

Liquidity channels and stability of shadow banking*

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Job Market Paper

Abstract

I focus on interaction between traditional and shadow banks before and after a financial crisis in a model that incorporates the central bank and the money market investors – who can run on the shadow banks. In case of a crisis, traditional banks may borrow from the central bank in an unlimited amount and then rescue struggling shadow banks. The banks and the investors act strategically and farsightedly. The equilibrium notion that is considered (K-level farsightedness) is an extension of pairwise and farsighted stability used in cooperative game theory. In equilibrium, financial market endogenously develops a core-periphery structure, such that the rescue liquidity to shadow banks is provided by core traditional banks. I show that core banks charge zero spread for their bailout services and earn a higher interest on investments. They default due to contagion when the counterparty shadow bank defaults. The paper suggests policies which increase stability of traditional banks, such as lowering the cap on exposure between shadow and traditional banks or raising the central bank liquidity support rate, but these tend to conflict with the stability of shadow banks. In order to minimize the expected number of defaulting banks in both sectors, government should control the number of core banks and the quality of the risky assets.

Keywords: Systemic risk, Economic networks, Financial networks, Shadow banking, Bank run, Central bank

JEL: D81, D85, G21, G23, G28, E58

*This paper would be impossible without the continuous support of F. Page, R. Becker, and F. Garcia. The author is also grateful for very helpful discussions with A. Ellul, B. Craig, and V. Skavysh.

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1 Introduction

In recent years, the presence of various regulatory arbitrages across the globe facilitated a development of shadow banking¹ – a network of financial intermediaries outside the traditional banking system. By 2014, the size of shadow banking grew to one half of the total financial intermediation in the United States and a quarter of the total financial intermediation in the world (IMF (2014).) Similar to traditional banks, shadow banks perform a general function of liquidity, credit, and maturity transformation. However, while the assets of traditional banks are relatively safe and financed with stable deposits, the assets of most shadow banks are risky and financed by short-term instruments from investment funds and other money market investors. Moreover, shadow banks are prone to investor runs, as these banks have no access to the lender of last resort and are not covered by deposit insurance. In the case when a money market run is caused by an overall market event, such as an asset market shock, shadow banks may require significant liquidity support from outside of the shadow banking.

In the recent 2007-2009 financial crisis, the problems of runs and of liquidity crunch were solved by the central bank and the U.S. government providing liquidity to shadow banks². This liquidity provision eliminated the run on money market funds and allowed the interest rates to stabilize. However, if the central bank always bails out shadow banks in the case of a crisis, it will create a moral hazard problem and increase inefficiency. Therefore, it is important to study different ways in which liquidity can reach shadow banks. In particular, I would like to answer the question of whether traditional banks would be willing to serve as liquidity conduits from the central bank to the shadow banks in case of a crisis caused by a sudden devaluation of risky assets.

¹I use the definition provided by the Financial Stability Board (2014): shadow banking is the “credit intermediation involving entities and activities (fully or partially) outside the regular banking system.” In this paper, I distinguish shadow banks from traditional banks based on these characteristics: shadow banks can invest in risky assets, have no access to the lender of last resort, do not provide deposits, and, as a result, are not covered by the deposit insurance.

²For example, Commercial Paper Funding Facility extended access to the discount window to issuers of commercial paper; Primary Dealer Credit Facility extended access to the discount window to primary dealers; Term Securities Lending Facility lent Treasury securities to primary dealers in exchange for less liquid collateral; Term Asset-Backed Securities Loan Facility extended cheap credit to large investors – including hedge funds and private equity firms – so that these could jump-start the ABS market.

While it is quite unlikely that stressed traditional banks would bail out shadow banks on their own, it is possible that the traditional banks would be willing to take additional counterparty risk in case when the central bank provides liquidity to them.

This liquidity provision scheme is similar to some extent to the AMLF³ program organized by the Federal Reserve Bank during the recent financial crisis. Within the AMLF program, the Federal Reserve provided liquidity support to struggling money market mutual funds (MMMFs) indirectly by lending to regulated financial institutions so that they could purchase asset-backed commercial paper (ABCP) from the MMMFs. This paper proposes a similar idea to encompass generic shadow banks and not just MMMFs.⁴

The model assumes that the sequence of crisis events is triggered with an asset market crash which happens with a certain probability. The shock hits the distribution of the risky asset returns by shifting it toward lower values. As a result, the shadow banks that invested in the risky assets are stressed by the market crash. Consequently, money market investors strategically run on liquidity and create a liquidity crunch problem for the shadow banks. As a result of this run, shadow banks are forced to seek liquidity support from traditional banks. Traditional banks, then, choose strategically whether to bail out the shadow banks or not. Traditional banks are unable, in most cases, to rescue shadow banks independently of the central bank because they are financially constrained themselves. In this model, traditional banks can serve as liquidity conduits from the central bank to shadow banks if they choose strategically to do so. This liquidity scheme is different from the direct liquidity support from the central bank to shadow banks since it reallocates counterparty risk from taxpayers to the traditional banks and, as a result, creates a monitoring mechanism inside the banking system. Under this monitoring mechanism, the long-term relationship formed between a traditional

³Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility. For additional details see http://www.federalreserve.gov/newsevents/reform_amlf.htm

⁴I should emphasize that this model is not meant to be a recreation of the 2007-2009 financial crisis, but rather a study of different financial scenarios which the central bank and the government may consider. One way in which this model differs from the aforementioned financial crisis is that traditional banks are assumed not to borrow from the money market and not to invest in risky assets directly, while during the financial crisis the balance sheets of traditional banks were directly affected by the deterioration of risky assets. Due to this assumption, the risks that traditional banks face at the time of the market crash are only associated with interbank exposures and shadow banks.

bank and a shadow bank before the crisis is determined by whether the traditional bank will bail out the shadow bank or not. I determine the conditions needed for a bail out to occur and find stable financial networks before and during the crisis period. Moreover, I characterize the contagion process and determine a set of banks (both traditional and shadow) that can default.

Given that the market anticipates liquidity support during the crisis, the economic incentives of both traditional and shadow banks before the crises are affected accordingly. I begin my analysis by considering one shadow bank and one traditional bank. I find that there are two regimes under which the traditional bank may transfer liquidity from the central bank to the shadow bank. Under the first regime, the traditional bank has a positive probability of default due to contagion while the shadow bank is highly exposed to the money market. Under the second regime, the traditional bank stays solvent while the shadow bank is less exposed to the money market. I provide conditions under which each regime takes place. Although the second regime is more favorable from the viewpoint of systemic risk, this regime is not observed when I consider a network of many banks rather than a model with two banks. The reason for this is that, in the financial network formation game, a large coalition of banks delegates the bail out function to one bank for each shadow bank. In turn, the banks with the bail out function get overexposed to the shadow banks and cannot avoid defaulting as a result of contagion.

In my main theorem, I show that the economic incentives are likely to induce a network of bank relations with the core-periphery structure. The core traditional banks, defined as the most interconnected banks, provide the rescue liquidity channels to shadow banks and transfer liquidity from traditional banking to shadow banking. Although the initial model characteristics of all banks are identical, the core traditional banks make larger profits than periphery traditional banks while the market is stable but are exposed to higher default risk due to excessive counterparty risk and financial contagion. The core banks charge zero spreads for their bail out services but get a higher interest on loans from shadow banks than the non-core banks. It is important to notice that the assumption of limited liability is crucial for this result to exist because limited liability mitigates the counterparty risk of the overexposed traditional banks.

I suggest mechanisms by which the government can control stability of banks within this model. In particular, I consider the effectiveness of caps on the exposure of traditional banks to shadow banks, central bank's liquidity support rate, and other policies. Policies which can increase the stability of the traditional banking sector include: (a) limiting the exposure from traditional to shadow banking, (b) increasing the central bank's liquidity support rate, and (c) discouraging investors from lending on the money market in favor of traditional bank deposits. However, some of these policies discourage traditional banks from rescuing shadow banks and, therefore, conflict with the stability of the shadow banking sector. If the regulators are also concerned with the stability of shadow banking, additional policies should be considered to reduce the overall riskiness of the banking industry. If the probability of risky asset default following a market crash is high, the regulators can minimize the expected number of defaults in both sectors by reducing the liquidity support. This can be done by reducing the number of core banks. Conversely, if the probability of risky asset default is low, the presence of many core financial institutions and indirect bail outs is beneficial for the stability of both banking sectors. Therefore, this result suggests that the liquidity provision scheme considered in this paper can be favorably applied in the real world if the shadow bank assets are of high quality.

Finally, this paper sheds light on the origins of the "too big to fail" problem. In particular, although in the model the exogenous parameters of all traditional banks are identical, large interconnected (core) banks form endogenously as a result of strategic behavior. To the best of my knowledge, this is the first paper which shows that large interconnected banks can arise as a result of interaction between shadow banks and traditional banks as they try to establish liquidity channels among themselves.

The remainder of the article is structured as follows. Section 2 provides a brief literature review of both empirical and theoretical papers. Section 3 supplies details of the model. Section 4 defines the notion of farsighted equilibrium. Section 5 presents theoretical results. Section 6 concludes the paper. Finally, the Appendices contain proofs of the theorems and additional insights.

2 Literature review

The paper is motivated by a series of market events that took place during the 2007-2009 financial crisis and relevant theoretical and empirical articles. While the majority of the considered empirical literature is focused on the period of great recession in the United States, this paper is relevant to any financial market with a sufficient regulatory arbitrage and a major liquidity risk. In particular, the fast development of shadow banking in such countries as China, South Korea, Turkey, and Argentina,⁵ verify that the problem of interaction between regulated and unregulated financial institutions becomes an international concern.

The structure of shadow banking is very complex and highly interconnected (Pozsar et al. (2010)). Therefore there are various ways in which the interaction between regulated and shadow banks can be considered. In this paper, I simplify the analysis to considering the interaction between safe regulated banks and risky unregulated (dealer) banks before and after the money market liquidity run.⁶ The money market runs are similar to the traditional bank runs that existed in the United States until 19th century and were stopped by providing the Federal Deposit Insurance to the depositors. The runs on unregulated banks during the recent time period have been documented by a number of empirical papers. I mention only few of them as an anecdotal evidence. Gorton and Metrick (2010, 2012) have showed that runs on bilateral repurchase agreements (repo) were at the heart of the financial crisis. Krishnamurthy, Nagel and Orlov (2011) also claimed that financial market experienced a contraction in the short-term funding. They found that the Money Market Funds have reduced the liquidity provision mainly through the liquidity crunch in the market of asset backed commercial papers (ABCP). Copeland, Martin and Walker (2014) concluded that runs on tri-party market are likely to happen precipitously, because tri-party investors prefer to withdraw the funding rather than change the terms of the repo contracts. Chernenko and Sunderam (2014) observed a “quiet run” in the money market during the European sovereign debt crisis.

