleagues. For example, in the case of a new communication technology, an agent cares whether his contacts, with whom he communicates, have adopt the technology or not. He may also care more if a close contact, with whom he communicates frequently, adopts the technology compared to a more distant contact. In the case of adopting safe sanitation, a household may care whether neighboring villagers have also adopted safe sanitation, and may care more whether neighbors who live close by and whose sanitation behavior is more important have adopted, than those who live in the same village but further away. The weighted network captures these scenarios and allows for a high degree of generality. For example, it can capture a situation where an individual cares both about the proportion of the population as a whole that have adopted and whether specific friends, family or colleagues have adopted.

Threshold of adoption. The threshold of adoption, $\underline{Q}$, characterizes an innovation. A low value of $\underline{Q}$ implies individuals are willing to adopt the innovation even when few others do so. A high value of $\underline{Q}$ implies individuals are only willing to adopt the innovation if many others do so. A large empirical literature studies innovations with the property that an individual's decision to adopt the innovation depends on his contacts adopting. In Section 6 we discuss the different channels this works through and how this determines the magnitude of the threshold of adoption, $\underline{Q}$. Drawing on this, we micro-found the parameter $\underline{Q}$ and show how it is determined by other parameters. In Section 4 we extend the model to incorporate heterogeneous preferences over technology and so heterogeneous thresholds of adoption.

Joint decision making. Friends may together decide to adopt WhatsApp and set up a group to arrange meet ups. Neighbors may be part of a sanitation program that encourages discussion and collective action, and together decide to adopt sanitation. The set-up presented restricts those who can make a joint decision to groups of well-connected individuals. While this may be a reasonable assumption, other stories are also plausible. For example, two individuals who do not know each other but have a friend in common may be able to act together to adopt a new technology. Alternatively large groups of individuals, even if they are well connected, may fail to act collectively. We relax the assumptions on who can and cannot make a joint decision in Section 4. The results can be generalized because it is the possibility of joint decision making itself, rather than who can make a joint decision, that drives our results. In Section 5 we also present results when who can and cannot make a joint decision is unknown.

3 Results

3.1 Where will the Innovation Spread?

The first question we ask is where a particular innovation will diffuse within a given population. A measure of interconnectedness, referred to as 'cohesion', determines diffusion. However, the direction of the effect is ambiguous: cohesion can both facilitate adoption or block adoption. In this subsection we start with definitions and basic intuition on how cohesion affects diffusion. We provide a formal result (Lemma 1) on what kind of networked groups are resilient to adopting the innovation. Lemma 1 gives a condition showing the trade-off between cohesion being a help and a hindrance. Finally we characterize exactly where an innovation will diffuse and where it will not (Proposition 1).

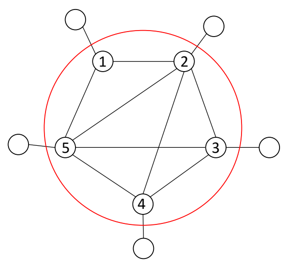
The *proportionate influence* of a subset of the population R on individual i is simply the proportion of his links that are within group R.

Definition 3 *The* proportionate influence *of a subset of the population* $R \subseteq N$ *on individual* i *is*

$$I_i(R) := \frac{\sum_{j \in R} w_{ij}}{\sum_{j \in N} w_{ij}}.$$

A group of individuals R is said to be $p-cohesive$ if *every* individual in R has at least proportion p of his links within R, and there is no larger value of p for which this is true (Morris, 2000). Cohesion is thus a measure of how insular a group is (or how strong the community is), since it is the extent to which individuals in the group have strong relations within the group relative to the outside world. Figure 3 illustrates a $2/3-cohesive$ group, so each individual has at least two thirds of his links within the group.

Figure 3: Proportionate Influence and Cohesion



Links shown correspond to a weight of 1, all other weights are 0. We consider group $R = \{1, 2, 3, 4, 5\}$ circled. Individual 1 has 2/3 of his links within group R. The proportionate influence of R on individual 1 is $I_1(R) = 2/3$. Individual 2 has 4/5 of his links within group R, hence $I_2(R) = 4/5$. Also $I_3(R) = 3/4$, $I_4(R) = 3/4$, $I_5(R) = 4/5$. Each individual in the group has at least two thirds of their links within the group, and so the group is 2/3-cohesive.

Before presenting the formal results we give some intuition for why and how cohesion determines the spread of an innovation.

A cohesive group blocks diffusion. An individual adopts an innovation if a high enough proportion of those he is linked to (more than $\underline{Q}$) adopt. Therefore, if the individual has a high proportion of his links within a given group, he is not willing to adopt until others in the group adopt. But if everyone in the group has a high proportion of their links within the group, then no one in the group is willing to adopt until others in the group adopt. It can therefore be difficult for an innovation to penetrate a sufficiently cohesive group.

A cohesive group facilitates diffusion. An individual's per period utility from adopting the innovation is increasing in the proportion of those he is linked to who also adopt. Therefore an individual who has a high proportion of his links within a particular group has a high benefit from adoption if the whole group adopts. If all members of the group have a high proportion of their links within the group, then all members get a high benefit if the group adopts the innovation. Thus a highly cohesive group, if it is able to make a joint decision, is willing to adopt the innovation as a group. To be clear, our results are not driven by the idea that certain network structures, such as cohesive groups, are better able to make joint decisions (although this may also be true). Rather, when joint decision making occurs, cohesive groups are more willing to adopt the innovation collectively.

The framework models a trade-off capturing two competing ideas: on the one hand, insular groups can be 'backwards' and resistant to change; on the other hand, insular groups can also be effective at instigating change together. This intuition is captured formally in the results below.

Recall that an innovation is summarized by its threshold $\underline{Q}$ which is the proportion of those an individual is linked to that need to adopt for him to be willing to adopt. For population N, network $\mathcal{L}$ and innovation with threshold $\underline{Q}$, Lemma 1 describes the characteristics of a group R that is resilient to adopting the innovation.

Lemma 1 *A set $R \subseteq N$ will never adopt technology $\underline{Q}$ (R is resilient) if both the following conditions are satisfied:*

(i)

$$I_i(R) \geq 1 - \underline{Q}, \text{ for all } i \in R$$

(ii) there does not exists a subset of two or more individuals, $T \in \Omega$, $T \subseteq R$, with

$$I_i(R) - I_i(T) < 1 - \underline{Q}, \text{ for all } i \in T.$$

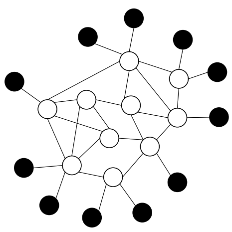
All proofs are found in the Appendix.[23] Lemma 1 says that a subset of the population R is resilient - will not adopt the new technology - if two conditions are satisfied. First, group R must be sufficiently cohesive. Each individual must have a high proportion of his links within group R (at least $1 - \underline{Q}$ by condition (i)), such that no individual is willing to adopt unless others in the group

[23]The Appendix provides proofs for the extended framework presented in Section 4.

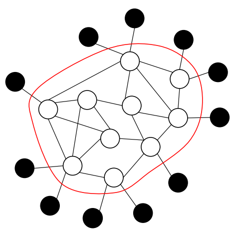
adopt. This ensures no single individual in the group is influenced to adopt by those outside the group. Second, there is no subset of individuals within group R which is itself sufficiently cohesive relative to group R. Condition (ii) ensures that there is no subset T within R which can make a joint decision and such that each individual in subset T has a high proportion of their links within T relative to their links within R (thus $I_i(T)$ should not be too high relative to $I_i(R)$, for each i in T, as stated in condition (ii)). Condition (ii) must hold so that no subset in R is willing to adopt.

The two conditions in Lemma 1 illustrate the trade-off. A group is resilient if: condition (i), the group is sufficiently cohesive, and condition (ii), there are no subgroups that are themselves sufficiently cohesive relative to the wider group. This is illustrated in Figure 4. Figure 4 also shows that it is not enough to find a cohesive group. To determine adoption one has to zoom in at each level and look for a cohesive group inside a cohesive group (communities inside communities), and so on.

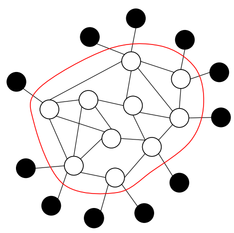
Figure 4: A Resilient Set



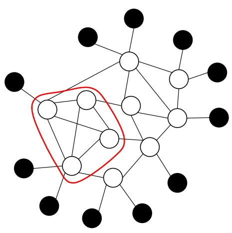
(a) Links shown have weight 1, all other weights are 0. Any set of individuals where each individual in the set has a link to all others (a clique) can make a joint decision.



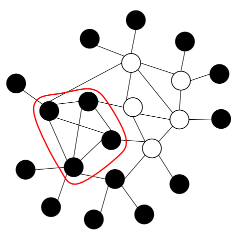
(b) Suppose the whole network has adopted the new technology, bar the group circled. Let $\underline{Q} = 2/3$. Is the circled set of non-adopters resilient?



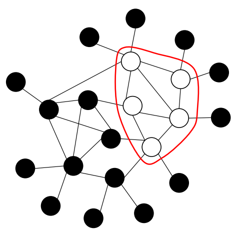
(c) The remaining non-adopters satisfy condition (i) since they form a $1/2-$*cohesive* group (circled). No individual is willing to adopt the innovation independently.



(d) A group of individuals may be willing to adopt collectively. There exists a group inside the set of non-adopters (circled) which is itself cohesive relative to the set of non-adopters. That is, each individual in the circled group has a high proportion of their links within that group relative to the set of non-adopters as a whole.



(e) The circled group can act jointly and is willing to adopt the innovation since each individual then has at least 3/4 of his links adopting. So condition (ii) is not satisfied. Following adoption by the group circled, an adjacent individual (filled node) is also willing to adopt.



(f) The remaining set of non-adopters (circled) forms a cohesive enough group that no individual alone is willing to adopt. Condition (i) is satisfied. Checking all groups that can act jointly, condition (ii) is also satisfied and no group is willing to switch. Thus the remaining set is resilient.

Proposition 1 states that for any given network and any innovation, the (Markov) process of diffusion converges to a unique absorbing state. Proposition 1 characterizes the set of final adopters to determine total adoption of the innovation. Lemma 1 gives the conditions for a set to be resilient to an innovation with threshold $\underline{Q}$. The largest resilient set for threshold $\underline{Q}$ is the largest (connected or disconnected) set that satisfies the two conditions in Lemma 1.[24] Proposition 1 says that no individual in the largest resilient set adopts the innovation, while everyone else will adopt.

Proposition 1 *For population N, network $\mathcal{L}$ and innovation with threshold $\underline{Q}$, the process will almost surely in finite time enter the unique absorbing state a, where*

$a_i = 0$ *for all $i \in N$ in the largest resilient set for threshold $\underline{Q}$*

$a_i = 1$ *otherwise.*

The proof is found in the Appendix and we here present a sketch of the proof. It is immediate from the previous discussion of Lemma 1 why no member of the largest resilient set will adopt. It is less immediate why all others adopt. Suppose the set of non-adopters is strictly larger than the largest resilient set. By assumption, the set of non-adopters does not satisfy both conditions for resilience. If the set does not satisfy both conditions for resilience in Lemma 1, then by definition there must exist either an individual who is willing to adopt, or group of individuals who can make a joint decision and are willing to adopt.

3.2 What kind of networks enable more diffusion?

What kind of network structures enable an innovation to spread? What kind of links promote diffusion? Is a society made up of cohesive groups, where individuals are part of insular communities, good at diffusing an innovation? Or instead, is a society where individuals have more disperse links better for diffusion? The previous result highlights the trade-off that cohesive groups can both help and hinder diffusion. Can we determine which effect dominates?

Providing analytic results that compare outcomes on different networks can be difficult. Networks are complex objects and two networks can look very different. We can run the above process on two networks and determine on which network an innovation diffuses further. However, if the two networks look different, it can be difficult to say anything concrete or meaningful about *why* an innovation diffused further through one network structure than the other. This becomes still more difficult when we want to say something general, about *any* network, without restricting to some class of network structures.

In this paper we take any given network and consider a marginal change in the network. We analyze whether a marginal increase in the strength of a link helps or hinders diffusion. More precisely, we consider whether a marginal increase in the strength of a link increases or reduces total diffusion in the population. This is analogous to the idea of adding or taking away a link in a binary network and asking how this affects diffusion. It turns out that this approach provides a meaningful and general result to understand how different links (and therefore different networks) facilitate diffusion.

[24] The largest resilient set for threshold $\underline{Q}$ is the union of all resilient sets for threshold $\underline{Q}$.

Figure 5: Strong and Weak Ties

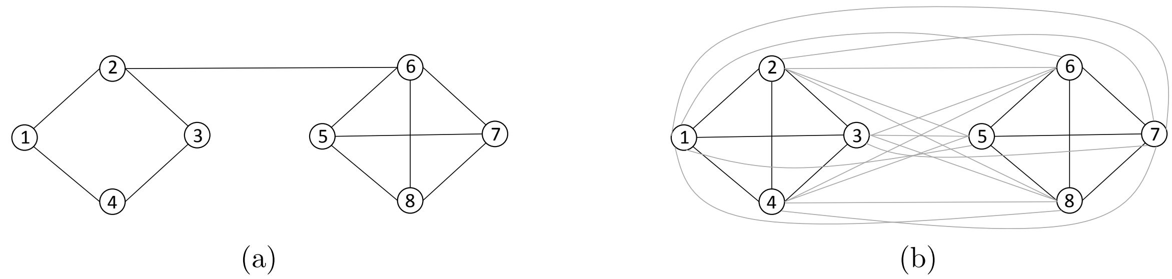


Figure 5a illustrates a network. Links shown have weight 1, all other weights are 0. Any set of individuals where each individual in the set has a link of weight at least α to all others can make a joint decision, where $0 < \alpha \leq 1$. We determine which pairs in the network in Figure 5a are strong ties and which are weak ties. This is shown in Figure 5b. Black lines connect pairs which are strong ties and grey lines connect pairs which are weak ties. We can use Definition 4 to see that all pairs in the set $\{5, 6, 7, 8\}$ are strong ties: at $\underline{Q} = 3/4$ the whole network is resilient, and so for set $T = \{5, 6, 7, 8\}$, for all $k \in T$, $I_k(R) = 1$ and $I_k(T) \geq 3/4$. Similarly, to see that the pair 1 and 3 is also a strong tie, observe that at $\underline{Q} = 1/2$ the set $R = \{1, 2, 3, 4\}$ is resilient, and so for the subset $T = \{1, 2, 3\}$, for all $k \in T$, $I_k(R) - I_k(T) \leq 1/2$. Analogously, all pairs in the set $\{1, 2, 3, 4\}$ are strong ties.

We show (Proposition 2) that for any network, the set of all links can be completely partitioned into exactly two 'types'. Links of one type, which we refer to as 'strong ties' have a very different effect on diffusion to links of the other type, which we refer to as 'weak ties'. Intuitively, strong ties are links which connect between two individuals within a 'sufficiently cohesive' group. Strong ties hinder the diffusion of an innovation with a low threshold of adoption $\underline{Q}$, but help in the diffusion of an innovation with a high threshold of adoption. In contrast, weak ties are all remaining links, so include links that connect between two different cohesive groups, making those groups less cohesive. Weak ties have the opposite effect to strong ties. Weak ties facilitate the diffusion of technologies with low thresholds of adoption, but hinder the diffusion of technologies with high thresholds. The results of Proposition 2 show that when the threshold of adoption is very low, increasing weak ties (connecting between cohesive groups) and reducing strong ties (reducing links within cohesive groups) will increase total adoption. For increasing thresholds of adoption, gradually reducing weak ties (disconnecting between cohesive groups) and increasing strong ties (building links within cohesive groups) will increase total adoption.

We first define the partition of links into strong and weak ties, before stating the effect of each type of link.

Definition 4 *For population N and network $\mathcal{L}$, any pair (i, j) is a:*

Strong tie: *if for some $\underline{Q}$, where R is the largest resilient set, i and j are part of a subset $T \subseteq R$ where $T \backslash \{j\}, T \backslash \{i\} \in \Omega$ such that*

$$I_k(R) - I_k(T) \leq 1 - \underline{Q}, \quad \text{for all } k \in T$$

Weak tie: *otherwise.*

Definition 4 states that any pair of individuals (i, j) is a strong tie if i and j are part of a set T where each individual in the set has a high proportion of his contacts within the set T relative to the proportion of his contacts within a wider cohesive set R. The set T must also potentially be able to make a joint decision.[25] That is, strong ties are always among cohesive groups, where each individual in the group has a high proportion of his links within that group compared to the proportion of his links within the wider cohesive community. Weak ties are all other ties. Thus links that connect between cohesive groups are weak ties. Figure 5 provides an example of a network and details which are strong and which are weak ties. The following proposition states the effect of these two types of tie, strong and weak, on diffusion.

Proposition 2 *For population N and network $\mathcal{L}$, if a pair (i, j) is a:*

Strong tie: *There exists a value Q_{ij} such that*

$$\text{for } \underline{Q} < Q_{ij}, \text{ total adoption is (weakly) decreasing in } w_{ij},$$
$$\text{for } \underline{Q} \geq Q_{ij}, \text{ total adoption is (weakly) increasing in } w_{ij}.$$

Weak tie: *There exists a value Q_{ij},*

$$\text{for } \underline{Q} < Q_{ij}, \text{ total adoption is (weakly) increasing in } w_{ij},$$
$$\text{for } \underline{Q} \geq Q_{ij}, \text{ total adoption is (weakly) decreasing in } w_{ij}.$$

Proposition 2 states that for any network, any link is categorized into one of two types, either as a 'weak tie' or as a 'strong tie'. Strong ties hinder diffusion at low thresholds $\underline{Q}$, but facilitate diffusion at high thresholds $\underline{Q}$. Weak ties facilitate diffusion at low thresholds but have the opposite effect at high thresholds. The proof is in the Appendix. The next paragraph gives intuition and Figures 6 and 7 illustrate the result.

We know there exists a trade-off: increasing the cohesion of a group makes it harder for an innovation to spread from outside into the group, but it also makes the group more willing to adopt the innovation if they can act collectively. Compare two technologies, one with a low and one with a high threshold of adoption. The innovation with a low threshold of adoption will spread relatively easily: once a few contacts adopt the innovation, an individual is willing to adopt it, and then so are his contacts, and so on. It spreads largely without the need for contacts to adopt collectively. In contrast, collective adoption is vital for the diffusion of an innovation with a high threshold of adoption (for example, suppose an individual needs 90% of his contacts to adopt before he is willing to adopt, clearly getting this innovation adopted requires contacts to cooperate). In summary, for technologies with a low threshold of adoption, cohesive groups are a hindrance to diffusion. Cohesive groups are not necessary for the diffusion of a very low threshold technology and, if the members of a

[25] A set T is the union of groups $T\backslash\{j\}$ and $T\backslash\{i\}$, both of which can make joint decisions. By increasing w_{ij}, potentially i and j can act jointly where they could not before. Thus the set T, which includes both i and j, may or may not be able to make a joint decision after the increase in w_{ij}. The results capture (albeit at the marginal level) the idea that a group may or may not be able to make a joint decision.

cohesive group are not able to act collectively, then such a group can block diffusion. In contrast, for innovations with higher thresholds of adoption, collective adoption becomes more important, and so increasingly cohesive groups become crucial for getting an innovation to spread. The next paragraphs provide more detailed intuition.

As discussed, strengthening any link results in a trade-off. From Section 3.1 we understand that cohesion within a group can block the spread of an innovation into that group, but it can also facilitate collective adoption by the group. Consider a strong tie. Strong ties are links within a cohesive group. Increasing a strong tie further increases cohesion within that group. Making the group more cohesive increases the benefit to the group of acting collectively to adopt the innovation. On the other hand, if the group cannot act jointly, then making the group more cohesive makes it more difficult for the innovation to spread into the group.[26] In contrast, the dominant effect of a weak tie is to act as a connector. Consider a weak tie which connects between two cohesive groups and makes them less cohesive. Increasing this weak tie makes it easier for the innovation to spread from one part of the network to another, but makes it more difficult for the individuals (or group) at either end of the link to adopt in the first place.

To see why a strong tie has a positive effect on diffusion at high thresholds and a negative effect at low thresholds, consider a cohesive group and a technology with a high enough threshold that it cannot spread into the group. Making this group more cohesive either has no effect (since the innovation cannot spread into the group anyway), or it helps diffusion if the group is now cohesive enough that, if they are able to act collectively, they are willing to adopt. Consider the same cohesive group and a low enough threshold such that the innovation can spread into the cohesive group without the need for the group to adopt collectively. Making this group more cohesive either has no effect (if the group can act together then it will adopt together anyway), or it can hinder the spread of the innovation into the group if it makes the group so cohesive that the innovation cannot spread and the cohesive group is not able to act collectively.

An analogous intuition holds for weak ties. Consider increasing a link that connects between two highly cohesive groups, and consider a technology with a low threshold that is adopted by one group but not the other. Increasing the connections between the two groups can enable the technology to spread from the first group into the second. However, for technologies with a high enough threshold, increasing connections between the two groups can instead make the first group unwilling to adopt.

Proposition 2 is a comparative static. In Section 5.1 we illustrate the result of Proposition 2 for a class of networks spanning from networks with disperse links to networks made up of highly cohesive groups. For innovations with low thresholds of adoption, networks with the most disperse links are optimal (allow for greater total adoption). As the threshold of adoption increases, networks with increasingly cohesive groups become optimal.

[26]Increasing the strength of such a link never helps the spread of innovation within the group. The reason is, since the group is so cohesive, once some individual or individuals in the group adopt, then the rest will adopt. Thus increasing the strength of a link, if the group cannot act jointly, will only hinder adoption by the group and will not help spread within the group. Strong ties never act as 'connectors' in this sense; instead, the dominant effect of a strong tie is always to make a group more 'robust'.

Figure 6: Addition of a Weak Tie

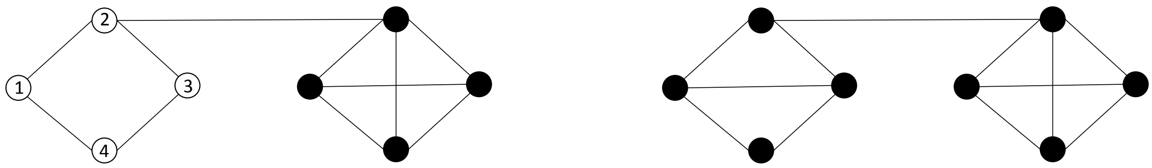
(a) (b)

Diffusion of an innovation with threshold $\underline{Q} = 3/5$. The addition of a weak tie reduces total diffusion. Figure 6a, shows the diffusion of this innovation in the network illustrated in Figure 5a. Group $\{5, 6, 7, 8\}$ can make a joint decision and will adopt the innovation since each individual has at least 3/4 of his neighbors adopting. The pair 2 and 3 can act jointly but do not adopt since 3 would only have 1/2 of his neighbors adopting. No other pairs will adopt. Figure 6b shows the addition of a link of weight $w_{36} = \gamma$ between individuals 3 and 6. Consider $\gamma = 1$ (a similar example can be show for any value of γ). In network 6b, although group $\{5, 6, 7, 8\}$ can act jointly, they choose not to adopt the innovation since individual 6 would have only 3/5 of his neighbors adopting.

(c) (d)

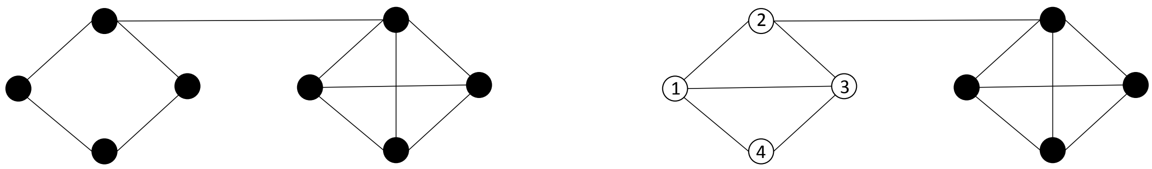
Diffusion of an innovation with a lower threshold, $\underline{Q} = 1/2$. The addition of a weak tie increases total diffusion. In Figure 6c, group $\{5, 6, 7, 8\}$ can act jointly and adopt. The pair 2 and 3 can act jointly but do not adopt since 3 would only have 1/2 of his neighbors adopt and requires strictly more than 1/2 to be willing to adopt. In Figure 6d, following the addition of a weak tie, if group $\{5, 6, 7, 8\}$ adopts then each member has at least 3/5 of his neighbors adopt and the group will adopt the innovation. The pair 2 and 3, will then act jointly to adopt the innovation since they then both have strictly more than 1/2 of their neighbors adopting. The innovation spreads to the whole network.

Figure 7: Addition of a Strong Tie



(a) (b)

Diffusion of an innovation with threshold $\underline{Q} = 1/2$. The addition of a strong tie increases total diffusion. Figure 7a is the same as Figure 6c. Group $\{5, 6, 7, 8\}$ adopt, but no others adopt. Figure 7b shows the addition of a strong tie of weight $w_{13} = \gamma$ between 1 and 3. Consider $\gamma = 1/2$ (a similar example can be show for any value of γ). Group $\{5, 6, 7, 8\}$ continues to adopt. There are then two possibilities: 1. If the new link enables 1 and 3 to make a joint decision (that is, if a link of weight 1/2 enables joint decision making) then group $\{1, 2, 3\}$ can act jointly and is willing to adopt the innovation since each individual in the group then has at least 3/5 of his neighbors adopting. 2. If the new link does not enable joint decision-making between 1 and 3, then total adoption is unchanged since no pair in group $\{1, 2, 3, 4\}$ is willing to adopt. Total adoption is unchanged or strictly increases in the network in Figure 7b.