⁵See a report on shadow banking worldwide at [and](#)

⁶I use the definition provided by The Financial Stability Board (2014): shadow banking is the “credit intermediation involving entities and activities (fully or partially) outside the regular banking system”.

The theoretical approach of this paper is consistent with the strategic network formation literature and the banking literature assuming that agents act strategically. The bank runs were first modeled by Diamond and Dybvig (1983) who showed that, in the absence of deposit insurance, runs can occur as a result of self-fulfilling expectations. Similar to their approach, I assume that the bank runs may be caused by the shift in investors' expectations about the asset payoff.

The fragility of the financial system was further developed in the literature with the help of the network approach. Allen and Gale (2000) were among the first to show that typical financial networks are fragile and subject to financial contagion. Acemogly, Ozdaglar, Tahbaz-Salehi (2015) found that exogenously given dense networks are more robust to small shocks, while less robust to large market shocks. Elliott, Golub, Jackson (2014) showed that the density of network has a concave effect on the fragility of the network. Allen, Babus, and Carletti (2009) developed an endogenous model of network formation and showed that the network structure plays an especially important role for the economic welfare in the market with the short term financing.

While many other papers have been written on the topic of financial networks, my results are most relevant to the recent work of Farboodi (2014) which provided a theoretical explanation for the existence of a core-periphery network structure. The author focused on the network formation based on risk sharing incentives, while I provide the model of network formation and financial contagion due to both liquidity risk and asset risk. The core-periphery result of my paper is also consistent with the empirical work of Craig and von Peter (2014) that showed the bank specialization and balance sheet characteristics determine the banks position in the network and lead to the core-periphery structure of the market.

The methodology that I use to find the equilibria comes from the cooperative game theory. In particular, I use a notion of farsighted stability, intuition for which was first introduced by Harsanyi (1974), and further developed by Chwe (1994). Farsighted behavior has been incorporated into strategic network formation by Page, Wooders and Kamat (2005). It was further developed by Dutta, Ghosal, and Ray (2005), Herings, Mauleon, and Vannetelbosch (2009), and Ray and Vohra (2015). I consider a special case of farsighted stability, where players can see only a finite (but large) number of

deviations ahead of them without exact restrictions on the number of steps. In particular, I use the level-K farsighted equilibrium notion introduced by Herings, Mauleon, and Vannetelbosch (2015).

3 Model

Regulatory arbitrage and model overview

The banking sector is populated by traditional banks $B = (1, \dots, N)$ and shadow banks $B^s = (N + 1, \dots, N + N^s)$. Traditional banks are regulated because of their central position in financing the real economy. This leads to three key differences between traditional and shadow banks. First, regulations require traditional banks to have only safe high quality assets, while shadow banks can make risky investments. Second, shadow banks may experience runs on their liquidity since their investors' money is not insured by the government. Third, traditional banks can count on government support in the form of the lender of last resort, while shadow banks do not have direct access to the central bank. These regulatory differences lead to markedly different behavior of the two types of banks and to a complex interplay between them.

All three differences make shadow banking less stable than traditional banks. Instability comes from both asset risk and liquidity risk. The liquidity risk occurs due to the fact that money market investors provide mostly short-term lending to the shadow banks. Given that the investors' money is not insured by the government, a significant shock to the asset return may cause the investors to strategically withdraw their funds before the maturity day. Despite the inherent instability of the shadow banks, the regulatory arbitrage created by the requirement on asset quality still makes it profitable for traditional banks to lend to shadow banks.⁷ This, in turn, makes traditional banks less stable. Massive runs on shadow banks may propagate to traditional banks through interbank loans. Thus, the contagion caused by the money market run may lead to three major scenarios: only shadow banks default, both traditional banks and shadow

⁷Although the liquidity transfer from traditional banking to shadow banking may take the form of debt, sponsorship support, or equity, I assume that the liquidity transfer is done only through interbank loans in order to simplify the model.

banks default, or the shock is absorbed by the market when traditional banks decide to re-finance shadow banks. In this paper, I provide the conditions for the endogenous bank runs, analyze how damaging the effect of runs can be, and describe how far the contagion avalanche spreads when banks strategically form their balance sheets. I also find the network of lending relationships which forms before and after the shock.

Time structure

The model has three time periods. At time $t = 1$, players make investment decisions and determine interbank relationships. At time $t = 2$, the risky asset market may crash with a certain probability. The expectations of the asset returns are updated following the event. Given the updated expectations, money market investors decide if they want to withdraw or to keep money in the shadow banks. At the same time, traditional banks decide if they want to provide liquidity support for the troublesome shadow banks. At time $t = 3$, the assets pay off, profits are delivered to the players, and the set of defaulted banks is determined.

Financial assets and market crash

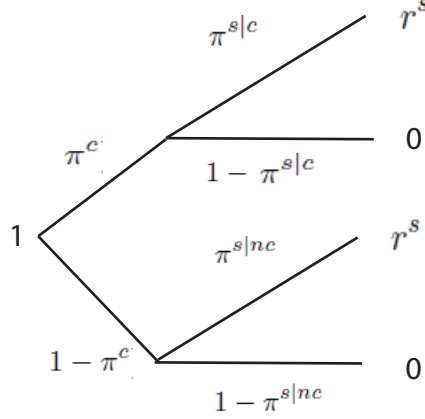
At time $t = 1$, shadow bank $i \in B^s$ invests in a risky long-term asset that pays off at time $t = 3$. The risky asset generates the gross return r^s in case of “success” and zero return in case of “failure”. The probability of success is determined by the market conditions at time $t = 2$. In particular, if a market crash is avoided at time $t = 2$, the asset pays off rate r^s with probability $\pi^{s|nc}$ and zero with probability $1 - \pi^{s|nc}$. If the market crash happens, the probability of success decreases to $\pi^{s|c}$, such that $\pi^{s|c} < \pi^{s|nc}$ (see Figure 1).

To shorten the mathematical expressions, I will use the joint probabilities rather than conditional probabilities when speaking about the market events:

$$\pi^{s,c} = \pi^{s|c}\pi^c, \quad \pi^{s,nc} = \pi^{s|nc}(1 - \pi^c),$$

$$\pi^{ns,c} = (1 - \pi^{s|c})\pi^c, \quad \pi^{ns,nc} = (1 - \pi^{s|nc})(1 - \pi^c).$$

Figure 1: The structure of the risky asset returns



I will also use the marginal probabilities:

$$\pi^s = \pi^{s,nc} + \pi^{s,c}, \quad \pi^{ns} = \pi^{ns,nc} + \pi^{ns,c},$$

$$\pi^c = \pi^{s,c} + \pi^{ns,c}, \quad \pi^{nc} = \pi^{s,nc} + \pi^{ns,nc},$$

By assumption, the unconditional expected return of the risky asset is greater than that of cash. I also assume that in the case of the market crash, the asset significantly depreciates. In other words:

Assumption 1. *The expected rate of return from the risky asset in the state of “no crash” exceeds the rate of return from deposits:*

$$r^s \pi^{s,nc} > r^{dep}.$$

In this paper, I assume that the risky assets of shadow banks have the same distribution, however each shadow bank invests in a different risky asset. It means that a default of one shadow bank is not necessarily accompanied by a default of another shadow bank.

Assumption 2. *Rates of return of two different shadow banks are not equal random variables.*

In addition, rather than consider many small shocks at $t = 2$, I consider one big market event, which affects the distribution of all risky assets in the same way.

Traditional banks

The main function of traditional banks $B = (1, \dots, N)$ is to facilitate financial intermediation between depositors and borrowers in the real economy. I assume that the rates on the loans and the deposits are set on the competitive basis at levels r^l and r^{dep} and that all banks issue the same amount of loans q^l to the real economy. For simplicity, the risk of traditional loans is reduced to zero. This assumption is consistent with the logic that economy's risky assets are being managed by the shadow banks. In this model, we do not consider a risk of traditional bank (deposit) run due to the deposit insurance provided by the regulators. Under these assumptions, we can say that the solvent traditional bank expects to gain profit v from performing the regulated activity:

$$v = (r^l - r^{dep})q^l.$$

For the rest of the paper, we will use v as a parameter of the model.

The traditional banks may also lend to any other bank with an endogenous interest rate. I assume that the interbank loans issued at time $t = 1$ pay off at $t = 3$. The bank may also serve as an intermediary by passing liquidity between banks. I designate a loan amount from bank $i \in B$ to bank $j \in B \cup B^s$ as q_{ij} and the corresponding interest rate as r_{ij} . Bank $i \in B$ may also provide additional liquidity support to $j \in B \cup B^s$ at time $t = 2$. I designate a support loan amount from bank $i \in B$ to bank $j \in B \cup B^s$ as q_{ij}^b and the corresponding interest rate as r_{ij}^b . The additional liquidity support may come to a traditional bank at $t = 2$ from two different sources: other banks or the central bank. The liquidity from the central bank can be borrowed in the unlimited amount at rate r^{cb} . I assume that financing through the central bank is expensive. In particular:

Assumption 3. *The expected rate of return of a risky asset given market crash is below one:*

$$r^s \pi^{s|c} < 1.$$

If the traditional bank is not able to repay the debt to the central bank or any other counterparty, it defaults. Limited liability condition is assumed for the defaulted banks, meaning that the utility payoff of the bankrupted bank is zero. If the bank possesses any cash at the time of default, it distributes the cash to the current creditors and defaults with zero cost of default. In reality, a default procedure of a traditional bank may be very complicated partially due to the fact that default costs are not zero and a deposit insurance company will get involved in the debt settlement process. In our model, we keep the default procedure simple: when repaying a debt, traditional bank favors current debt to long-term debt and depositors to banks. If the amount of available liquidity is not sufficient to compensate depositors, the rest is covered by the government. If multiple traditional and shadow banks demand liquidity, the defaulting bank pays them in the proportion to their corresponding debt obligations, according to the order that we mentioned earlier.