(c) (d)

Diffusion of an innovation with a lower threshold, $\underline{Q} = 2/5$. The addition of a strong tie reduces total diffusion. In Figure 7c, group $\{5, 6, 7, 8\}$ adopts. The pair 2 and 3 can act jointly and are willing to adopt since both individuals then have 1/2 of their neighbors adopting. The innovation spreads through the network. In 7d, after the addition of the strong tie, group $\{5, 6, 7, 8\}$ will still adopt. There are two possibilities: 1. If 1 and 3 can act jointly, then group $\{1, 2, 3\}$ make a joint decision to adopt. 2. If 1 and 3 cannot act jointly, then the pair 2 and 3 will not adopt since individual 3 has only 2/5 of his neighbors adopt the innovation and he requires strictly more than 2/5 to be willing to adopt. Total adoption is unchanged or has strictly decreased in the network in Figure 7d.

4 Heterogeneous Individuals and Arbitrary Joint Decision Making

In this section we relax some of the assumptions of the model. Alternatively, the reader can move to Section 5, where we address the case where who acts jointly is unknown and also provide simulations. The model can be generalized along at least two important dimensions. First, individuals may have heterogeneous preferences over technologies. Some individuals may never adopt a new technology, even if everyone else does so. Some people may value a particular technology much more than others and will be willing to adopt even if not many others do so. This implies idiosyncratic thresholds of adoption, denoted by $\underline{Q}_i$. Second, we previously assumed that only those with a strong enough relationship could act jointly, and they would always manage to act jointly. While this may be a sensible assumption, it rules out the possibility of joint adoption with a friend of a friend. It also rules out the possibility of that large groups fail to act collectively. Since we are not aware of any consensus on how the ability to act jointly relates to the network structure, it is important to show that our results hold if arbitrary sets of individuals can make joint decisions. In this section we show that the previous findings hold when individuals are heterogeneous and arbitrary sets of individuals can make joint decisions. This nests the canonical framework found in Morris (2000) and Acemoglu, Ozdaglar, and Yildiz (2011), since it subsumes the case of no joint decision-making.

The model is unchanged from Section 2.1 with two generalizations. Now elements of the set Ω are arbitrary can be independent of the network. This allows for flexible assumptions on who can make a joint decision with whom. Let $T \subseteq N$, $T \in \Omega$, denote a subset of individuals in the population that can act jointly. We make one assumption: any subset of a set $T \in \Omega$ is also an element of the set Ω. That is, any subset of a group that can act jointly can itself act jointly. As above, all individuals can make adoption decisions in isolation and so the set Ω necessarily includes each individual as a separate element. The model captures two (possibly related) social functions. The network captures who influences who in the decision to adopt. The set Ω captures who is able to make a joint decision with whom.

Second, individuals may have heterogeneous utility from technologies. Individual i's per period utility from maintaining the current technology, $a_i^t = 0$, is a constant, individual-specific value $u_i(a^t) = \underline{v}_i$. Individual i's per period utility from adopting the innovation, $a_i^t = 1$, is given by $u_i(a^t) = v_i(Q_i(a^t))$ which is individual-specific, and continuous and strictly increasing in the proportion of i's links who also take action 1 at time t

$$Q_i(a^t) = \frac{\sum_{j \in N} w_{ij} a_j^t}{\sum_{j \in N} w_{ij}}.$$

An individual's per period utility from adopting the innovation is greater than his utility from maintaining the status quo when the proportion of his links that adopt is greater than $\underline{Q}_i$, where $\underline{Q}_i$ solves $v_i(\underline{Q}_i) = \underline{v}_i$.[27] Thresholds can now be heterogeneous across individuals. The value $\underline{Q}_i$ is assumed

[27] Assuming a solution exists. This assumption is without loss of generality as the behavioral implications of the

to be composed of two components, an innovation-specific component $\underline{Q} \in \mathbb{R}$ which is common to all $i \in N$, and an idiosyncratic component $\underline{\theta}_i \in \mathbb{R}$, such that $\underline{Q}_i = \underline{Q} + \underline{\theta}_i$. Denote the vector of idiosyncratic components by $\underline{\theta}$.

Characterization

The conditions for a set to be resilient change to accommodate possibly heterogeneous thresholds:

Lemma 2 *A set $R \subseteq N$ (is resilient) will never adopt a technology with innovation-specific component $\underline{Q}$ and idiosyncratic components $\underline{\theta}$, if both the following conditions are satisfied:*

(i)

$$I_i(R) \geq 1 - \underline{Q} - \underline{\theta}_i, \text{ for all } i \in R;$$

(ii) there does not exist a subset $T \in \Omega$, $T \subseteq R$, with

$$I_i(R) - I_i(T) < 1 - \underline{Q} - \underline{\theta}_i, \text{ for all } i \in T.$$

The conditions are analogous to Lemma 1, but adjust for the heterogeneous thresholds. Condition (i) says that in a resilient group each individual must have a high proportion of his links within the group, as before, but the exact proportion for any individual will also depend on their idiosyncratic component θ_i. Consider for example a group where each individual has a low idiosyncratic component and so a low threshold of adoption $\underline{Q}_i$. Each individual in the group is willing to adopt the innovation even if few others do. Compare this to a group where individuals are relatively opposed to the new technology and have high idiosyncratic components and so high thresholds $\underline{Q}_i$. For condition (i) to be satisfied in both cases, the group with low idiosyncratic components must be weakly more cohesive than the group with higher idiosyncratic components. A similar intuition applies to condition (ii): a subset T where each individual in that subset has high idiosyncratic components can be more cohesive relative to the set R and still maintain resilience, compared to a subset with low idiosyncratic components.

The generalization of the characterization in Proposition 1 is then immediate.

Proposition 3 *For population N, network $\mathcal{L}$, and technology with innovation-specific component $\underline{Q}$ and idiosyncratic components $\underline{\theta}$, the process will almost surely in finite time enter the unique absorbing state a, where*

$a_i = 0$ *for all $i \in N$ in the largest resilient set at $\underline{Q}$ and $\underline{\theta}$,*

$a_i = 1$ *otherwise.*

nonexistence of a solution can be perfectly captured by values of $\underline{Q}_i$ outside of the interval $[0, 1]$.

Comparative Statics

Analogous to Definition 2, for population N, network $\mathcal{L}$ and technology with innovation-specific component $\underline{Q}$ and idiosyncratic components $\underline{\theta}$, 'total adoption' is defined as the number of individuals who adopt the technology in the unique absorbing state a. We examine how total adoption changes when we change the following parameters of the model: the innovation, the extent of joint decision-making in the population, and the network.

Corollary 1 *For population N and network $\mathcal{L}$, all else equal:*

- *Total adoption of technology $(\underline{Q}, \underline{\theta})$ is (weakly) higher than total adoption of technology $(\underline{Q}', \underline{\theta}')$, where $\underline{Q} \leq \underline{Q}'$ and $\theta_i \leq \theta_i'$ for all $i \in N$;*
- *Total adoption of a technology $(\underline{Q}, \underline{\theta})$ is (weakly) higher under Ω than Ω' , where $\Omega' \subseteq \Omega$.*

Corollary 1 states that total adoption is decreasing in $\underline{Q}$. That is, the more an innovation requires others to adopt for it to be profitable, the harder it is to diffuse. The diffusion model captures a coordination problem in adopting innovations: we may all be better off adopting the innovation, but coordinating a change in regime may be difficult. Clearly, the higher is $\underline{Q}$, the more severe the coordination issue and the harder is diffusion. In contrast, total adoption is increasing in the ability of groups within the population to make joint decisions. This is because such groups essentially solve their local coordination problem.

Next we consider the main comparative static. What happens to diffusion if we increase the weight of a link in a given network? The previous results generalize. We show that all pairs can be partitioned into exactly two sets with the same effects presented in Proposition 2.

In the model in Section 2, the network and who can make a joint decision are related. Increasing w_{ij} above α enables i and j to act jointly. The idea is that individuals with strong connections are more likely to be able to act jointly. Consider two family members who talk a lot. If one adopts a new communication technology this may strongly influence the other's decision to adopt. But, in addition, these two family members may also be likely to discuss the new technology and coordinate. If this is true, then different networks may facilitate different degrees of joint decision-making. For example, well connected groups may be more likely to be able to act jointly than a disparate group of individuals with few links between them. In this section we allow arbitrary groups of individuals to act jointly. However, our results hold even when we allow for the possibility that the network and who can make joint decisions are related.

Precisely, the results in Proposition 4 hold under Assumption 1 which simply says that increasing the strength of a link between two individuals i and j, holding all else fixed, may enable i and j to act jointly. Assumption 1 allows for two possibilities. 1. Joint decision-making in the population does not change with an increase in the weight of a link w_{ij}. 2. An increase in w_{ij} can increase joint decision-making between i and j (and as a result between groups that could already make a joint decision with i and j separately but not i and j together) but does not affect other individuals.

Assumption 1 *For population N, network $\mathcal{L}$ and set of groups that can make a joint decision Ω, following an increase in w_{ij}, the set of groups that can make a joint decision in the new network includes Ω and possibly new groups from the set*

$$\{T : \{i,j\} \subset T, \quad T\backslash\{j\} \in \Omega, \text{ and } T\backslash\{i\} \in \Omega\}.$$

The following definition partitions the set of all pairs into strong and weak ties and Proposition 4, analogous to Proposition 2, gives their effect. The only difference from the partition given in Definition 4 is the incorporation of the idiosyncratic components θ_i and their interaction with cohesion, described above. Also different sets of groups can act jointly. For population N, network $\mathcal{L}$, set of groups that can act jointly Ω, and idiosyncratic components $\underline{\theta}$, we have the following:

Definition 5 *Any pair (i, j) is a:*

Strong tie: *if, for some $\underline{Q}$, where R is the largest resilient set, i and j are part of a subset $T \subseteq R$ where $T\backslash\{j\}, T\backslash\{i\} \in \Omega$ such that*

$$I_k(R) - I_k(T) \leq 1 - \underline{Q} - \theta_k, \quad \text{for all } k \in T$$

Weak tie: *otherwise.*

Proposition 4 holds under Assumption 1. For population N, network $\mathcal{L}$, set of groups that can act jointly Ω, and idiosyncratic components $\underline{\theta}$, we have the following:

Proposition 4 *If a pair (i, j) is a:*

Strong tie: *There exists a value Q_{ij} such that*

for $\underline{Q} < Q_{ij}$, total adoption is (weakly) decreasing in w_{ij},

for $\underline{Q} \geq Q_{ij}$, total adoption is (weakly) increasing in w_{ij}.

Weak tie: *There exists a value Q_{ij},*

for $\underline{Q} < Q_{ij}$, total adoption is (weakly) increasing in w_{ij},

for $\underline{Q} \geq Q_{ij}$, total adoption is (weakly) decreasing in w_{ij}.

5 When who can Act Jointly is Unknown

We know that joint decision-making between individuals sometimes occurs when people adopt a new technology or innovation. Suppose we want to evaluate the extent to which a particular innovation is likely to be adopted in a given population. We observe the population and the network structure, we also know that some people may act jointly, but we do not know who they are. Can we say anything about what networks are better for diffusing the innovation when we do not know which groups can act jointly? Are our previous results robust to this more difficult problem?

The model is unchanged from above. We know the network, which consists of a finite set of individuals $N = \{1, 2, \ldots, n\}$, a set of links $\mathcal{L}$, and weights w_{ij}. As above, there is some set of groups, denoted Ω, where each group is able to coordinate amongst itself and decide whether or not to adopt the innovation together. The only difference is that we do not know which these groups are. We continue to make the (arguably uncontroversial) assumption that all individuals can make adoption decisions independently so are in the set Ω. For simplicity, we also assume the set Ω stays fixed following a marginal increase in w_{ij}, but this can be extended to the more general case (Assumption 1). The rest of the model is unchanged. We consider homogenous preferences and therefore a homogenous threshold $\underline{Q}$ for all individuals in the population.[28]

Figure 8: A Strongly-Cohesive Group

(a) The group $\{1, 2, 3, 4\}$ satisfies the conditions of a strongly-cohesive group. Each member of the group has a high proportion of his links within the group and the group is evenly connected with no discernable subgroups.

(b) After removing the link between 1 and 3, the group $\{1, 2, 3, 4\}$ does not satisfy the conditions of a strongly-cohesive group. Consider the set $\{1, 2, 4\}$. Any member of this set has at most $1/4$ of his links to 3. Individual 3 has $2/3$ of his links to the set $\{1, 2, 4\}$. Since the set $\{1, 2, 4\}$ is relatively disconnected from 3, 3 does not have a high enough proportion of his links to this group to satisfy the definition of strong cohesion. In other words, $\{1, 2, 4\}$ and 3 are discernable subgroups.

We define what we call a *strongly-cohesive group.* Very informally, a strongly-cohesive group is a discernable community which does not comprise any discernable sub-communities. Precisely:

Definition 6 *A set R is 'strongly-cohesive' if:*

- *for each $T \subset R$, find $\hat{Q}$ that satisfies $I_i(R) - I_i(T) \leq 1 - \hat{Q}$ for all $i \in T$ with equality for some i, then any subset $T' \subseteq R \backslash T$ has $I_i(R) - I_i(T') > \hat{Q}$ for some $i \in T'$;*
- *and there is no larger set $R' \supset R$ which satisfies the above.*

A strongly cohesive set is defined by the network only and is independent of the set Ω which is unknown. The definition of a strongly-cohesive set is similar but stronger than the idea of cohesion.

[28]This can be extended to allow for heterogeneous thresholds of adoption.

A strongly-cohesive group is a group in which each individual in the group either has a high proportion of his links within the group, or other individuals in the group have a high proportion of their links to him. Second, there can be no subset of the group in which individuals have a low proportion of their links out of the subset and the rest of the set has a low proportion of their links into the subset. The idea is similar to cohesion but takes into account both links out and links in for an individual in the group. It also ensures that the strongly cohesive set is relatively evenly connected, such that there are no discernable sub-communities. That is, within the strongly-cohesive set there are no subsets which are 'poorly connected' to the rest of the group. Figure 8 gives an example.

The following proposition shows that links between two individuals within a strongly-cohesive group act like strong ties. Increasing such links increases total diffusion for innovations with high thresholds of adoption, but reduces diffusion for innovations with low thresholds of adoption. In contrast links connecting between two distinct strongly-cohesive groups act like weak ties. Increasing such links weakly decreases total diffusion for innovations with high thresholds of adoption, but increases diffusion for innovations with low thresholds of adoption.

Proposition 5 *For population N, network $\mathcal{L}$ and an unknown set of groups that can make joint decisions Ω, if i and j are part of a strongly-cohesive set, then the link (i, j) is a strong tie and there exists a value Q_{ij} such that*

for $\underline{Q} < Q_{ij}$, total adoption is (weakly) decreasing in w_{ij},

for $\underline{Q} \geq Q_{ij}$, total adoption is (weakly) increasing in w_{ij}.

If i and j are part of distinct strongly-cohesive sets, then the link (i, j) is a weak tie and there exists a value Q_{ij} such that

for $\underline{Q} < Q_{ij}$, total adoption is (weakly) increasing in w_{ij},

for $\underline{Q} \geq Q_{ij}$, total adoption is (weakly) decreasing in w_{ij}.

We find that links within a strongly-cohesive group are strong ties, while links that connect between two strongly-cohesive groups are weak ties. In a strongly-cohesive set, individuals in the set are so influenced by the others that once an individual or any part of the group is willing to adopt, the innovation will spread to the rest of the group. Therefore any link within a strongly-cohesive group acts as a strong tie. Increasing a link within a strongly-cohesive group has one of the following two effects. If the link connects between individuals that act jointly to adopt, this will increase the benefit to that group of adopting the innovation and so facilitate diffusion. Otherwise, since the link makes the group more insular, it makes it weakly more difficult for the innovation to penetrate the group. Why does a link within a strongly-cohesive group never act as 'a connector' to enable the innovation to spread better within the group? The reason is that, by definition, a strongly-cohesive group is connected in such a way that once the innovation enters the group it spreads everywhere in that group, and so the role of 'a connector' is redundant. In contrast, links connecting different strongly-cohesive sets are weak ties. These links connect between two groups which are connected in such a way that once any individual in either group is willing to adopt it spreads through the whole of that group. Links out of a group can make it harder for the innovation to penetrate the group in the first place, however, once one of the groups adopts, this link will make it easier to spread from one group to the other.

5.1 Simulations

(a) **Four individuals in a small-world network with no rewiring.** The cohesion of group $\{1, 2, 3\}$ is $1/2$. The cohesion of group $\{1, 2, 4\}$ and group $\{1, 2, 3, 4\}$ is also $1/2$.

(b) **Four individuals in a small-world network after a link is rewired.** The link between 1 and 3 is deleted and a new link is added between 3 and some other individual in the network. The cohesion of group $\{1, 2, 3\}$ and group $\{1, 2, 3, 4\}$ is now $1/4$.

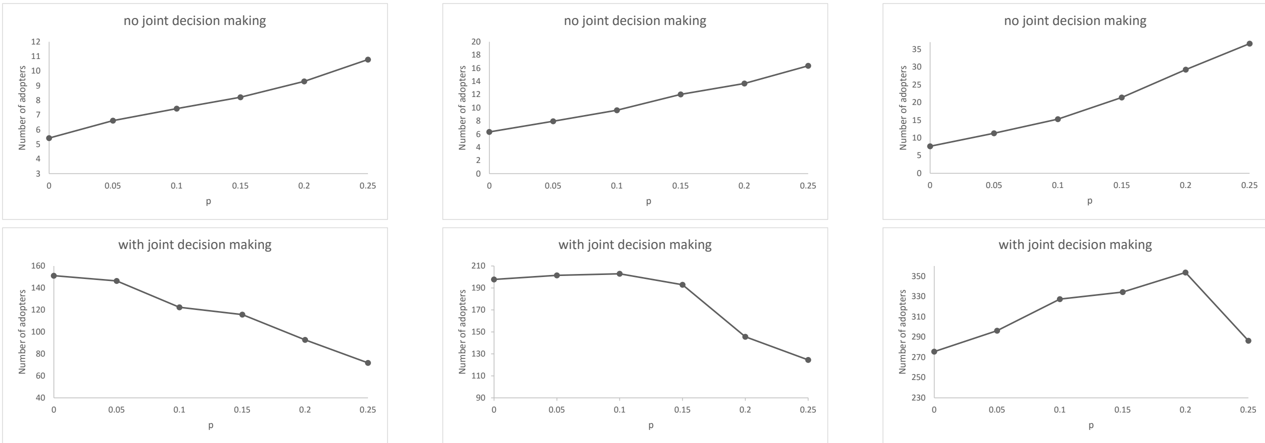
Figure 9: Example of four individuals in a small-world network

We test our model by simulating diffusion over a well-studied class of networks known as 'small-world' networks (Watts and Strogatz (1998)). Small-world networks are binary, symmetric networks with two properties found in many real networks. The first is *small average path lengths*, such that any two randomly chosen individuals tend to be connected to each other via relatively few links. The second is *high clustering*, that is, individuals tend to have links in common with those they are linked to. In a classic paper, Watts and Strogatz (1998) consider random rewiring in small-world networks. They start with a highly clustered network. They then randomly remove links and add new ones. This process gradually breaks up tightly knit groups, creating networks with more disperse links. Acemoglu, Ozdaglar, and Yildiz (2011) show that reduced clustering is associated with reduced cohesion of groups within a network. Therefore, as the amount of rewiring in these small-world networks increases, this is associated with reduced cohesion of groups in the network. We follow their example of using this class of networks to consider diffusion across comparable networks but which move from networks made up of cohesive groups to networks with disperse ties. Figure 9 shows a group of 4 individuals in a small-world network and what happens to the cohesion of these 4 individuals following rewiring.

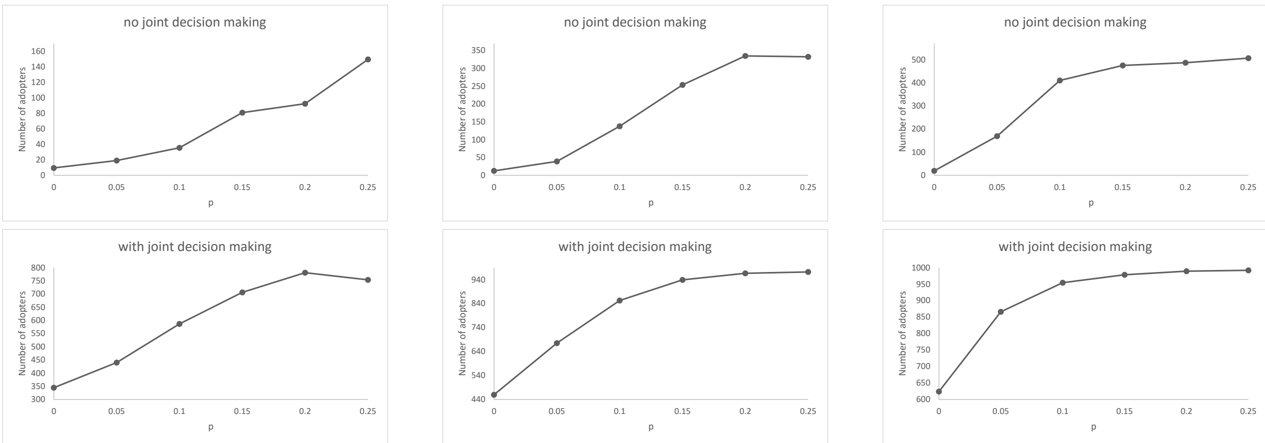
Precisely, we start with a ring lattice with 1000 individuals and 4 links per individual (Figure 9a shows what the connections for 4 neighboring individuals look like in this network - this is repeated throughout the network). The rewiring probability is a probability p for each link that the link is deleted and reconnected between two random individuals.[29] A higher value of p results in more

[29]A link will not be rewired to connect between two individuals who already have a link or to connect an individual

Figure 10: **Simulations across different networks**



(a) (b) (c)



(d) (e) (f)

Figure 10a plots average total adoption for an innovation with a high threshold of adoption across the class of networks we consider (from networks with rewiring probability $p = 0$ to rewiring probability $p = 0.3$). The top graph in Figure 10a plots average total adoption when joint decision-making does not occur during the diffusion process. The bottom graph in Figure 10a plots average total adoption when joint decision-making does occur during the diffusion process. Moving from Figure 10a through to Figure 10f, we repeat the same process but for innovations with increasingly lower thresholds of adoption. Figure 10f shows diffusion for the innovation with the lowest threshold of adoption. When no joint decision-making takes place (the top graph in Figure 10a to Figure 10f) the lines are upward sloping: the relationship between total adoption and cohesion is negative, for all thresholds of adoption. When joint decision-making occurs, at high thresholds of adoption the slope of the line is instead negative, see Figure 10a and Figure 10b. The relationship between total adoption and cohesion is positive and the optimal network is the most cohesive network in the class of networks we consider. When joint decision-making occurs, as the threshold of adoption decreases, the optimal network is less and less cohesive. For the lowest threshold of adoption we consider, in Figure 10f, the slope of the line is positive and the least cohesive society we generate allows the most adoption.

rewiring and so breaks up more links inside clustered groups and rewires them randomly to generate more disperse ties. We thus create a class of networks with different rewiring probabilities, where higher rewiring probabilities are associated with fewer cohesive groups and more disperse ties.

In the simulations, we consider a given innovation and how this will diffuse throughout the class of networks generated. For that given innovation, we generate an idiosyncratic threshold for each individual, $\underline{Q}_i$, from a uniform distribution. To consider innovations with higher thresholds, we select individual thresholds from a uniform distribution with a higher support. We generate 100 networks for each rewiring probability we consider. That is, we generate 100 of the most cohesive type of network, 100 of the next most cohesive, and so on. For each network we randomly select some groups of 3 or more completely connected individuals who are able to make joint adoption decisions. We seed the network so that 1 individual is selected to adopt in period 0 and then run the diffusion process described in the framework above. We compare how this given innovation diffuses across more or less cohesive networks in our class. We repeat the whole process again for a range of innovations with different thresholds.

Figure 10 plots the findings from the simulations. Figure 10a shows the average number of adopters of a given innovation, for each network in the class (from no rewiring, $p = 0$, which is the most cohesive, to rewiring with probability $p = 0.3$, which is the least cohesive network we consider), and for the case of no joint decision-making (top graph) and the case with joint decision-making (bottom graph). Figure 10b does the same for a given innovation with a lower threshold of adoption than that in Figure 10a. Figure 10c, Figure 10d, and so on, show the simulations for innovations with increasingly lower thresholds of adoption. The relationship between rewiring and adoption is always positive when joint decision-making does not take place; that is, cohesion hurts diffusion. When joint decision-making occurs, for high thresholds (e.g. Figure 10a) the relationship between rewiring and adoption goes in the opposite direction, it is negative: cohesion helps diffusion. However, we see the peak of the ‘with joint-decision making’ plots gradually moving to the right as rewiring increases. As the threshold decreases, less cohesive and increasingly disperse networks generate the most adoption.

6 Discussion: Innovation, Society, and Policy

The model identifies two key parameters that determine diffusion. The first, related to the innovation, is the threshold of adoption (the proportion of contacts that must adopt an innovation before an individual is willing to adopt it). The second, related to the network, is the extent to which the society is made up of cohesive groups. In this section we consider evidence on these two parameters. Which societies are comprised of more cohesive groups? What kind of innovations have higher or lower thresholds? We then consider the policy implications for governments wanting to encourage adoption of technologies. As well as the implications for firms trying to diffuse a product.

with himself.

6.1 Measures of Societal Structure

What evidence is there on the extent to which people tend to interact within a cohesive group versus having loosely knit ties? Does this vary and, if so, how does this vary across societies? If this is an important feature of different societies then our findings have consequences, not only for technological progress and specialization, but also for policy and marketing strategies of firms.