Traditional bank maximizes expected profit from interaction with both financial market and real economy. We assume that the amount of liquidity passing from traditional banking to shadow banking is regulated by the government. In particular, the amount of exposure from traditional bank $i \in B$ to all shadow banks B^s backed by depositors should not exceed a certain threshold \bar{q} set by the regulator. In this paper, we will consider different values of parameter \bar{q} to test the efficiency of government regulation.

To eliminate unrealistic equilibria, we assume that traditional banks get a loan from the central bank only if there is a positive probability that the loan will be repaid.

Shadow banks

The utility of a shadow bank $i \in B^s = (N + 1, \dots, N + N^s)$ is defined as the expected profit generated at time $t = 3$. Shadow banks finance their investments using funding from private banks $B \cup B^s$ and money market investors \mathbf{M}_i . A loan between shadow bank $i \in B^s$ and private bank $j \in B \cup B^s$ in the amount of q_{ij} is priced at the individual rate r_{ij} , which is determined endogenously. The number of money market investors is assumed to be large, so the shadow bank $i \in B^s$ surrounded by money market investors \mathbf{M}_i has the same borrowing rate r_i^m for all money market investors. It is equivalent

to saying that a particular money market investor $M \in \mathbf{M}_i$ decides to join or not to join the investment fund that lends to shadow bank $i \in B^s$ at rate r_i^m . The amount ϕ_i borrowed by bank $i \in B^s$ from the money market and the money market interest rate r_i^m are determined endogenously, and connected through the money market supply schedule $r_i^m(\phi_i)$. Hereafter, I will say interchangeably that $i \in B^s$ chooses an optimal r_i^m given the supply schedule $\phi_i(r_i^m)$ or $i \in B^s$ chooses an exposure to the money market ϕ_i and interest rate $r_i^m(\phi_i)$ clears the supply schedule. The specifics of the supply schedule are provided in the following section.

Shadow banks default when they are not able to repay current creditors. It is assumed that a bank always has a small amount of cash ε available. The additional amount of emergency liquidity can be obtained from the interbank loans. Given a positive amount of cash, a shadow bank first uses available cash to repay current liabilities, and defaults with no cash in hand. Long-term risky asset is assumed to be illiquid. Given that for a defaulting bank the size of current liabilities exceeds the size of available liquidity, the debt will be repaid to creditors at random with the probabilities proportional to the debt sizes. Once the bank announces bankruptcy, it repays zero to the rest of creditors. This simplifying assumptions eliminate unnecessary complications regarding bankruptcy procedure, which is not the focus of this paper.

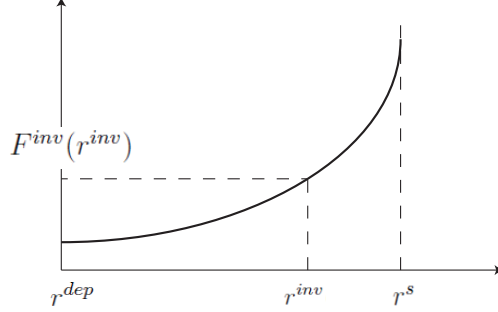
Money market investors

Each shadow bank $i \in B^s$ can borrow from an associated set of investors $\mathbf{M}_i \in [0, \infty)$. Each of the investors $M \in \mathbf{M}_i$ is endowed with the liquidity of measure one and maximizes its expected return at the end of the game. Investor $M \in \mathbf{M}_i$ chooses between lending to the shadow bank and investing elsewhere at the rate $r^{inv}(M)$, which I will call the individual risk-free rate. Investors \mathbf{M}_i are ranked according to their individual risk-free rates $r^{inv}(M)$ in ascending order. For simplicity, we assume that the individual risk-free rates have cumulative exponential distribution⁸ defined as:

⁸The exponential property of the money market supply is not crucial in the model. The crucial condition is that the distribution of investors types $F^{inv}(x)$ is assumed to be log-concave.

Log-concavity is a very weak assumption, which is satisfied for many distributions defined for valid x , including uniform distribution, large number of Gamma distributions, logistic distribution, exponential distribution, and many others. The log-concavity assumption is equivalent to the assumption that the

Figure 2: Distribution F^{inv} of money market investor types



$$F^{inv}(r^{inv}) = F_0 \exp\left(\frac{r^{inv} - r^{dep}}{\lambda}\right) \quad (1)$$

for $r^{inv} \geq r^{dep}$, and $F^{inv}(r^{dep}) = F_0$ when rate $r^{inv} = r^{dep}$ (see Figure 2). Function $F^{inv}(r^{inv})$ defines the volume provided by the money market investors with individual risk-free rate r^{inv} or lower.

An important feature of the model is the maturity mismatch in shadow bank operations. While the risky asset has to be held until $t = 3$, the money market investors can withdraw their liquidity at $t = 2$. I assume that withdrawing the funds incurs a penalty in the form of opportunity cost. In particular, if one unit of liquidity was invested at time $t = 1$, a maximum of one unit can be withdrawn at time $t = 2$. The exact amount withdrawn will depend on the amount of cash available at the bank at time $t = 2$. Liquid bank $i \in B^s$ repays to the running creditors in full. If bank $i \in B^s$ does not have

inverse Mills ratio of distribution F^{inv} is decreasing. This assumption is required to guarantee that the solution of the first order conditions provides a unique maximum point.

I first remind that inverse Mills ratio is defined as the ratio of probability distribution function to the cumulative distribution function:

$$h = \frac{dF^{inv}/dt}{F^{inv}}.$$

The inverse Mills ratio is sometimes called selection hazard rate and is used in statistics to account for the selection bias and in engineering to determine the probability that a system collapses. In statistics literature, it is assumed that h is decreasing (see Heckman (1979)). I deal with the extreme case when hazard rate is constant ($h = 1/\lambda$) to solve the model explicitly.

enough liquidity to repay the creditors at $t = 2$, it defaults and distributes available liquidity among those who withdraw. Therefore, the actions of each investor are to choose among “invest” and “not invest” at time $t = 1$ and among “run” and “not run” at time $t = 2$ if the market crash happens.

The supply schedule of the money market, $\phi(r_i^m)$, can be derived from the cumulative distribution (1). Given money market rate r_i^m and the strategies of all market players, it is possible to determine the expected rate that investors face at time $t = 1$. Without loss of generality, we denote this rate as $E[r^{m,i}(M)]$ for each investor. It is reasonable to consider $E[r^{m,i}(M)] > r^{dep}$. Therefore, the number of investors that are willing to lend to bank i at this rate at time $t = 1$ is determined as

$$\phi(r_i^m) = \int_{-\infty}^{\infty} \mathbf{1}_{r^{inv} \leq E[r^{m,i}(r^{inv})]} dF^{inv}(r^{inv}),$$

where the expected rate at time $t = 1$ proposed to investor M with type r^{inv} depends on the strategies of the whole network. We specify that

$$E[r^{m,i}(r^{inv})] = r_i^m \pi^s$$

when M does not run and bank i stays solvent in case of crash at $t = 2$, and

$$E[r^{m,i}(r^{inv})] = r_i^m \pi^{s,nc}$$

when M does not run and bank i defaults in case of crash at $t = 2$.

When investor M runs on liquidity, and bank i stays solvent, the expected payoff that it gets is equal to

$$E[r^{m,i}(r^{inv})] = 1\pi^{s,c} + r_i^m \pi^{s,nc},$$

and when the bank defaults, the outcome will depend on the amount of cash available at the shadow bank at time $t = 2$. It is clear that no liquidity support will be provided to the shadow bank unless the liquidity support is sufficient to stop the run, therefore the utility of the running investor when the bank defaults will be

$$E[r^{m,i}(r^{inv})] = r_i^m \pi^{s,nc} + \varepsilon.$$

4 Notion of Farsighted Stability

The model that I consider is a cooperative game: an interest rate and a loan volume cannot be set unilaterally and require consent of both counterparties. Therefore, we will need to abandon the standard notion of Nash equilibrium and proceed with the cooperative concept of equilibrium. I first require the coalitions to be farsighted.

The farsighted behavior in game theory was first introduced by Harsanyi (1974) as a critique of the (non-farsighted) stability concept proposed by von Neumann and Morgenstern (1944) for cooperative games. Chwe (1994) has formulated the first notion of farsighted stability. Simply put, farsightedly stable set can be defined as the sequence of game profiles (exposures and contracts) such that no feasible coalition of players will deviate to a different profile. Moreover, the reason why a coalition will not deviate is either because at least one of the members of a coalition would be worse off as a result of this deviation or because such a deviation would cause a chain of counter-reactions from other coalitions and lead to the outcome which makes the first coalition worse off. I am convinced that the farsighted behavior is the most appropriate notion of stability that can be applied to over-the-counter financial markets. The reason behind this fact is that a bargaining process between a set of lenders and a set of borrowers is cooperative and farsighted in its nature. For example, an interest rate specified by a pair lender-borrower is conditional on the alternative offers of other traders and can only be determined by considering the sequences of counter-offers that banks can make while bargaining.

The notion of farsighted stability can be applied to a general class of network formation games as defined by Page, Wooders and Kamat (2005). Therefore, the notion of farsighted stability in networks can be considered to be an extension of the concept of pairwise stability developed by Jackson and Wolinsky (1996) and of the concept of a core discussed in Gillies (1959).

The majority of papers dealing with the farsighted stability focus on the perfect

farsightedness of players.⁹ It means that a coalition may make a non-favorable move even if it expects that a large number of other coalitions will proceed with sequential moves, which will finally lead to the favorable outcome for all participating coalitions. In reality, farsightedness of agents is limited, meaning that a player can only consider a limited number of coalitional moves ahead. The evidence that subjects have an intermediate level of farsightedness is intuitive from observing everyday interactions. It was also documented in the lab experiment by Kirchsteiger, Mantovani, Mauleon and Vannetelbosch (2013). To account for the intermediate level of farsightedness, I use the notion of level-K farsightedness, developed by Herings, Mauleon, and Vannetelbosch (2015), which allows players to be farsighted only K coalitional moves ahead of them. I will use level-K farsightedness without the specification of K by just assuming that K is large and finite, meaning that players may see only K finite number of steps ahead.

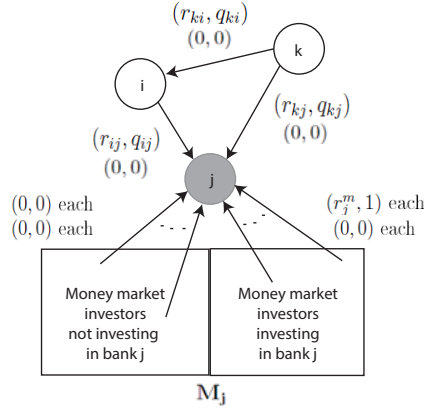
In order to strictly define the farsightedly stable equilibrium for the game, I first define the financial network and determine the set of possible deviations that can be made by various coalitions.¹⁰

In this paper, a financial network is equivalent to the game profile and is defined as a directed multilayered graph with each connection representing a set of contracts between two financial agents. We will reasonably assume that no changes are being made in the network in case of “no crash”. As a result, the contracts and exposures can be presented as a two-layer network: one layer for $t = 1$, and one layer for $t = 2$ in case of “crash”, with each link representing an interest rate and an exposure measure (see Figure 3 for an example). In Figure 3, it is clear that each connection between two parties has two layers. For banks, the first layer captures the exposure at $t = 1$, and the second layer captures the financial support at $t = 2$. For money market investors, the first layer captures exposure at $t = 1$ and the second layer captures exposure at $t = 2$. The exposure between two arbitrary banks $i, j \in B \cup B^s$ is measured as a loan size q_{ij} for the layer at $t = 1$ and q_{ij}^b for the layers at $t = 2$. The exposure between

⁹The papers that consider farsighted stability include a paper by Herings, Mauleon, and Vannetelbosch (2009), an earlier paper of Dutta, Ghosal, and Ray (2005), and the most recent paper by Ray and Vohra (2015).

¹⁰The network formation game is defined using the same nomenclature as in Page, Wooders, and Kamat (2005): feasible coalitions, feasible deviations, farsighted dominance relationship, and the farsighted equilibrium concept.

Figure 3: Example of a multilayered network with a full market run and the absence of liquidity support



investor $M \in \mathbf{M}_j$ and $j \in B^s$ is measured as a liquidity measure (either 0 or 1) invested at $t = 1$ for the layer at $t = 1$ and as a liquidity measure (either 0 or 1) kept at the bank at $t = 2$ for the layer at $t = 2$. For the purpose of consistency in notation, we assume that the rate is zero when the exposure is zero and the existence of one contract is enough for the connection to exist.

The network is considered feasible if it satisfies the assumptions of Section 3.

In the game, there are only the following types of coalitions that are considered feasible:

Definition 1. The set of feasible coalitions \mathbf{S} is defined as a set of all possible pairs (i, j) and elements (i) from the set of banks $i, j \in B \cup B^s$ and a set of all possible elements (M) from the set of investors $M \in \mathbf{M}_k$ surrounding bank $k \in B^s$.

Under this definition, no syndicated loans are allowed, meaning that a coalition can only be formed by a maximum of two banks. However, it does not mean that a bank cannot have multiple counterparties. Furthermore, a farsighted deviation can be performed by multiple banks if it can be presented as a sequence of pairwise deviations.

I also do not allow outside investors to negotiate with shadow banks directly. This is done to simplify the model and also to capture the fact that most investments are operated by large investment funds that gather liquidity from smaller investors. Nev-

ertheless, every small investor $M \in \mathbf{M}_i$ is allowed to take an individual investment decision regarding its unit of liquidity. The strategies of investors are as follows. At $t = 1$, an investor $M \in \mathbf{M}_i$ decides either to invest in $i \in B^s$ with nominal rate r_i^m or invest in the investor's alternative option. At time $t = 2$, the investor either withdraws or keeps liquidity in the fund.

If a pair of banks forms a new lending relationship, the rate is determined by the parties of the contract, while a breach of the contract can be done unilaterally by either lender or borrower. This rules are consistent with the pairwise network formation rules defined by Jackson and Wolinsky (1996). A consent of shadow bank $i \in B^s$ and the participating money market investors is necessary in order for bank i to borrow amount ϕ_i from the money market, while no consent of shadow bank is necessary for the investors to withdraw their funds from $i \in B^s$. To summarize the set of feasible deviations, I define them properly:

Definition 2. Given current financial network, the following deviations are considered feasible:

- a) a unilateral decision of either $i \in B \cup B^s$ or $j \in B \cup B^s$ is sufficient to deviate to $(r_{ij}, q_{ij}) = (0, 0)$, and a bilateral decision of coalition (i, j) is necessary to deviate to $(r_{ij}, q_{ij}) \neq (0, 0)$;
- b) a unilateral decision of either $i \in B \cup B^s$ or $j \in B \cup B^s$ is sufficient to deviate to $(r_{ij}^b, q_{ij}^b) = (0, 0)$, and a bilateral decision of coalition (i, j) is necessary to deviate to $(r_{ij}^b, q_{ij}^b) \neq (0, 0)$;
- c) a unilateral decision of investor $M \in \mathbf{M}_i$ is sufficient to deviate to any strategy of $M \in \mathbf{M}_i$, $i \in B^s$;
- d) a unilateral decision of bank $i \in B^s$ is sufficient to deviate to any rate r_i^m .

To define the equilibrium concept, I will denote a set of feasible coalitions with bold letter \mathbf{S} , as it is done in Definition 1, and a set of feasible game profiles (multilayered networks) with bold \mathbf{X} . It is important to note that specification of the bargaining mechanism is not necessary to be able to answer the questions raised at the beginning of the paper. Instead, I define a weak notion of equilibrium which determines the sets of possible outcomes that can and cannot happen independently of the bargaining procedure.

In order to strictly define farsighted consistency, I first define direct dominance.

Definition 3. A feasible game profile $x' \in \mathbf{X}$ directly dominates a feasible game profile $x \in \mathbf{X}$,

$$x' \succ x,$$

if there is a coalition $S \in \mathbf{S}$, such that a deviation from x to x' is feasible for coalition S , and each coalition member $i \in S$ benefits from this deviation:

$$E[u_i(x')] > E[u_i(x)].$$

In order to understand the following definitions, consider the simple example: a pair of lender $i \in B$ and borrower $j \in B^s$ deviate to favorable terms of liquidity support (r_{ij}^b, q_{ij}^b) . If both banks benefit from this deviation, we say that new strategy profile (directly) dominates the old strategy profile. In order to incorporate farsightedness, notice that in this example a traditional bank may be willing to deviate to the new lending terms only if the shadow bank reduces exposure ϕ_j . Therefore, a deviation by two coalitions $((i, j), i)$ is necessary. The order of the coalitional moves is not principal in the game, because only the final outcome influences the payoffs. If the farsighted deviation by $((i, j), i)$ from profile x to profile x' benefits both coalitions, we can say there exists a farsightedly improving path from the initial network x to the new network x' .

To understand the intuition behind the farsighted stability, consider the same example with two banks: $i \in B$ and $j \in B^s$. Recall that the traditional bank may agree to provide liquidity support at a more favorable rate if the shadow bank will agree to reduce the liquidity risk. However, there is no direct mechanism which forces the shadow bank to keep a certain exposure ϕ_j . As a result, shadow bank may deviate further following the deviation of $((i, j), i)$, which will make traditional bank i even worse comparing to the initial outcome. Being farsighted, the traditional bank $i \in B$ will try to prevent the hold up by restricting the lending terms with $j \in B^s$.

In the example that we consider there are only few coalitional moves. However, when the number of players goes to infinity, it is necessary to restrict the number of coalitions

that can cooperate to the finite number. This is the reason behind the assumption of level-K farsightedness.

We now give the strict definition of farsightedly improving path:

Definition 4. A feasible game profile (network) $x' \in \mathbf{X}$ level-K farsightedly dominates a feasible profile $x \in \mathbf{X}$,

$$x' \succ_K x,$$

if there is a sequence of coalitions $(S^1, \dots, S^K) \in \mathbf{S}$ and a corresponding sequence of outcomes $(x^0, x^1, \dots, x^K) \in \mathbf{X}$, where $x^0 = x$ and $x^K = x'$, such that each coalition S^k , $k = 1, \dots, K$ is feasible to make a move $x^{k-1} \xrightarrow{S^k} x^k$, and all coalitions (S^1, \dots, S^K) benefit from the final outcome:

$$E[u_i(x^K)] > E[u_i(x^{k-1})] \text{ for all } i \in S^k, k = 1, \dots, K$$

Using other terminology, relationship $x' \succ_K x$ means that there is a farsightedly improving path of length K from x to x' .

Now I can define the notion of level-K farsighted stability for the model of financial market:

Definition 5. A set of game profiles (networks) $\mathbf{FS} \in X$ is level-K farsightedly stable if the following criteria hold:

(1) any strategy profile x' , which results from the feasible deviation $x \xrightarrow{S} x'$, $x' \succ x$, $x \in \mathbf{FS} \in \mathbf{X}$, is farsightedly dominated by another strategy profile x'' , which creates a threat for at least one coalition member $i \in S$: $E[u_i(x'')] < E[u_i(x)]$, and either

- a) $x'' \in \mathbf{FS}$ and $x'' \succ_K x'$ for some $k \leq K - 2$, or
- b) $x'' \notin \mathbf{FS}$ and $x'' \succ_K x'$ for $k = K - 1$ but not $k < K - 1$.

(2) set \mathbf{FS} is reachable from any profile $x' \notin \mathbf{FS}$ via a finite (or infinite) sequence of improving paths of a kind $x^{(n+1)} \succ_{k_n} x^{(n)}$, with each path being k_n farsighted, for some $k_n \leq K$.

For simplicity, we will refer to the first criteria (1) as a criteria of internal stability and the second criteria (2) as a criteria of external stability (Herings, Mauleon, and

Vannetelbosch (2015) call these criteria level-K deterrence of external deviations and level-K external stability in order to distinguish them from the earlier definitions of farsighted stability. They also select the minimal set out of all level-K farsightedly stable sets. In this paper, we do not try to find the smallest set, but rather characterize it. Here and after, I will refer to the set of level-K farsightedly stable outcomes as simply the (farsightedly stable) equilibrium or the stable set.