In fact, this dimension of society has been extensively documented. Societies made up of closely knit groups, with interactions mainly within the group and few outside, are referred to as collectivist. Societies where individuals have more disperse connections are referred to as individualist. Avner Greif, in important work in economics showing the effect of culture on institutions, describes differences across societies as follows:[30]

'In collectivist societies the social structure is "segregated", in the sense that each individual interacts socially and economically mainly with members of a particular religious, ethnic, or familial group...In Individualistic societies, the social structure is "integrated", in the sense that economic transactions are conducted among people from different groups, and individuals frequently shift from one group to another.'

Such differences in the degree to which individuals in a society interact within an insular groups is considered an aspect of a society's culture. Gorodnichenko and Roland (forthcoming) point out that 'the individualism-collectivism distinction is considered by cross-cultural psychologists to be the main dimension of cultural variation'.

Such differences have been documented even across historic societies. Cohesive, large, kinship groups were part of most early societies (Greif and Tabellini (2010)). Greif and Tabellini (2010) document how such groups persisted in China and how clans remained part of the structure of Chinese society even in late imperial China. In contrast, in Europe, tribal tendencies were undone by the Church so that by the 9th century 'large kinship groups remained only on Europe's social and geographical margins'.

The best known modern measure of collectivism was collected by Hofstede (2003). Like Avner Greif, his description of this dimension of social structure mirrors the network definition we present in the theoretical results above:

'Individualism on the one side versus its opposite, collectivism, is the degree to which individuals are integrated into groups. On the individualist side we find societies in which the ties between individuals are loose...On the collectivist side, we find societies in which people from birth onwards are integrated into strong, cohesive in-groups, often extended families...'

Hofstede (2003) surveyed IBM employees across various countries. He then used factor analysis to construct the collectivist/individualist index. Although the above definition refers to relations between individuals, Hofstede (2003) did not map out social connections but instead asked survey questions. Details can be found in Fogli and Veldkamp (2013) and Gorodnichenko and Roland (forthcoming), as well as references to follow-up work which links the index to social networks. Table 1 gives some examples of the individualism scores of various countries in the Hofstede Index. The

[30] Greif (1993), Greif (2006).

higher the score the more individualist the country.

Table 1: Hofstede Index of Individualism for a sample of countries, from $0 - 100$ where 100 is most individualist

Country	Score
United States	91
Great Britain	89
Canada	80
Netherlands	80
Italy	76
France	71
Germany	67
Israel	54
Spain	51
Japan	46
Brazil	38
Mexico	30
Malaysia	26
China	20
South Korea	18
Indonesia	14

There are clearly difficulties in collecting and developing a country-specific measure of social structure. Recent work in economics has used creative approaches to develop new variables relevant to this dimension of social structure, but which avoid some of the problems associated with Hofstede's Index. Fogli and Veldkamp (2013) use the difference in prevalence between human and zoonotic disease to capture the network structure. The idea is that communicable diseases influence the 'prevalence of collectives' in a society since such groups have greater protection against the spread of disease and so will be less likely to die out in environments with a high disease burden.[31] Gorodnichenko and Roland (forthcoming) use genetic research suggesting that certain genes can affect the prevalence of collectivist culture and showing a correlation between the two. This works through the effect of a gene in increasing the intensity of stress when faced with social rejection, leading to collectivist cultures that protect against social rejection. They also consider a second gene that increases the risk of depression, with the argument that this gene encourages individuals to embed themselves to a greater extent in strong communities to again offer some protection.

A literature in economics on the family, also captures, although in a different way, the idea of a cohesive group.[32] This literature shows that some societies have particularly strong relations between group members, where the group, in this case, is the family. As a result, societies with strong family relations typically have weaker relations between non-family members. Societies with strong family ties have a similar interpretation to that of the clan, these are societies made up of insular, tightly-knit groups. Indeed family relations can vary dramatically 'at one extreme, nuclear families are those

[31] This is supported by results in network design. Goyal and Vigier (2014) suggest that when attack resources are higher (here the disease burden) then optimal networks involve segregating a population into more groups.

[32] See Alesina and Giuliano (2014) for a review of the literature.

in which children are emancipated from their parents and leave the household at the time of marriage or before. At the opposite extreme, the extended family typically consists of three generations living together and cooperating' (Alesina and Giuliano (2010)). Alesina and Giuliano (2010) document the strength of family ties across different countries and create an index of how this varies.

Given societies differ systematically along the dimension suggested by our theory, our findings have consequences not only for technological progress and specialization, but also for government policy and marketing strategies of firms. Government policies to induce adoption of a wide spectrum of technologies are ubiquitous, both across the developing and developed world. Which technologies governments should promote depends on how effectively that effort translates into adoption. Firms have limited resources to spend on marketing a product and maybe limited in the number of markets they can enter. Which markets they should enter is an important consideration.

Our findings suggest different technologies will spread differently in different societies and by potentially different means. A government or other body wanting to encourage adoption of technologies, be it through subsidies or other programmes, should choose carefully *which technologies* to 'promote' depending on the structure of society. Technologies with high thresholds will be very difficult to diffuse in 'individualist' societies with disperse social connections. This is true even if joint decision-making amongst individuals is relatively prolific. In collectivist societies, except in relation to technologies with very low thresholds, diffusion of a technology can only take place if accompanied by joint decision-making. Devoting resources to encouraging cooperation in more collectivist societies and among insular groups (if it is possible to do so) will aid diffusion.

A firm or other organization wanting to increase adoption of a certain technology has a slightly different problem. They must choose carefully in *which societies* to spend scarce resources. Technologies with high thresholds of adoption will be very difficult to diffuse in individualist societies, and should be targeted at collectivist societies. Targeting a technology with a low threshold of adoption at an individualist society will result in higher take-up of the product.

6.2 Threshold of Adoption of an Innovation

This paper considers innovations where an individual's utility from adopting the innovation depends on others adopting. We review evidence on the different channels this works through. We use this to provide micro-foundations to determine how thresholds of adoption differ across different technologies and why. In this section we simply state the results, while the formal micro-foundations are found in the Appendix. We discuss how this relates to findings and policy implications.

Value of the Network versus Independent Value

Consider a technology whose value derives from the network of users (its usefulness depends on others using the technology) and which has independent value (it has some value independent of whether others use the technology). The higher the independent value of the technology, the lower its threshold of adoption.

Example: Communication Technologies. At one extreme are technologies whose value derives completely from who else is using them. This can be a feature of communication technologies and suggests a high threshold of adoption.[33] Messaging applications such as WhatsApp provide a good example. The value of a messaging application comes solely through communication with contacts who also use the application. Individuals tend to use only one messaging application (rather than use different applications with different contacts), and hence messaging applications are argued to have a high threshold since it is difficult to get people to switch to new applications without their contacts.[34] The adoption of mobile phones in developing countries also provides a good example. In developing countries, which typically do not have expansive fixed-line networks, and where internet services on phones were not widely used during the adoption phase, the benefit of using a mobile phone comes solely from others using one too.[35]

The value of some communication technologies may depend on the network of other users, but also have independent value. The higher the independent value, the lower the threshold of adoption. For example, mobile phone technology in developed countries was introduced alongside extensive fixed-line networks. A mobile phone therefore had some value even if none of an individual's contacts adopted one. Of course, the utility of adopting a mobile phone is still increasing as contacts adopt a mobile phone, since it allows both users to communicate wherever they are, as well as for new forms of communication (e.g. text messages). This implies a lower threshold of adoption than technologies whose value depends solely on the network. Another example is choice of mobile phone operator. In many countries, operators differentiate the cost of calls to users of the same operator and the cost to users of a different operator. Then the benefit of an operator depends on the proportion of an individual's contacts who also use the same operator. However, having a particular operator has independent value, even if an individual's contacts all use different operators, since communication is still possible across operators, it is just more costly. Analysis of choice of network provider in the UK finds 'considerable inertia [in an individual's choice of provider],...but is heavily influenced by the choices of others in the same household' (Birke and Swann (2006)). Similarly, computers have both network benefits, as well as independent value. Goolsbee and Klenow (2002) show that individuals are more likely to buy their first home computer when a larger share of their family and friends do, and that this is tied to the use of email and internet.

The framework in this paper predicts that communication technologies whose value depends solely on the networks of users will be adopted to a greater extent in societies formed of more cohesive groups, compared to communication technologies with some independent value which will be adopted to a greater extent in societies with less cohesive groups and more disperse ties. The fax

[33]Technologies can also have indirect benefits from other users. Direct benefits occur when the value of the good comes from other users, while indirect benefits occurs when more consumers induces a response from the supply side. In this paper we are concerned with direct benefits. For work on direct benefits but from the supply side see Economides and Himmelberg (1995).

[34]'Why Telegram has become the hottest messaging app in the world' www.theverge.com. Of course, an interesting question, that we do not ask here, is why firms do not make their messaging application compatible with other messaging applications. For interesting work on this see Economides and Skrzypacz (2003).

[35]Björkegren (2015) looks at adoption in Rwanda where 'usage and availability of mobile internet data during the period of interest was negligible'.

machine is one such technology whose value depends solely on the network of users. In the 1980s, the fax machine gradually became the mode of business communication, superseding telex (a network of printers that sent long-distance text based messages). However, widespread adoption of the fax machine did not occur in the US where it was invented. The fax machine was invented in 1843 and continuously improved upon by companies in the United States, but it failed to reach a mass market. In the 1980s however, widespread adoption occurred in Japan. Japan ranks as one of the most collectivist high-income countries in the world. The fax machine spread so widely in Japan it became a standard not only in businesses but also homes. It was only after the fax machine was adopted extensively in Japan that it finally became a widespread technology in the West.[36]

Complementarities in Adoption versus Pure Externalities

Consider a technology with complementarities in adoption (the utility from adopting the innovation is increasing as others adopt) and with pure externalities (the utility for an individual who does not adopt the innovation is increasing as others adopt it). The higher the complementarities in adoption, the lower the threshold of adoption. The higher the pure externalities, the higher the threshold of adoption.

Example: Health Technologies. Subsidies and other interventions to increase adoption of various health technologies are common across the developing world.[37] As discussed in the introduction, adopting safe sanitation may not have particularly high benefit if neighboring households practice open defecation and flies continue to carry germs from nearby. A household may then want to adopt safe sanitation only when some proportion of neighboring households do so. Guiteras, Levinsohn, and Mobarak (2015) show that when some members of communities in Bangladesh are given subsidies for hygienic latrines, this increases ownership both among those individuals and their neighbors. In the same way, using a bed net to prevent malaria may have fewer benefits if a low proportion of others in the community use bed nets since then the risk of malaria in the community can remain high.[38] Dupas (2014) finds that individuals are more likely to adopt a bed net if they receive a subsidy or if they do not receive a subsidy but their neighbors do. Another example is deworming treatment. Treating a child may have higher returns for that child if other children at school also get treatment, since it reduces the child's likelihood of reinfection (Miguel and Kremer (2004)).

A number of influences could play a role in an individual's decision to adopt such health technologies. First, how valuable the health technology is to an individual when none of his contacts adopt it. Second, the degree of complementarity associated with the health technology: how much does an individual's benefit from adopting the health technology increase as his contacts adopt it. Third, the degree of pure externality: how much does the individual's utility from not using the technology increase as others adopt it. We show in the Appendix that the threshold of adoption $\underline{Q}$ is decreasing

[36]Information on the fax machine from Beise (2003).

[37]See Dupas (2014), Guiteras, Levinsohn, and Mobarak (2015).

[38]See Killeen, Smith, Ferguson, Mshinda, Abdulla, Lengeler, and Kachur (2007) for details on malaria transmission and bed net usage in communities.

as the value to an individual of adopting the health technology when none of his contacts adopt it increases. The threshold of adoption $\underline{Q}$ is decreasing in the degree of complementarity and increasing in the degree of pure externality.

One implication of our framework is that health technologies with high pure externalities (where individuals benefit from others adopting even if they personally do not adopt), will not only be extremely difficult to diffuse but, without providing extra incentives, will be adopted only in insular tightly knit communities. In urban areas, where living conditions are typically more inter-connected and individuals are less likely to be living in insular neighborhoods, our findings imply it will be difficult to get health technologies adopted. Consider the example of sanitation. Suppose a household will not adopt sanitation unless a high proportion of neighbors adopt. In an area with many inter-connected households, a given household does not want to adopt until his neighbors adopt, but his neighbors do not want to adopt unless their neighbors adopt. In an urban setting, where individuals are not part of insular groups, the neighbors of a household's neighbors are not his neighbors. This means that for any household to want to adopt, a high proportion of the population as a whole must adopt. This is not because the behavior of a large proportion of the population influences a single household, rather this is a result of the indirect effect of each household needing a high proportion of his neighbors adopting and the structure of the network.

In an experiment studying the adoption of an online health technology, Centola (2010) is able to analyze directly whether the health technology will spread further on some types of network than others. He assigns participants in his study 'health buddies' and looks at whether they sign up to an online health forum (the health technology), which rates different health resources. Those who sign up to the online health forum receive ratings and feedback about health resources from their buddies. Centola (2010) shows that participants are more likely to sign up when more of their health buddies sign up. He then shows that the health technology spreads further in networks where health buddies form clustered groups, compared to networks where individuals have more random ties to different health buddies.