The approach we use to find the equilibrium is the following. First, I will consider the special cases of direct dominance between the outcomes. Then, I will build a larger picture and show how the farsighted behavior of players changes their incentives to deviate.

5 Stable Financial Networks and Liquidity Channels

Endogenous bank run

In this section, we consider the behavior of money market investors \mathbf{M}_j , $j \in B^s$ that have invested at time $t = 1$ at nominal rate $r^{dep} \leq r_j^m \leq r^s$.

First, notice that when no crash hits the market, the investors that invested at time $t = 1$ expect to get a payoff $r_j^m \pi^{s|nr}$, which is even higher than the one they expected at time $t = 1$. Therefore, our assumption that investors do not withdraw in the high state of the world is strategically rational. We now will consider the strategies when market crash happens. The following proposition claims that the complete run is inevitable under Assumption 3:

Proposition 1. *In the equilibrium, a market crash triggers a run of all money market investors.*

Proof. In order to prove this proposition, we need to show that the two conditions in Definition 5 are satisfied for an equilibrium set FS . This proposition does not characterize the equilibrium set precisely, but rather narrows down the set of strategies that can be stable. Therefore, I assume that there is an equilibrium set FS , which does not have any farsighted deviations by a coalition of banks, and only focus on the deviations involving money market investors.

I first prove condition (1) of Definition 5, which can be described as a criteria of internal stability. We want to show that an investor that decides to keep liquidity in the bank, when every other investor runs, does not benefit from such a deviation. It immediately follows from the fact that the investor is not able to change the solvency status of the shadow bank: defaulting bank will stay defaulting, solvent bank will stay solvent. Therefore, an investor of defaulting bank prefers to get a payoff of order ε when it runs comparing to the payoff of zero when it does not run. An investor of a solvent bank prefers to get a payoff of one when it runs to the payoff $r^s \pi^{s|c} < 1$ when it does not run.

To prove condition (2), we need to show that from any strategy profile $x' \notin FS$ outside of the set FS , there is a sequence of level-K farsightedly improving paths that lead to FS . We first consider a profile x' , where bank i defaults at $t = 2$. Then it becomes clear that the investors will deviate one-by-one to the profile in FS , where all investors run on liquidity, because each deviating investor gets a payoff that exceeds zero.

Now consider a profile x' , where a non-empty subset of investors run on liquidity but bank i stays solvent. It is only possible when the required liquidity has been granted to the shadow bank. In this situation, all running investors get a payoff of 1, while all non-running investors get a payoff of $r^s \pi^{s|c} < 1$. At least one non-running investor will deviate to run, which will guarantee him a payoff of slightly below one but above $r^s \pi^{s|c}$. This deviation will turn the shadow bank into the defaulting bank and we can use the proof in the previous paragraph in order to show that there is a level-K farsighted path to set FS .

Finally, consider a profile x , where investors do not run and as a result shadow bank stays solvent. Then similar to the previous case, at least one investor has incentives to deviate to run and benefit from the additional payoff increase of $1 - r^s \pi^{s|c} > 0$. This will lead to the default of the bank and to the sequential level -K farsighted deviations that lead to set FS . \square

Therefore, if the money market rate is r_j^m and liquidity support is provided to the shadow bank, the size of money market is

$$\phi(r_j^m) = F^{inv}(\pi^{s,c} + r_j^m \pi^{s,nc}),$$

and if the liquidity support is not provided,

$$\phi(r_j^m) = F^{inv}(r_j^m \pi^{s,nc}),$$

In both cases, the analogue of inverse hazard rate is $\phi'/\phi = \pi^{s,nc}/\lambda$ ¹¹.

Stable equilibrium: two banks

In this section, I find a farsightedly stable equilibrium for one shadow and one traditional banks: $i \in B$, $j \in B^s$. I first consider the optimal strategies when $q_{ij} = \bar{q} = 0$, meaning that traditional bank i does not have a long-term exposure to shadow bank j .

Case $\bar{q} = 0$

If the liquidity support is provided, the banks expected utility functions have the following parametrical form¹²:

$$E[u_i] = ((r_{ij}^b - r^{cb})\phi_j + v)\pi^{s,c} + (v - r^{cb}\phi_j)\pi^{ns,c}1_{\{v - r^{cb}\phi_j \geq 0\}} + v\pi^{nc}, \quad (2)$$

$$E[u_j] = \phi_j ((r^s - r_j^m)\pi^{s,nc} + (r^s - r_{ij}^b)\pi^{s,c}), \quad (3)$$

and if the liquidity support is not provided, the utilities are

$$E[u_i] = v,$$

$$E[u_j] = \phi_j (r^s - r_j^m)\pi^{s,nc}.$$

¹¹It is assumed that ε is sufficiently small such that $F^{inv}(y + \varepsilon) = F^{inv}(y)$ for any y

¹²Expression $1_{\{v - r^{cb}\phi_j \geq 0\}}$ denotes an indicator function, which is one when the expression in the brackets is true.

It can be shown that under Assumptions 1 and 3, the equilibrium takes the following form:

Proposition 2. *In the game with two banks $i \in B$ and $j \in B^s$, given $\bar{q} = 0$, the equilibrium is characterized in the following way (see Figure 4):*

1) *Traditional bank is indifferent between providing and not providing liquidity support: $E[u_i] = v$.*

2) *One of two regimes can be observed when the liquidity support is provided:*

a) *Under the first regime, money market rate and volume are*

$$r^{m,large} = r^s \left(1 + \frac{\pi^{s,c}}{\pi^{s,nc}}\right) - r^{cb} \frac{\pi^{s,c}}{\pi^{s,nc}} - \frac{\lambda}{\pi^{s,nc}},$$

$$\phi^{large} = F^{inv}(\pi^{s,c} + r^{m,large} \pi^{s,nc}),$$

and the rate at which the liquidity support is provided is

$$r_{ij}^b = r^{cb} + \frac{v}{\phi^{large}} \frac{\pi^{ns,c}}{\pi^{s,c}}.$$

Under this regime, traditional bank has a positive probability $1 - \pi^{s|c}$ of default at time $t = 3$ due to contagion.

b) *Under the second regime, money market rate and volume are*

$$r^{m,small} = r^{m,large} - r^{cb} \frac{\pi^{ns,c}}{\pi^{s,nc}},$$

$$\phi^{small} = F^{inv}(\pi^{s,c} + r^{m,small} \pi^{s,nc}),$$

and the rate at which the liquidity support is provided is

$$r_{ij}^b = r^{cb} \left(1 + \frac{\pi^{ns,c}}{\pi^{s,c}}\right).$$

Under this regime, a default of shadow bank at $t = 3$ does not trigger a contagion.

4) *In the case when*

$$r^{cb} \leq \pi^{s|c} + r^s \pi^{s|c}. \quad (4)$$

liquidity support is always provided. The liquidity support is provided under the first

regime for $v \leq v^*$, and under the second regime for $v \geq v^*$, where the tipping point is

$$v^* = \frac{\lambda}{\pi^{ns,c}}(\phi^{large} - \phi^{small}).$$

4) If condition (4) is not satisfied, liquidity support is only provided under the first regime for all v : $v \leq v^{supp}$,

$$v^{supp} = \frac{\pi^{s,c}}{\pi^{ns,c}}(r^s - r^{cb})\phi^{large}.$$

5) When the liquidity support is not provided by bank i , the amount that bank j borrows from the money market is

$$\phi^{non-c} = F^{inv}(r^{m,non-c}\pi^{s,nc})$$

and the money market rate is

$$r^{m,non-c} = r^s - \frac{\lambda}{\pi^{s,nc}}.$$

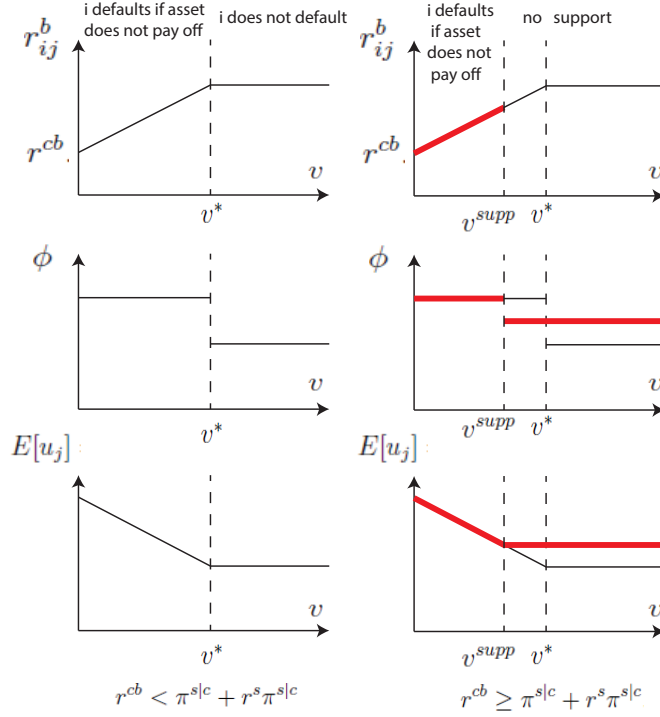
The proof of Proposition 2 is given in the Appendix. Here, I would like to emphasize certain equilibria characteristics. The following is true:

Corollary 1. *Ceteris paribus, equilibrium exposure ϕ_j of shadow bank $j \in B^s$ to money market decreases with an increase in profit v or an increase in central bank's support rate r^{cb} .*

This result can be further extended: a sufficient increase in the central bank's support rate r^{cb} leads to the termination of liquidity support. The results are consistent with the idea of monitoring. If a shadow bank wants to be subsidized, it is required to keep a limited amount of risk. Therefore, in cases when liquidity support is expensive (high r^{cb} and high v), shadow bank prefers to take additional liability risk and default with certainty when a market crash happens.

As Proposition 2 states, a traditional bank also charges a positive spread for its bail out services if shadow bank is willing to stay under protection. It is clear from Figure 3 that an increase in v and interest rate r^{cb} make traditional bank more risk averse, which leads to the default of shadow banking.

Figure 4: Equilibrium network characteristic given $\bar{q} = 0$



To estimate the efficiency, we find the sum of two utility functions $E[u_i] + E[u_j]$ for two regimes. Under the first regime, the pairwise payoff is

$$E[u_i] + E[u_j] = \phi^{large} \lambda + v(1 - \pi^{ns,c}),$$

and under the second regime, the pairwise payoff is

$$E[u_i] + E[u_j] = \phi^{small} \lambda + v.$$

When the liquidity support is not provided, the pairwise payoff is

$$E[u_i] + E[u_j] = \phi^{non-c} \lambda + v.$$

As a result, a substitution effect can be observed when the profit of traditional bank

from the safe banking activity decreases: once v passes a certain threshold, shadow banking significantly enlarges in size and diminishes the welfare that is generated by traditional banking.

Following the same logic, an interesting observation can be made: in two markets with equal number of liquidity providers,

$$q^l + \phi = \text{const}, \quad (5)$$

banking sector can generate different social welfare depending on the policy implications¹³. As a result, the equal size markets (5) with higher lending via traditional banking q^l and lower lending via shadow banking ϕ are be more socially efficient.

Case $\bar{q} > 0$

We now consider stable network when $\bar{q} > 0$, and as a result $q = q_{ij} \geq 0$, meaning that traditional bank i can have a long-term exposure to shadow bank j . Under Assumptions 1 and 3, the equilibrium is the following:

Proposition 3. *In the game with two banks, $i \in B$ and $j \in B^s$, and $\bar{q} > 0$, the equilibrium is characterized in the following way (see Figure 5):*

- 1) *Exposure constraint is binding: $q_{ij} = \bar{q}$.*
- 2) *One of the two regimes can be observed when the liquidity support is provided:*
 - a) *Under the first regime,*

$$\phi_j = \phi^{\text{large}}, \quad r_j^m = r^{\text{m,large}}, \quad r_{ij}^b = r^{\text{cb}},$$

for ϕ^{large} and $r^{\text{m,large}}$ defined in Proposition 2.

Under this regime, traditional bank has a positive probability $1 - \pi^{\text{slc}}$ of default at time $t = 3$ due to contagion.

- b) *Under the second regime,*

$$\phi_j = \phi^{\text{small}}, \quad r_j^m = r^{\text{m,small}}, \quad r_{ij}^b = \frac{r^{\text{cb}}}{\pi^{\text{s,c}}},$$

¹³To see it, draw a downward sloping line $v + (r^l - r^{\text{dep}})\phi = \text{const}$ in Figure 4.

Under this regime, a default of shadow bank at $t = 3$ does not trigger a contagion.

3) In the case when

$$(r^s \pi^s - r^{dep}) \bar{q} \geq \lambda (\phi^{non-c} - \phi^{small}) + r^{cb} \phi^{small} \pi^{ns,c}, \quad (6)$$

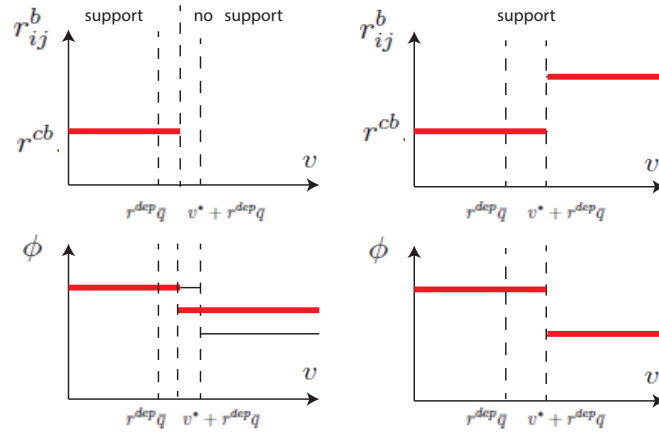
liquidity support is always provided. The liquidity support is provided under the first regime for $v \leq r^{dep} \bar{q} + v^*$, and under the second regime for $v \geq r^{dep} \bar{q} + v^*$, where v^* is defined in Proposition 3.

4) If condition (6) is not satisfied, liquidity support is only provided under the first regime for all v : $v \leq r^{dep} \bar{q} + v^{dev}$, where

$$v^{dev} = r^s \frac{\pi^{s,c}}{\pi^{ns,c}} \bar{q} + (\phi^{large} - \phi^{non-c}) \frac{\lambda}{\pi^{ns,c}}.$$

5) When the liquidity support is not provided, bank j borrows from the money market amount ϕ^{non-c} at $r^{m,non-c}$. Under this regime, market crash leads to a default of traditional bank if and only if $v \leq r^{dep} \bar{q}$, and to a default of shadow bank under any parametrization.

Figure 5: Equilibrium characteristics given $\bar{q} > 0$



The pairwise optimal expected payoff is

$$E[u_i] + E[u_j] = (r^s - r^{cb})\phi\pi^{s,c} - r^{cb}\phi\pi^{ns,c} + \phi(r^s - r^m)\pi^{s,nc} + v,$$

when $v \geq r^{cb}\phi$ and

$$E[u_i] + E[u_j] = (r^s - r^{cb})\phi\pi^{s,c} + \phi(r^s - r^m)\pi^{s,nc} + v(1 - \pi^{ns,c}),$$

when $v < r^{cb}\phi$.

We observe that there are natural barriers for the traditional banks not to get exposed to shadow banks. However, when shadow banks invest in the asset with relatively high quality, traditional banks are willing to get exposed to the shadow banking even when it increases their probability of default.

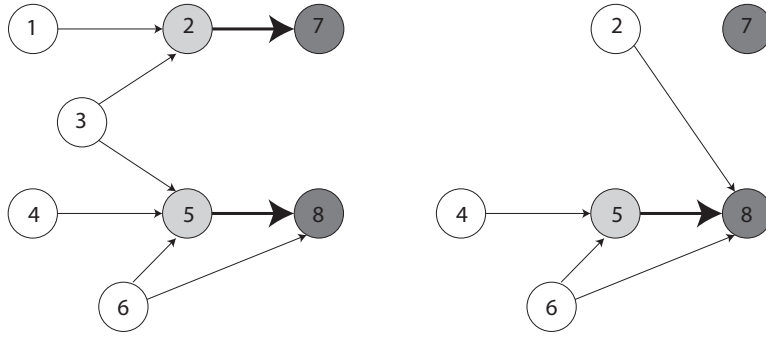
Stable equilibrium: multiple banks

We consider a situation when there are many traditional and shadow banks: $B = (1, \dots, N)$ and $B^s = (N + 1, \dots, N + N^s)$. In this section, we consider the case when $v \geq r^{dep}\bar{q}$. When $v < r^{dep}\bar{q}$, all traditional banks default if the crisis hits the market, because they are over-exposed to the shadow banking. Since we are interested in the cases, when a liquidity can be transferred from the central bank to the shadow banks with the help of traditional banks, we consider the case when the traditional banks are sufficiently regulated in the sense of tight cap on the shadow banking exposure ($\bar{q} \leq \frac{v}{r^{dep}}$). Then the following result holds:

Proposition 4. *In the equilibrium with a sufficient number of traditional banks B and shadow banks B^s , the liquidity support is provided via only few traditional banks $B^{core} \subset B$, $\dim(B^{core}) \leq \dim(B^s)$ in the amount of $\phi_j = \phi^{large}$ at rate $r_{ij}^b = r^{cb}$ each. Each core bank $i \in B^{core}$ also serves as an intermediary from a subset of traditional banks to one shadow bank in B^s , such that the total amount that it transfers at $t = 1$ exceeds $v - r^{dep}\bar{q} - r^{cb}\phi^{large}$ with the lending rate $r_{ij} \geq r^s$ and the borrowing rate $r_{ki} \leq r^s$, for all connected banks $k \in B$. The traditional banks get the same expected utility from the financial activities as non-core banks, with the core banks getting higher utility in case of market stability.*

The example of the stable shadow banking network is provided in Figures 7a,b. In Figure 7a, traditional banks $2, 5 \in B$ are the core banks, and traditional banks $1, 3, 4, 6 \in B$ are not in the core. Liquidity support of shadow banks $7, 8 \in B^s$ is realized through core banks with the support of the central bank. Traditional banks which are not in the core can lend to both traditional banks and shadow banks as soon as the liquidity transferred through the core is sufficiently large to support the activity of the core banks.

Figure 6: Example of the equilibrium network



In Figure 7b, the number of traditional banks is not sufficient to support all shadow banks in the case of market crash, therefore only one bank $8 \in B^s$ will be supported. Notice that the exposure of supported shadow bank to the money market is larger than the exposure of the non-supported bank:

$$\phi_7 < \phi_8,$$

and the interest rate that a supported shadow bank offers to the investors is also greater;

$$r_7^m < r_8^m,$$

where

$$\phi_7 = F^{inv}(r^{m,non-c}\pi^{s,nc}), r_7^m = r^{m,non,c}$$

$$\phi_8 = F^{inv}(\pi^{s,c} + r^{m,large}\pi^{s,nc}), r_8^m = r^{m,large}.$$

It is the case, because of both supply and demand forces, money market investors invest more in the shadow banks which are more stable, and shadow banks borrow more on the money market when the liquidity support is cheap.

It is also clear that the shadow banks that are being supported by the traditional banks are larger in size than those with no support. But it is necessary to say that even though shadow bank $8 \in B^s$ earns a higher expected return on investments than shadow bank $7 \in B^s$, bank $7 \in B^s$ is indifferent between lending to $8 \in B^s$ and investing on its own since it is equally likely default due to the bank run with or without the intermediation.

6 Policy recommendations

The results of the previous section can be used for policy recommendations. In order to determine the correct policy, we should determine the goal that government is trying to achieve by imposing the regulation. Since we consider a partial equilibrium model, the sum of utilities $\sum_{i \in B \cup B^s} u_i$ is not a good predictor for social welfare. Instead, government may be concerned about the stability of traditional banks only, or the welfare of investors and traditional banks altogether. There are many different agendas that can be achieved by the regulators. In this paper, I avoid any normative statements about the policy, and proceed with the positive results.

As it can be seen from propositions and the detailed proofs, in order to increase the stability of the traditional banks during the crisis, the beneficial policies include: limiting exposure \bar{q} between traditional banks and shadow banks to some small but positive value¹⁴, encourage investors to lend via traditional deposits rather than money markets (increase of v and increase of λ), increase the cost of shadow bank bail out rate

¹⁴When two banking sectors do not get exposed to each other before the crisis, liquidity support becomes expensive and does not benefit traditional banks. In other words, long-term exposure between a traditional bank and a shadow bank serves as a mechanism to price the liquidity risk farsightedly. Therefore, at least a small exposure between two sectors is necessary for the most efficient equilibrium.

r^{cb} ¹⁵. While these measures may prevent a default of traditional banks, they may also disincentivize traditional banks from bailing out and lending to shadow banks, leading to the defaults of shadow banks in the states of a market crash.