7 Conclusion

We developed a model of diffusion to incorporate an additional feature of adoption behavior observed in the real world. Family, friends, neighbors, and other contacts do not just influence each other's adoption decision, they also discuss and make adoption decisions together. We showed which societal structures allow for greater diffusion of an innovation. Accounting for joint decision-making behavior is important since it alters our understanding of what kind of network structures allow for greater diffusion. The extent to which an innovation will diffuse depends systematically on a particular dimension of network structure: the extent to which a society is more or less 'community-based'. For innovations with low thresholds of adoption, less community-based societies enable more diffusion. As the threshold of adoption increases, gradually more and more community-based societies become superior at diffusing the innovation.

These findings suggest technology specialization across different societies and imply that governments, development organizations, and firms that aim to promote or subsidize adoption of technologies should carefully consider both technologies and markets. That is, a government wanting to promote adoption of new technologies in a particular population should choose carefully which technologies to subsidize. A firm marketing its product should choose carefully in which populations and markets to spend resources.

A lot of detailed work in the development literature studies how best to get populations to adopt various technologies. Applying our predictions in such micro-level studies would enable a careful test of the theory as well as provide evidence on how to target technologies at particular communities.

References

Acemoglu, D., A. Ozdaglar, and E. Yildiz (2011): "Diffusion of Innovations in Social Networks," *Proc. of IEEE Conference on Decision and Control.*

Alesina, A., and P. Giuliano (2010): "The Power of the Family," *Journal of Economic Growth*, 15(2), 93–125.

——— (2014): "Family Ties," *Handbook of Economic Growth. Edited by Philippe Aghion and Steven N Durlauf.*

Ambrus, A., M. Mobius, and A. Szeidl (2014): "Consumption Risk-Sharing in Social Networks," *American Economic Review*, 104(1), 149–82.

Anderlini, L., and A. Ianni (1996): "Path Dependence and Learning from Neighbors," *Games and Economic Behavior*, 13(2), 141–177.

Bala, V., and S. Goyal (1998): "Learning from Neighbours," *Review of Economic Studies*, 65(3), 595–621.

Bandiera, O., and I. Rasul (2006): "Social Networks and Technology Adoption in Northern Mozambique," *Economic Journal*, 116(514), 869–902.

Banerjee, A., A. G. Chandrasekhar, E. Duflo, and M. O. Jackson (2013): "The Diffusion of Microfinance," *Science*, 341(6144).

——— (2014): "Gossip: Identifying Central Individuals in a Social Network," *MIT Department of Economics Working Paper No. 14-15.*

Beise, M. (2003): "Lead Markets: Drivers of the Global Diffusion of Innovations," Discussion Paper Series 141, Research Institute for Economics & Business Administration, Kobe University.

Birke, D., and G. Swann (2006): "Network effects and the choice of mobile phone operator," *Journal of Evolutionary Economics*, 16(1), 65–84.

Björkegren, D. (2015): "The Adoption of Network Goods: Evidence from the Spread of Mobile Phones in Rwanda," *mimeo.*

Blume, L. E. (1993): "The Statistical Mechanics of Strategic Interaction," *Games and Economic Behavior*, 5(3), 387–424.

——— (1995): "The Statistical Mechanics of Best-Response Strategy Revision," *Games and Economic Behavior*, 11(2), 111–145.

Centola, D. (2010): "The Spread of Behavior in an Online Social Network Experiment," *Science*, 329(5996), 1194–1197.

Conley, T. G., and C. R. Udry (2010): "Learning about a New Technology: Pineapple in Ghana," *American Economic Review*, 100(1), 35–69.

Dupas, P. (2014): "Short-Run Subsidies and Long-Run Adoption of New Health Products: Evidence From a Field Experiment," *Econometrica*, 82(1), 197–228.

Easley, D., and J. Kleinberg (2010): *Networks, Crowds, and Markets: Reasoning About a Highly Connected World.* Cambridge University Press, New York, NY, USA.

Economides, N., and C. Himmelberg (1995): "Critical Mass and Network Evolution in Telecommunications," *in G. Brock (Ed.) Toward a competitive telecommunications industry: Selected papers from the 1994 telecommunications policy research conference*, p. 4763.

Economides, N., and A. Skrzypacz (2003): "Standards Coalitions Formation and Market Structure in Network Industries," *mimeo.*

Ellison, G. (2000): "Basins of Attraction, Long-Run Stochastic Stability, and the Speed of Step-by-Step Evolution," *Review of Economic Studies*, 67(1), 17–45.

Fogli, A., and L. Veldkamp (2013): "Germs, Social Networks and Growth," *NBER Working Paper, w18470.*

Foster, A. D., and M. R. Rosenzweig (1995): "Learning by Doing and Learning from Others: Human Capital and Technical Change in Agriculture," *Journal of Political Economy*, 103(6), 1176–1209.

Galeotti, A., and S. Goyal (2009): "Influencing the influencers: a theory of strategic diffusion," *RAND Journal of Economics*, 40(3), 509–532.

Galeotti, A., S. Goyal, M. O. Jackson, F. Vega-Redondo, and L. Yariv (2010): "Network Games," *Review of Economic Studies*, 77(1), 218–244.

Goolsbee, A., and P. J. Klenow (2002): "Evidence on Learning and Network Externalities in the Diffusion of Home Computers," *Journal of Law and Economics*, 45(2), 317–43.

GORODNICHENKO, Y., AND G. ROLAND (2011): "Understanding the Individualism-Collectivism Cleavage and its Effects: Lessons from Cultural Psychology," *Invited paper at the XVIth Congress of the International Economic Association.*

GORODNICHENKO, Y., AND G. ROLAND (forthcoming): "Culture, Institutions and the Wealth of Nations," *Review of Economics and Statistics.*

GOYAL, S. (1996): "Interaction Structure and Social Change," *Journal of Institutional and Theoretical Economics*, 152(3), 472–494.

——— (2011): "Learning in Networks," *in the Handbook of Social Economics (edited by Benhabib, Bisin, Jackson). North Holland Press.*

GOYAL, S., AND A. VIGIER (2014): "Attack, Defence, and Contagion in Networks," *Review of Economic Studies*, 81(4), 1518–1542.

GREIF, A. (1993): "Contract Enforceability and Economic Institutions in Early Trade: the Maghribi Traders' Coalition," *American Economic Review*, 83(3), 525–48.

——— (2006): "Institutions and the Path to the Modern Economy: Lessons from Medieval Trade," *Cambridge University Press.*

GREIF, A., AND G. TABELLINI (2010): "Cultural and Institutional Bifurcation: China and Europe Compared," *American Economic Review*, 100(2), 135–40.

——— (2015): "The Clan and the Corporation: Sustaining Cooperation in China and Europe," *mimeo.*

GUITERAS, R., J. LEVINSOHN, AND A. M. MOBARAK (2015): "Encouraging sanitation investment in the developing world: A cluster-randomized trial," *Science*, 348(6237), 903–906.

HOFSTEDE, G. (2003): *Culture's Consequences: Comparing Values, Behaviors, Institutions and Organizations Across Nations.* SAGE Publications, Inc, 2nd edn.

JACKSON, M. O., AND D. LÓPEZ-PINTADO (2013): "Diffusion and Contagion in Networks with Heterogeneous Agents and Homophily," *Network Science*, 1(1).

JACKSON, M. O., T. RODRIGUEZ-BARRAQUER, AND X. TAN (2012): "Social Capital and Social Quilts: Network Patterns of Favor Exchange," *American Economic Review*, 102(5), 1857–97.

JACKSON, M. O., B. ROGERS, AND Y. ZENOU (2015): "The Economic Consequences of Social Network Structure," *CEPR Discussion Paper 10406.*

JACKSON, M. O., AND L. YARIV (2007): "Diffusion of Behavior and Equilibrium Properties in Network Games," *American Economic Review*, 97(2), 92–98.

——— (2011): "Diffusion, Strategic Interaction, and Social Structure," *in the Handbook of Social Economics (edited by Benhabib, Bisin, Jackson). North Holland Press.*

Karlan, D., M. Mobius, T. Rosenblat, and A. Szeidl (2009): "Trust and Social Collateral," *The Quarterly Journal of Economics*, 124(3), 1307–1361.

Kets, W., G. Iyengar, R. Sethi, and S. Bowles (2011): "Inequality and network structure," *Games and Economic Behavior*, 73(1), 215–226.

Killeen, G. F., T. A. Smith, H. M. Ferguson, H. Mshinda, S. Abdulla, C. Lengeler, and S. P. Kachur (2007): "Preventing Childhood Malaria in Africa by Protecting Adults from Mosquitoes with Insecticide-Treated Nets," *PLOS Medicine*, 4(7).

López-Pintado, D. (2008): "Diffusion in complex social networks," *Games and Economic Behavior*, 62(2), 573–590.

Miguel, E., and M. Kremer (2004): "Worms: Identifying Impacts on Education and Health in the Presence of Treatment Externalities," *Econometrica*, 72(1), 159–217.

Morris, S. (2000): "Contagion," *Review of Economic Studies*, 67(1), 57–78.

Munshi, K. (2004): "Social learning in a heterogeneous population: technology diffusion in the Indian Green Revolution," *Journal of Development Economics*, 73(1), 185–213.

Munshi, K., and J. Myaux (2006): "Social norms and the fertility transition," *Journal of Development Economics*, 80(1), 1–38.

Newton, J., and S. D. Angus (2015): "Coalitions, Tipping points and the Speed of Evolution," *Journal of Economic Theory*, 157, 172187.

Rogers, E. M. (2003): *Diffusion of Innovations*. Simon and Schuster International, 5th edn.

Valente, T. W. (1995): *Network Models of the Diffusion of Innovations*. Hampton Press, NJ.

Watts, D. J., and S. H. Strogatz (1998): "Collective Dynamics of 'Small-World' Networks," *Nature*, 393, 440–442.

Young, H. P. (1996): "The Economics of Convention," *Journal of Economic Perspectives*, 10(2), 105–122.

——— (2011): "The Dynamics of Social Innovation," *PNAS*, 108(4), 2128521291.

Young, H. P., and M. A. Burke (2001): "Competition and Custom in Economic Contracts: A Case Study of Illinois Agriculture," *American Economic Review*, 91(3), 559–573.

Appendix

Proof of Proposition 3

First note that there always exists at least one resilient set. Specifically, the empty set is always resilient. Let S_1, S_2 be two resilient sets. Consider $S = S_1 \cup S_2$. We show that S is resilient. Assume that it is not. That is, there exists $T \in \Omega$, $T \subseteq S$ such that for all $i \in T$, $I_i(S) - I_i(T) < 1 - \underline{Q}_i$. T must intersect S_1 and or S_2. Assume w.l.o.g. that $T \cap S_1 =: T_1 \neq \emptyset$. As $S_1 \supseteq T_1$, for all $i \in T_1$ we have that:

$$\begin{aligned}
I_i(S_1) - I_i(T_1) &= \frac{\sum_{j \in S_1} w_{ji} - \sum_{j \in T_1} w_{ji}}{\sum_{j \in N} w_{ji}} \\
&= \frac{\sum_{j \in S_1 \cup (T \setminus T_1)} w_{ji} - \sum_{j \in T_1 \cup (T \setminus T_1)} w_{ji}}{\sum_{j \in N} w_{ji}} \\
&= \frac{\sum_{j \in S_1 \cup T} w_{ji} - \sum_{j \in T} w_{ji}}{\sum_{j \in N} w_{ji}} \leq \frac{\sum_{j \in S} w_{ji} - \sum_{j \in T} w_{ji}}{\sum_{j \in N} w_{ji}} = I_i(S) - I_i(T) < 1 - \underline{Q}_i
\end{aligned}$$

which contradicts S_1 being resilient. Thus there exists a unique maximal resilient set S^*, in which all resilient sets S are set included $S \subseteq S^*$. For any $i \in R \subseteq N$ for any R which satisfies Lemma 2, then $i \in S^*$. For all other i, then $i \notin S^*$.

Denote

$$S^t := \{i \in N : a_i^t = 0\}.$$

Take any $t-1$ where $S^{t-1} \supseteq S^*$. Note that $S^0 \supseteq S^*$. Let $T \in \Omega$ be the coalition selected in period t according to a distribution $\Xi(.)$ which has full support on Ω. If $T \cap S^* = \emptyset$, then clearly $S^t \supseteq S^*$. Assume that $T \cap S^* \neq \emptyset$. Denote $T_1 := T \cap S^*$. Note that by the definition of Ω, $T_1 \in \Omega$. Seeking a contradiction, suppose

$$u_i(a_T = \mathbf{1}, a_{N \setminus T} = a_{N \setminus T}^{t-1}) > u_i(a^{t-1}) \quad \text{for all} \quad i \in T.$$

Then, for all $i \in T$,

$$\underline{Q}_i < Q_i(a_T = \mathbf{1}, a_{N \setminus T} = a_{N \setminus T}^{t-1}) \leq \frac{\sum_{j \notin S^t \setminus T} w_{ji}}{\sum_{j \in N} w_{ji}} \leq \frac{\sum_{j \notin S^* \setminus T} w_{ji}}{\sum_{j \in N} w_{ji}},$$

giving

$$\begin{aligned}
1 - \underline{Q}_i > 1 - \frac{\sum_{j \notin S^* \setminus T} w_{ji}}{\sum_{j \in N} w_{ji}} &= \frac{\sum_{j \in S^* \setminus T} w_{ji}}{\sum_{j \in N} w_{ji}} = \frac{\sum_{j \in S^*} w_{ji} - \sum_{j \in T \cap S^*} w_{ji}}{\sum_{j \in N} w_{ji}} \\
&= I_i(S^*) - I_i(T \cap S^*) = I_i(S^*) - I_i(T_1),
\end{aligned}$$

which contradicts resilience of S^*. So, there must exist $i \in T$ with $u_i(a_T = \mathbf{1}, a_{N \setminus T} = a_{N \setminus T}^{t-1}) \leq u_i(a^{t-1})$, implying $a^t = a^{t-1}$ and $S^t = S^{t-1}$. So, for all t, $S^t \supseteq S^*$.

Now, as S^* is the maximal resilient set, if $S^t \neq S^*$, there must exist $T \in \Omega$, $T \subseteq S^t$ such that for all $i \in T$,

$$1 - \underline{Q}_i > I_i(S^t) - I_i(T) = \sum_{j \in S^t} w_{ji} - \sum_{j \in T} w_{ji} = \sum_{j \in S^t \backslash T} w_{ji},$$

implying

$$\underline{Q}_i < \sum_{j \notin S^t \backslash T} w_{ji} = Q_i(a_T = \mathbf{1}, a_{N \backslash T} = a^{t-1}_{N \backslash T})$$

which implies $u_i(a_T = \mathbf{1}, a_{N \backslash T} = a^{t-1}_{N \backslash T}) > u_i(a^{t-1})$. The probability of such a T being selected by $\Xi(.)$ is bounded below by a strictly positive number, so with probability 1 such a set will be selected in finite time and will choose to protest, giving $S^\tau \subset S^t$ for some $\tau > t$.

Iterating, with probability one, a state a^t, $t \in \mathbb{N}_+$, will be reached such that $S^t = S^*$. By the arguments in the first paragraph of this proof, such a state must be absorbing.

Proof of Proposition 4

For population N, network $\mathcal{L}$ with weights w, set of coalitions Ω, common threshold $\underline{Q}$ and idiosyncratic components $\underline{\theta}$:

(i) Denote by $S^*(w, \underline{Q}, \theta, \Omega)$ the set of individuals who do not adopt under the unique absorbing state.

(ii) Define the threshold for adoption by i

$$\Lambda_i(w, \underline{\theta}, \Omega) = \min\{\underline{Q} : i \in S^*(w, \underline{Q}, \underline{\theta}, \Omega)\}.$$

(iii) Define an adoption path

$$S^0, S^1, ..., S^t, ...,$$

where S^t is the set of non-adopting nodes at any time t. A maximal adoption path is a path with $S^t = S^*(w, \underline{Q}, \theta, \Omega)$ for some finite t.

(iv) Define the maximal possible set of feasible coalitions after an increase in w_{ij}

$$\Omega(\max) \equiv \Omega \cup \{T : \{i, j\} \subset T, \quad T \backslash \{j\} \in \Omega, \text{ and } T \backslash \{i\} \in \Omega\}.$$

(v) Without loss of generality suppose $\Lambda_i(w, \underline{\theta}, \Omega(\max)) \geq \Lambda_j(w, \underline{\theta}, \Omega(\max))$.

Lemma 3 *For any $r \in N$, under network $\mathcal{L}$, with coalitions Ω, idiosyncratic components $\underline{\theta}$, and $\underline{Q} = \Lambda_r(w, \underline{\theta}, \Omega)$, there exists a coalition $T \in \Omega$ where $T \subseteq S^*(w, \underline{Q}, \underline{\theta}, \Omega)$ such that*

$$\forall\, k \in T, \quad I_k(S^*(w, \underline{Q}, \underline{\theta}, \Omega)) - I_k(T) \leq 1 - (\underline{Q} + \theta_k). \tag{1}$$

Select some such T which satisfies (1). If $r \in T$ then stop, and T terminates the iteration. If $r \notin T$

then there exists a coalition $T' \subseteq S^*(w, \underline{Q}, \underline{\theta}, \Omega) \backslash T$ *such that*

$$\forall\, k \in T', \quad I_k(S^*(w, \underline{Q}, \underline{\theta}, \Omega)\backslash T) - I_k(T') \leq 1 - (\underline{Q} + \theta_k). \tag{2}$$

Select some such T' *which satisfies (2). If* $r \in T'$ *then stop, and* T' *terminates the iteration. If* $r \notin T'$ *then repeat.*

By definition of $\Lambda_r(w, \underline{\theta}, \Omega)$, r does not adopt at $\underline{Q} = \Lambda_r(w, \underline{\theta}, \Omega)$, but adopts for all $\underline{Q} < \Lambda_r(w, \underline{\theta}, \Omega)$. For all $\underline{Q} < \Lambda_r(w, \underline{\theta}, \Omega)$, there must exist an adoption path such that at time t, $S^t = S^*(w, \underline{Q}, \underline{\theta}, \Omega)$, since any path up to some S^t available under $\underline{Q}$ is also available under all $\underline{Q}$ lower. Further, since the resilient set at all $\underline{Q} < \Lambda_r(w, \underline{\theta}, \Omega)$ does not include r, then the resilient set for all $\underline{Q} < \Lambda_r(w, \underline{\theta}, \Omega)$, must be a strict subset of $S^*(w, \underline{Q}, \underline{\theta}, \Omega)$.

To show that (1) must hold, suppose that when $\underline{Q} = \Lambda_r(w, \underline{\theta}, \Omega)$, for all $T \subseteq S^*(w, \underline{Q}, \Omega, \underline{\theta}, \Omega)$ there exists $k \in T$ such that

$$I_k(S^*(w, \underline{Q}, \underline{\theta}, \Omega)) - I_k(T) > 1 - (\underline{Q} + \theta_k).$$

Then this continues to hold at $\underline{Q} = \Lambda_r(w, \underline{\theta}, \Omega) - \epsilon$ for ϵ small enough. By definition of resilience, then $S^*(w, \underline{Q}, \underline{\theta}, \Omega)$ is resilient for some $\underline{Q} < \Lambda_r(w, \underline{\theta}, \Omega)$. A contradiction.

Now, for any $\underline{Q} < \Lambda_r(w, \underline{\theta}, \Omega)$, some T which satisfies (1) is selected at time $t+1$ with positive probability, and so there exists a feasible adoption path with $S^{t+1} = S^*(w, \underline{Q}, \Omega, \underline{\theta}, \Omega)\backslash T$. If $r \in S^{t+1}$, by the argument above, the resilient set is a strict subset of S^{t+1} for all $\underline{Q} < \Lambda_r(w, \underline{\theta}, \Omega)$, and there exists $T' \subseteq S^{t+1}$ such that

$$\forall\, k \in T', \quad I_k(S^{t+1}) - I_k(T') \leq 1 - (\Lambda_r(\tilde{w}) + \theta_k). \tag{3}$$

The argument repeats itself until the feasible adoption path has r adopt. □

<u>For weighted network w, coalitions $\Omega(\max)$, suppose $\nexists T$ where $\{i, j\} \subseteq T$ that satisfies (1)</u>

For weighted network w, coalitions $\Omega(\max)$, first take the case where $j \notin T$ for any possible terminating coalition in Lemma 3.

Then $S^*(w, \underline{Q}, \theta, \Omega) = S^*(w, \underline{Q}, \theta, \Omega(\max))$ for all $\underline{Q}$ and $\Lambda_k(w, \underline{\theta}, \Omega) = \Lambda_k(w, \underline{\theta}, \Omega(\max))$ for all k. Under w and $\Omega(\max)$, for all $\underline{Q} < \Lambda_i(w, \underline{\theta}, \Omega(\max))$, there exists a maximal path of adoption such that i adopts as part of a coalition T where $j \notin T$ and for all $\underline{Q} \geq \Lambda_i(w, \underline{\theta}, \Omega(\max))$, there exists a maximal path that does not include i or j. Clearly the same such path is available under w and Ω, thus $S^*(w, \underline{Q}, \theta, \Omega) \subseteq S^*(w, \underline{Q}, \theta, \Omega(\max))$ for all $\underline{Q}$. Since a reduction in the set of feasible coalitions (weakly) decreases total adoption, $S^*(w, \underline{Q}, \theta, \Omega) \supseteq S^*(w, \underline{Q}, \theta, \Omega(\max))$, for all $\underline{Q}$.

Next, show that under w, Ω, at $\underline{Q} = \Lambda_i(w, \underline{\theta}, \Omega)$ that $S^*(w, \underline{Q}, \theta, \Omega)$ remains resilient following a marginal increase in w_{ij}. By definition of resilience, under w for all $T \subseteq S^*(w, \underline{Q}, \theta, \Omega)$, $T \in \Omega$, there exists $k \in T$ such that

$$I_k(S^*(w, \underline{Q}, \theta, \Omega)) - I_k(T) = \frac{\sum_{l \in S^*(w,\underline{Q},\theta,\Omega) \setminus T} w_{lk}}{\sum_l w_{lk}} \geq 1 - \underline{Q} - \theta_k. \tag{4}$$

To show $S^*(w, \underline{Q}, \theta, \Omega)$ remains resilient, we show that the inequality in (4) holds following a marginal increase in w_{ij} not only for all $T \in \Omega$, but also $T \in \Omega(\max)$, since new coalitions may become feasible. For $k \neq i, j$ there is no change in (4) following a marginal increase in w_{ij}. For $k = i, j$ and T such that $\{i, j\} \nsubseteq T$ then the left hand side of (4) increases after a marginal increase in w_{ij} and (4) continues to hold. By assumption, for $k = i, j$ and for any $T \in \Omega(\max)$ such that $\{i, j\} \subseteq T$, then by Lemma 3, (4) holds strictly for some k, and so for a small enough marginal increase in w_{ij} (4) continues to hold.

Next examine the change in $\Lambda_j(w, \underline{\theta}, \Omega)$ if $\Lambda_i(w, \underline{\theta}, \Omega) > \Lambda_j(w, \underline{\theta}, \Omega)$. Under w, Ω, for all $\Lambda_j(w, \underline{\theta}, \Omega) \leq \underline{Q} < \Lambda_i(w, \underline{\theta}, \Omega)$, there exists an initial part of an adoption path $S^0, .., S^{t'}$ such that i adopts action 1 at time t'. For any $\underline{Q} < \Lambda_j(w, \Omega)$, there exists a continuation to this path denoted $S^{t'+1}, ..., S^{t''}, ...$ such that j adopts action 1 at time t'', and then a further extension to a maximal path of adoption. For all $k \in T$ for all coalitions T which adopt action 1 on this path at each time t:

$$I_k(S^{t-1}(w, \underline{Q}, \underline{\theta}, \Omega)) - I_k(T) = \frac{\sum_{l \in S^{t-1}(w,\underline{Q},\underline{\theta},\Omega) \setminus T} w_{lk}}{\sum_l w_{lk}} < 1 - (\underline{Q} + \theta_k). \tag{5}$$

Consider a marginal increase in w_{ij}. For any $\underline{Q} < \Lambda_j(w, \Omega)$, the maximal path of adoption described above continues to exist. For all $k \neq i, j$, (5) continues to hold since there is no change. For $k = i$, for w_{ij} small enough (5) holds, since before the marginal increase it holds for some $\underline{Q} > \Lambda_j(w, \Omega)$. Hence for a small enough increase in w_{ij} this path remains feasible.

Thus, for some small enough marginal increase in w_{ij} there exists $\epsilon \geq 0$ such that, for all $\underline{Q} < \Lambda_i - \epsilon$, i will adopt and, for all $\underline{Q} \geq \Lambda_i - \epsilon$, i will not adopt. It is then immediate from the above that for all $\underline{Q} < \Lambda_i - \epsilon$ total adoption is weakly higher and for all $\underline{Q} \geq \Lambda_i - \epsilon$ total adoption is weakly lower.

The remaining possibility is that for network w, coalitions $\Omega(\max)$ that $\{i, j\} \subseteq T$ for some terminating coalition in Lemma 3 (although by assumption, there is no such T that satisfies (1) where $\{i, j\} \subseteq T$). Following a marginal increase in w_{ij}, the inequality in Lemma 3 continues to hold for T where $\{i, j\} \subseteq T$. But for a small enough marginal increase in w_{ij} this coalition will not adopt until some coalition T that satisfies (1) adopts. Therefore under w and $\Omega(\max)$, adoption is invariant to a marginal increase in w_{ij}. The same holds if $\{i, j\} \subseteq T$ for some terminating coalition in Lemma 3 when the set of coalitions is Ω. If not, then we are in the situation above and the above threshold holds.