If the regulator would like to minimize the total number of defaults, it should take into account that a liquidity support given by a traditional bank to a shadow bank does not always lead to an increase in financial stability in the long-run. Under certain conditions, the risk may not be mitigated, but rather covered until the risky asset pays off. The default of both liquidity supporter and the supported shadow bank is especially likely to happen after the crisis when the traditional bank has a large exposure to the shadow bank before the crisis and the risky asset is of a low quality. Therefore, we focus on the goal to minimize the expected number of defaulted banks following the crisis, rather than the number of defaulted banks during the crisis. As was shown above, a decrease of \bar{q} to some small positive value will lead to fewer defaults. We can estimate the number of defaults when the large financial market is regulated such that $\bar{q} \leq \frac{v}{r^{dep}}$:

Corollary 2. *When there is a sufficient number of traditional banks B and shadow banks B^s , and $\bar{q} \leq \frac{v}{r^{dep}}$, the expected number of defaults is*

$$(2\pi^{ns,c} - \pi^c)|B^{core}| + |B^s|\pi^c.$$

This corollary implies that, when $\pi^{ns|c} > 1/2$, the expected number of defaults is minimized when the number of core banks is minimized. This result is intuitive because the core banks are getting exposed to the systemic risk at a high probability rate when the conditional probability of asset default is high. To decrease the number of core banks, the regulator should keep the profit $v - r^{dep}\bar{q} - r^{cb}\phi^{large}$ high during the crisis events. When $\pi^{ns|c} < 1/2$, the opposite actions are required.

The most beneficial conditions of the bail out are possible when traditional banks have a low cap on the exposure to the shadow banks and the critical liquidity support is provided via core traditional banks. In this case the burden from the majority of banks

¹⁵The reader should be cautioned that this bailout rate is not equivalent to the standard discount rate of the central bank. The bail out rate in the model is strictly the rate at which the central bank is willing to lend to traditional banks for the purpose of rescuing shadow banks. Therefore, the results of this paper should not be taken to imply that the central bank ought to regulate the discount rate in any way.

is taken by the core traditional banks. However, in this case, core traditional banks become systemically unstable. This is the first paper to my knowledge that emphasizes the fact that “too big to fail” banks can arise as a result of interaction between shadow banks and traditional banks as they try to establish liquidity channels. I suspect that, in reality, an extensive growth of such institution is likely to be accompanied by their growth in other banking activities. This will only exaggerate the “too big to fail” problem that becomes the issue when the core traditional banks serve as liquidity conduits for non-core traditional banks. Therefore, further research is necessary to evaluate which regulation maximizes social welfare when considering the costs of “too big to fail.”

7 Conclusion

In this paper, I consider a stable financial network before and after a crisis when traditional banks are allowed to serve as liquidity conduits between the central bank and shadow banking. I characterize the contagion following the asset price shock and determine a set of banks (both traditional and shadow) that can default as a result of a credit crunch. I also show that financial exposures between banks form a network with the core-periphery structure. The core traditional banks, defined as the most interconnected banks, provide the rescue liquidity channels to shadow banks and transfer liquidity from traditional banking to shadow banking. Although the initial model characteristics of all banks are identical, the core traditional banks make larger profits than periphery traditional banks while the market is stable, but are exposed to higher default risk due to their intermediary function, risk taking, and financial contagion. Finally, I provide a mechanism by which the government can control risks and efficiency of traditional banks. In particular, I consider the effectiveness of caps on the exposure of traditional banks to shadow banks and of controlling the discount interest rate.

In the future, the model can be extended to include capital requirements, heterogeneous shadow banks, different asset classes, and other variables as seen in the real world. I should note that this model takes a micro approach to the financial networks and systemic risk. In order to solve the model explicitly and distinguish important

forces from unimportant ones, a number of variables were not considered. The model explains well the interaction between traditional banks and shadow banks, but may lack predictive power regarding the interaction within shadow banking. Therefore, further research is necessary in this direction. Moreover, it is important to compare the model in this paper to the model where loans from the central bank are used not for lending to shadow banks but for purchases of toxic assets from the shadow banks or, even, purchases of the shadow banks themselves. I would also like to extend my policy analysis to more substantial results by performing a welfare analysis of the network. Such analysis may one day reveal an answer for how best to prevent and contain future financial crises.

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Appendices:

Proof of Proposition 2

To prove the theorem, we first consider deviations within the set of strategies characterized by $q_{ij} = 0$, $q_{ij}^b > 0$. In other words, we assume that bank $i \in B$ provides liquidity support to bank $j \in B^s$ in case of market crash, but does not get exposed to the shadow bank before the crisis.

The inability of traditional bank to control the balance sheet of the shadow bank will lead to a unilateral choice of exposure ϕ_j by shadow bank j . When shadow bank j wants to be supported by i , its optimal response is to choose r_j^m that maximizes utility $E[u_j]$ and keeps traditional bank interested in the provision of the support:

$$\begin{aligned} \max_{r_j^m} \phi(r_j^m) \left((r^s - r_j^m) \pi^{s,nc} + (r^s - r_{ij}^b) \pi^{s,c} \right) \\ \text{s.t. } E[u_i] \geq v, \end{aligned} \tag{7}$$

where the

$$E[u_i] = ((r_{ij}^b - r^{cb}) \phi_j + v) \pi^{s,c} + (v - r^{cb} \phi_j) \pi^{ns,c} 1_{\{v - r^{cb} \phi_j \geq 0\}} + v \pi^{nc},$$

$$\phi(r_j^m) = F^{inv}(\pi^{s,c} + r_j^m \pi^{s,nc}). \quad (8)$$

First, we show that constraint (7) is binding, so that the traditional bank does not get any profit from the provision of liquidity support. To show this, we solve the optimization problem with no constraint (7) in place. Given the log-concave properties of distribution $F^{inv}(x)$, the model can be solved explicitly. The shadow bank demands liquidity from the money market in the amount of $\phi_j = F^{inv}(\pi^{s,c} + r_j^m \pi^{s,nc})$ at rate

$$r_j^m = r^s \left(1 + \frac{\pi^{s,c}}{\pi^{s,nc}}\right) - r_{ij}^b \frac{\pi^{s,c}}{\pi^{s,nc}} - \frac{\lambda}{\pi^{s,nc}}. \quad (9)$$

Given the strategy of shadow bank j , the highest utility that i can achieve is

$$\max_{r_{ij}^b} E[u_i] = -\lambda \phi_j + v(\pi^{nc} + \pi^{s,c} + \pi^{ns,c} 1_{\{\phi_j \leq v/r^{cb}\}}).$$

Independent on parameter values, the maximum utility that i can achieve does not exceed the bank's breaching value v . It means that the utility function of bank i given constraint (7) is not sufficient for i to financially support bank j , when j does not control its money market exposure:

$$E[u_i(r_j^m(r_{ij}^b), r_{ij}^b)] \leq \max_{r_{ij}^b} E[u_i] < v.$$

This contradiction proves that constraint (7) is binding and bank i is indifferent between providing financial support and breaching. Moreover, it becomes clear that rate r_{ij}^b is chosen in such way that expected utility $E[u_j]$ is maximized and utility $E[u_i]$ is kept constant. If this is not the case, a sequence of deviations exists which leads to this outcome.

The constrained solution of problem

$$\max_{r_j^m, r_{ij}^b} E[u_j] = \phi_j(r_j^m) \left((r^s - r_j^m) \pi^{s,nc} + (r^s - r_{ij}^b) \pi^{s,c} \right) \quad (10)$$

$$\text{s.t. } E[u_i] = v$$

is such that the two regimes are possible depending on the parameters. In the first regime, shadow bank is more exposed to money market than in the second regime:

$$\phi^{large} > \phi^{small},$$

and the liquidity in the money market is more expensive:

$$r^{m,large} > r^{m,small}.$$

The regime switching occurs at point $v = v^*$, such that the utility function $E[u_j]$ under both regimes is identical:

$$v^* = \frac{\lambda}{\pi^{ns,c}} (\phi^{large} - \phi^{small}). \quad (11)$$

The regimes are described below:

a) When $v \leq v^*$, the liquidity support is provided at rate

$$r_{ij}^b = r^{cb} + \frac{v}{\phi^{large}} \frac{\pi^{ns,c}}{\pi^{s,c}}$$

and the money market exposure is $\phi^{large} = F^{inv}(\pi^{s,c} + r^{m,large}\pi^{s,nc})$ where

$$r^{m,large} = r^s \left(1 + \frac{\pi^{s,c}}{\pi^{s,nc}}\right) - r^{cb} \frac{\pi^{s,c}}{\pi^{s,nc}} - \frac{\lambda}{\pi^{s,nc}}.$$

Under this regime, traditional bank defaults with a positive probability. In particular, a default of bank $i \in B$ occurs at time $t = 3$ as a contagious response to the default of bank $j \in B^s$.

Under this regime, the money market rate and size are greater than the rate and size of the shadow bank that acts non-cooperatively and misses a liquidity support:

$$r^{m,non-c} = r^s - \frac{\lambda}{\pi^{s,nc}}, \quad (12)$$

$$\phi^{m,non-c} = F^{inv}(r^{m,non-c}\pi^{s,nc}). \quad (13)$$

Under the first regime, the expected payoff of bank j is

$$E[u_j] = \phi^{large} \lambda - v \pi^{ns,c},$$

and under the non-cooperative case

$$E[u_j] = \phi^{non-c} \lambda,$$

Therefore, the coalition benefits from the liquidity support when:

$$(\phi^{large} - \phi^{non-c}) \frac{\lambda}{\pi^{ns,c}} \geq v$$

When $r^{cb} < \pi^{s|c} + r^s \pi^{s|c}$, the support is always provided because

$$(\phi^{large} - \phi^{non-c}) \frac{\lambda}{\pi^{ns,c}} > \frac{\lambda}{\pi^{ns,c}} (\phi^{large} - \phi^{small}) > v,$$

when $r^{cb} > \pi^{s|c} + r^s \pi^{s|c}$, the support is provided when:

$$v \leq (\phi^{large} - \phi^{non-c}) \frac{\lambda}{\pi^{ns,c}}.$$

b) When $v \geq v^*$, the liquidity support rate is

$$r_{ij}^b = r^{cb} \left(1 + \frac{\pi^{ns,c}}{\pi^{s,c}} \right),$$

which exceeds the central bank's lending rate by $\frac{\pi^{ns,c}}{\pi^{s,c}}$ percentage points.