For network w, coalitions $\Omega(\max)$, suppose $\exists T$ that satisfies (1) where $\{i, j\} \subseteq T$

It follows immediately from Lemma 3 that $\Lambda_i(w, \underline{\theta}, \Omega(\max)) = \Lambda_j(w, \underline{\theta}, \Omega(\max))$. For network w and coalitions $\Omega(\max)$, for all $\underline{Q} < \Lambda_i(w, \underline{\theta}, \Omega(\max))$ there exists an initial part of a maximal path $S^0, ..., S^*(w, \Lambda_i(w, \underline{\theta}, \Omega(\max)), \underline{\theta}, \Omega(\max))$. By Lemma 3, there exists a continuation to this path such that a coalition T where $\{i, j\} \subseteq T$ adopts on the path. For all $k \in T$ for each coalitions T which adopts action 1 at each time t:

$$I_k(S^{t-1}(w, \underline{Q}, \underline{\theta}, \Omega(\max))) - I_k(T) = \frac{\sum_{l \in S^{t-1}(w, \underline{Q}, \underline{\theta}, \Omega(\max)) \setminus T} w_{lk}}{\sum_l w_{lk}} < 1 - (\underline{Q} + \theta_k). \tag{6}$$

Following an increase in w_{ij}, for $k \neq i, j$ the inequality (6) holds. For $k = i, j$ then on this path i and j both adopt as part of the same coalition T and the left hand side of inequality (6) decreases. Thus the same path remains feasible following a marginal increase in w_{ij} with $\Omega(\max)$ held fixed.

The equality $\Lambda_i(w, \underline{\theta}, \Omega) = \Lambda_j(w, \underline{\theta}, \Omega)$ follows from $\Lambda_i(w, \underline{\theta}, \Omega(\max)) = \Lambda_j(w, \underline{\theta}, \Omega(\max))$. If for network w and coalitions Ω there exists T that satisfies (1) where $\{i, j\} \subseteq T$, then by the same argument as above adoption cannot decrease following a marginal increase in w_{ij}. If not, but some such coalition becomes available following an increase in w_{ij} then as above. If not, and no such coalition becomes available following an increase in w_{ij}, then by the same arguments as the section above, $\Lambda_i(w, \underline{\theta}, \Omega) = \Lambda_j(w, \underline{\theta}, \Omega)$ is weakly decreasing in w_{ij}.

For $\underline{Q} < \Lambda_i(w, \underline{\theta}, \Omega)$ total adoption weakly decreases following a marginal increase in w_{ij}. Fix $\underline{Q} < \Lambda_i(w, \underline{\theta}, \Omega)$, then for all $T \subseteq S^*(w, \underline{Q}, \underline{\theta}, \Omega)$ there exists $l \in T$ such that

$$I_l(S^*(w, \underline{Q}, \underline{\theta}, \Omega)) - I_l(T) = \frac{\sum_{h \in S^*(w, \underline{Q}, \underline{\theta}, \Omega) \setminus T} w_{hl}}{\sum_h w_{hl}} \geq 1 - Q_l. \tag{7}$$

Following a marginal increase in w_{ij}, for any $l \neq i, j$ then (7) is invariant to a marginal increase in w_{ij}. Since $i, j \notin S^*(w, \underline{Q}, \underline{\theta}, \Omega)$ then for all $T \subseteq S^*(w, \underline{Q}, \underline{\theta}, \Omega)$ there exists $l \in T$ such that (7) holds, and further the marginal increase in w_{ij} does not create new coalitions contained in $S^*(w, \underline{Q}, \underline{\theta}, \Omega)$.

For $\underline{Q} \geq \Lambda_i(w, \underline{\theta}, \Omega)$ total adoption weakly increases following a marginal increase in w_{ij}. For any $\underline{Q} > \Lambda_i(w, \underline{\theta}, \Omega)$, take any feasible adoption path $S^0, ..., S^t, ...$ At each time t where a coalition T adopts the innovation, for all $k \in T$

$$I_k(S^{t-1}) - I_k(T) = \frac{\sum_{l \in S^{t-1} \setminus T} w_{lk}}{\sum_l w_{lk}} < 1 - (\underline{Q} + \theta_k). \tag{8}$$

For fixed S^{t-1} and any $k \neq i, j$ this expression is invariant to a marginal increase in w_{ij}. Therefore, since i, j do not adopt on this path, this path remains feasible following a marginal increase in w_{ij}.

Proof of Proposition 5

Given population N, network w, coalitions Ω, and a strongly-cohesive set R, let $\tilde{Q}$ be the highest value of $\underline{Q}$ at which no individual in the set R adopts, but some individual in R adopts for all $\underline{Q} < \tilde{Q}$. That is at $\underline{Q} = \tilde{Q}$: 1. in the absorbing state, $a_i = 0$ for all $i \in R$, and 2. in the absorbing state, for some set $T_1 \subset R$ and some coalition $T \in \Omega$, where $T_1 \subseteq T$ and $T \backslash T_1 \cap R = \emptyset$

$$\frac{\sum_{j \notin R} a_j w_{ij}}{\sum_j w_{ij}} + \frac{\sum_{j \in T} w_{ij}}{\sum_j w_{ij}} \geq \bar{Q}, \text{ for all } i \in T, \tag{9}$$

with equality for some $i \in T_1$. The whole set R will then adopt in the absorbing state for all $\underline{Q} < \tilde{Q}$. Suppose not, and let $\tilde{T} \subset R$ be the maximal set of individuals that do not adopt in the absorbing state for some $\underline{Q} < \tilde{Q}$. By definition of a strongly-cohesive set, $I_i(R) - I_i(\tilde{T}) > \tilde{Q}$ for some $i \in \tilde{T}$. But then since $a_j = 1$ for all $j \in R \backslash \tilde{T}$ in the absorbing state for any $\underline{Q} < \tilde{Q}$, then when $i \in \tilde{T}$ is selected to update he chooses $a_i = 1$. A contradiction.

Consider a marginal increase in w_{ij} for i and j both part of such a set T_1. The left hand side of (9) increases and so the inequality continues to be satisfied. Then the set R continues to adopt in the absorbing state following a marginal increase in w_{ij} and total adoption does not decrease, for all $\underline{Q} < \tilde{Q}$. Similarly, since R does not adopt in the absorbing state for any $\underline{Q} \geq \tilde{Q}$, then if no member of R adopts following a marginal increase in w_{ij} then there is no change in the absorbing state. However, if (9) becomes strict for all $i \in T$ following a marginal increase in w_{ij}, then T_1 will adopt in the absorbing state at $\tilde{Q}$ and total adoption strictly increases at this value.

Consider a marginal increase in w_{ij} for i and j where neither i nor j are part of any such set T_1 that satisfies (9). From above, the whole set R continues to adopt for all $\underline{Q} < \tilde{Q}$ since the inequality $I_i(R) - I_i(\tilde{T}) > \tilde{Q}$ remains strict following a small enough marginal increase in w_{ij}. The set R will never adopt for any $\underline{Q} \geq \tilde{Q}$ since (9) is never satisfied with a strict inequality for any such T_1. There is no change in total adoption at any $\underline{Q}$.

Consider a marginal increase in w_{ij} where i is part of some set T_1 that satisfies (9) and j is not part of the same set T_1. The left hand side of (9) weakly decreases for i and the condition may no longer hold. If some other set continues to satisfy this condition following a marginal increase in w_{ij}, then the effect is the same as a marginal increase in w_{ij} for i and j where neither are part of any such set T_1 (above). If no other group satisfies the condition following a marginal increase in w_{ij}, then total adoption may decrease for some $\underline{Q}$ less than but approaching $\tilde{Q}$. There is no change for the remaining $\underline{Q} \geq \tilde{Q}$. Set $Q_{ij} = \tilde{Q}$, then total adoption is weakly increasing for $\underline{Q} \geq \tilde{Q}$ and weakly decreasing for $\underline{Q} < \tilde{Q}$.

For some other strongly-cohesive set R', let $\tilde{\tilde{Q}}$ be the highest value of $\underline{Q}$ at which no individual in the set R' adopts in the absorbing state, but some individual in R' adopts in the absorbing state for all $\underline{Q} < \tilde{\tilde{Q}}$. Suppose $\tilde{Q} > \tilde{\tilde{Q}}$ and consider a marginal increase in w_{ij}. Suppose condition (9) continues to be satisfied for some set T_1, then by the same argument above, the set R adopts in the absorbing state for all $\underline{Q} < \tilde{Q}$ but not for $\underline{Q} \geq \tilde{Q}$. Suppose condition (9) is no longer satisfied for any set T_1, then as above, following a small enough marginal increase in w_{ij} total adoption may decrease for some $\tilde{\tilde{Q}} < \underline{Q} < \tilde{Q}$, where $\underline{Q}$ approaches $\tilde{Q}$. If some member of R adopts the whole set adopts. There is no change for $\underline{Q} \geq \tilde{Q}$. What happens to the set R'? If j is part of some T_1 that satisfies (9) for R', then the left hand side of (9) weakly increases for j and the condition is possibly satisfied with strict equality for all members of the coalition. Then the set T_1 will adopt at $\underline{Q} = \tilde{\tilde{Q}}$ and total adoption weakly increases for all $\underline{Q} \leq \tilde{\tilde{Q}}$ following a marginal increase in w_{ij}. Set $\tilde{\tilde{Q}} \leq Q_{ij} < \tilde{Q}$, then total adoption is weakly increasing for $\underline{Q} \leq Q_{ij}$ and weakly decreasing for $\underline{Q} > Q_{ij}$.

Suppose instead $\tilde{Q} = \tilde{\tilde{Q}}$. Then condition (9) holds for at least one set $T_1 \subseteq R$ and at least one set $T_1' \subseteq R'$. Suppose there is no set $T \in \Omega$ which satisfies condition (9) and contains both sets $T_1 \subseteq R$ and $T_1' \subseteq R'$. By the same argument as above, a marginal increase in w_{ij} either has no effect on total adoption or total adoption may decrease for some $\underline{Q}$ less than but approaching $\tilde{Q}$. If instead there exists a set $T \in \Omega$ which satisfies condition (9) and contains both sets $T_1 \subseteq R$ and $T_1' \subseteq R'$, then again as above, a marginal increase in w_{ij} may result in adoption in R and R' at $\underline{Q} = \tilde{Q}$.

Micro-founding the Threshold of Adoption $\underline{Q}$

We micro-found the parameter $\underline{Q}$ to understand how its value is affected by other parameters. Assume homogeneous individuals.

1. Network effects versus independent value.

Fix the utility for individual i from the current technology at some value $\underline{u} > 0$. Suppose the value of the new technology derives solely from interaction with other users (for example messaging applications). The value to i of adopting the new technology if none of his links adopt ($Q_i = 0$), must be no greater than zero, denoted $v(Q_i = 0) \leq 0$. Assume this is increasing at rate $\partial v(Q_i)/\partial Q_i > 0$. Then $\underline{Q}$ is the unique value of Q_i that satisfies $v(Q_i) = \underline{u}$. Hold fixed the technology but assume it now has some value independent of whether others use the same technology or not. Let this value to individual i be $\delta > 0$. Then utility from the new technology is $v(Q_i) + \delta$ with $\partial v(Q_i)/\partial Q_i > 0$ unchanged. Now $\underline{Q}$ is the unique value of Q_i that satisfies $v(Q_i) + \delta = \underline{u}$. $\underline{Q}$ is strictly lower.

2. Complementarities and pure externalities.

The utility of individual i when he adopts the technology depends on the proportion of contacts who have adopted, $v(Q_i)$. Similarly i's utility when he does not adopt the technology can depend on the proportion of contacts who have adopted the new technology, $u(Q_i)$. Suppose $u(0) > v(0)$ so that i will not adopt if no one else adopts. Then $\partial v(Q_i)/\partial Q_i > 0$ measures how the utility from adopting the technology increases as others adopt, which we refer to as the degree of complementarity in the technology. While $\partial u(Q_i)/\partial Q_i > 0$ measures how the utility when i does not adopt the new technology increases as others adopt the new technology. This represents pure externalities. Suppose $\partial v(Q_i)/\partial Q_i < \partial u(Q_i)/\partial Q_i$ for all Q_i. Then i will never adopt the new technology. Let us consider the relevant case for our framework where $\partial v(Q_i)/\partial Q_i \geq \partial u(Q_i)/\partial Q_i$ for all Q_i and there exists a unique value of $\underline{Q}$ that satisfies $v(\underline{Q}) = u(\underline{Q})$ that occurs for $\underline{Q} \leq 1$. A weak increase in the degree of complementarity for $\partial v(Q_i)/\partial Q_i$ for all Q_i weakly reduces the threshold of adoption $\underline{Q}$, while a weak increase in pure externalities $\partial u(Q_i)/\partial Q_i$ for all Q_i weakly increases the threshold of adoption $\underline{Q}$.