The money market exposure is $\phi^{small} = F^{inv}(\pi^{s,c} + r^{m,small} \pi^{s,nc})$, such that

$$r^{m,small} = r^s \left(1 + \frac{\pi^{s,c}}{\pi^{s,nc}} \right) - r^{cb} \left(1 + \frac{\pi^{ns,c}}{\pi^{s,c}} \right) \frac{\pi^{s,c}}{\pi^{s,nc}} - \frac{\lambda}{\pi^{s,nc}}.$$

Under the second regime, the expected payoff of bank j is

$$E[u_j] = \phi^{small} \lambda,$$

which is greater than the payoff in the non-cooperative case $E[u_j] = \phi^{non-c} \lambda$ if the lending rate of the central bank is sufficiently low:

$$r^{cb} < \pi^{s|c} + r^s \pi^{s|c}. \quad (14)$$

If condition (??) is not satisfied, shadow bank will prefer to act independently on the traditional bank and get overexposed to the money market. Intuitively from Figure 4, a financial support will only be provided when:

$$v \leq v^{supp},$$

where

$$v^{supp} = \frac{\pi^{s,c}}{\pi^{ns,c}} (r^s - r^{cb}) \phi^{large}$$

$$\frac{\lambda}{\pi^{s,c}} (\phi^{large} - \phi^{small}) = (r^s - r^{cb}) \phi^{large}$$

As a result, under Assumptions 1 and 3 financial support is not provided for the cases when traditional bank has a large exposure v or the risky asset delivers low returns. It is also the case that, ceteris paribus, increase in interest rate r^{cb} may terminate the provision of liquidity support.

The logic provided above is sufficient to show that the resulting equilibrium candidate portrayed in Figure 4 is level-K farsighted equilibrium. The internal stability follows from the facts that there are no beneficial deviations (either direct or farsighted). When the liquidity is not provided, a deviation to any network with a positive liquidity support will lead eventually to lower pairwise expected payoff dominated by the stable set. When the liquidity support is provided, a coalition of bank (i, j) will not proceed with a different contract $(q_{ij}^b = \phi_j, r_{ij}^b)$, because the equilibrium money market rate r_j^m and quantity ϕ_j are pairwise efficient, while any changes in r_{ij}^b will lead to a redistribution of surplus from j to i . It is also the case that none of the banks will breach the existing contract.

Finally, external stability follows from the initial observations in the proof that ϕ_j is chosen unilaterally. Therefore any outcome with $E[u_i] > v$ will be dominated by an outcome with $E[u_i] = v$ as a result of deviation by j . Also, from any outcome with non-equilibrium (r_j^m, ϕ_j) there is a dominance path managed by coalition (i, j) ,

such that r^b and ϕ reach optimal values and the payoffs of both banks increase. As we already showed this deviation will be followed by the deviation of j , which will lead to the stable set. We have shown that from any non-equilibrium outcome, there is a sequence of deviation that leads to the level-K farsightedly stable set.

Proof of Proposition 3

Consider a situation when traditional bank $i \in B$ is exposed to shadow bank $j \in B^s$ at time $t = 1$ and the liquidity support is provided at time $t = 2$. For now assume that the exposure q_{ij} is fixed at level q_{ij} and $0 < q_{ij} \leq \bar{q}$. If the network connection exist, r_{ij} and r_{ij}^b are such that both banks stay solvent in case of “success”. In fact, if this is not the case, the bail out is not meaningful.

Then the banks expect the following payoffs

$$E[u_i] = ((r_{ij} - r^{dep})q_{ij} + v)\pi^s + \phi_j(r_{ij}^b - r^{cb})\pi^{s,c} + \dots$$

$$\dots + (v - r^{dep}q_{ij})\pi^{ns,nc}1_{\{v - r^{dep}q_{ij} \geq 0\}} + (v - r^{dep}q_{ij} - r^{cb}\phi_j)\pi^{ns,c}1_{\{v - r^{dep}q_{ij} - r^{cb}\phi_j \geq 0\}}$$

$$E[u_j] = (r^s - r_{ij})q_{ij}\pi^s + \phi_j((r^s - r_j^m)\pi^{s,nc} + (r^s - r_{ij}^b)\pi^{s,c})$$

The first thing that we should notice is that the presence of transfer $q_{ij} > 0$ makes it possible for the banks to choose a pairwise optimal level of exposure to the money market ϕ_j . If the exposure ϕ_j is not pairwise optimal, banks can always adjust r_{ij} and r_{ij}^b , such that both counterparties benefit from the change.

The equilibrium exposure ϕ_j is pairwise efficient only when bank j has no incentives to change it unilaterally. In other words, there is no deviation by bank j that cannot be blocked by bank i . The unilateral deviation by j can only be prevented when the equilibrium rate r_{ij}^b is chosen such that the utility function of bank i does not depend on the exposure ϕ_j . In order to find stable r_{ij}^b , notice that the pairwise optimal ϕ_j is

equal to

$$r^{m,large} = r^s \left(1 + \frac{\pi^{s,c}}{\pi^{s,nc}}\right) - r^{cb} \frac{\pi^{s,c}}{\pi^{s,nc}} - \frac{\lambda}{\pi^{s,nc}},$$

under the first regime $v \leq v^* + r^{dep}q_{ij}$, and

$$r^{m,small} = r^s \left(1 + \frac{\pi^{s,c}}{\pi^{s,nc}}\right) - r^{cb} \left(\frac{\pi^{ns,c}}{\pi^{s,nc}} + \frac{\pi^{s,c}}{\pi^{s,nc}}\right) - \frac{\lambda}{\pi^{s,nc}},$$

under the second regime $v \geq v^* + r^{dep}q_{ij}$, where v^* is defined in (11). Therefore, it is required that

$$r_{ij}^b = r^{cb}$$

under the first regime, and

$$r_{ij}^b = r^{cb} \left(1 + \frac{\pi^{ns,c}}{\pi^{s,c}}\right)$$

under the second regime.

Until now we kept q_{ij} as given. Next we determine what $q_{ij} \leq \bar{q}$ is pairwise optimal. Due to the fact that payoff is transferable, the pairwise optimal q_{ij} is the one that maximizes the sum of two utility functions. The structural change in the pairwise payoff occurs in two points: first at $q_{ij} = \frac{v}{r^{dep}}$, when bank i becomes insolvent in case of “no crash, “no success”, and second at $q^* : v = v^*(q^*)$, when traditional bank becomes insolvent in case of “crash, no success”. Clearly, $q^* \leq \frac{v}{r^{dep}}$. The pairwise payoff of two banks, as a function of q_{ij} , is continuous and convex. It means that the optimal q_{ij} is either zero or \bar{q} . On the interval $0 \leq q_{ij} \leq q^*$ the slope of $E[u_i] + E[u_j]$ is positive according to Assumption 1: $r^s > r^{dep}/\pi^s$, so it is positive for all feasible q_{ij} . Therefore, total expected utility increases with an increase in q_{ij} , so constraint $q_{ij} \leq \bar{q}$ is binding: $q_{ij} = \bar{q}$.

The last condition that we need to check is whether banks i and j have incentives to breach. From the previous calculations, we derive the total expected payoff when $v \leq r^{dep}\bar{q}$:

$$E[u_i] + E[u_j] = (r^s - r^{dep})\bar{q}\pi^s + \phi^{large}\lambda + v\pi^s,$$

when $r^{dep}\bar{q} \leq v \leq v^* + r^{dep}\bar{q}$:

$$E[u_i] + E[u_j] = (r^s - r^{dep})\bar{q}\pi^s - r^{dep}\bar{q}\pi^{ns,nc} + \phi^{large}\lambda + v(1 - \pi^{ns,c}),$$

and when $v \geq v^* + r^{dep}\bar{q}$

$$E[u_i] + E[u_j] = (r^s - r^{dep})\bar{q}\pi^s - r^{dep}\bar{q}\pi^{ns} + \phi^{small}(\lambda - r^{cb}\pi^{ns,c}) + v, \quad (15)$$

The pairwise payoff in the case of empty network is:

$$E[u_i] + E[u_j] = \phi^{non-c}\lambda + v. \quad (16)$$

We first show that no deviation to $q_{ij}^b = 0$ is beneficial, when q_{ij} does not change. The formula for the pairwise expected payoff given $q_{ij}^b = 0$ and $q_{ij} = \bar{q}$ is

$$E[u_i] + E[u_j] = (r^s - r^{dep})\bar{q}\pi^{s,nc} + (v - r^{dep}\bar{q})(1 - \pi^{s,nc})1_{v-r^{dep}\bar{q} \geq 0} + \phi^{non-c}\lambda + v\pi^{s,nc},$$

which is always greater than (16) because according to Assumption 1 $r^s\pi^{s,nc} - r^{dep} \geq 0$.

Therefore, in order to show that there are no deviations from the equilibrium, we only need to check deviations to $q_{ij}^b = 0$.

Given that pairwise payoff function has convex properties and segment (15) is parallel to $\phi^{non-c}\lambda + v$, in order to show that no deviation to $q_{ij} = q_{ij}^b = 0$ is possible, it is sufficient to show under which conditions (15) intersects $v = 0$ above $\phi^{non-c}\lambda$. It is exactly the case when:

$$(r^s\pi^s - r^{dep})\bar{q} + \phi^{small}(\lambda - r^{cb}\pi^{ns,c}) \geq \phi^{non-c}\lambda. \quad (17)$$

If condition (17) is not satisfied, the liquidity support is only provided when $v \leq r^{dep}\bar{q} + v^{dev}$:

$$v^{dev} = r^s \frac{\pi^{s,c}}{\pi^{ns,c}} \bar{q} + (\phi^{large} - \phi^{non-c}) \frac{\lambda}{\pi^{ns,c}}.$$

It immediately follows that $r^{dep} \bar{q} < v^{dev} + r^{dep} \bar{q} < v^* + r^{dep} \bar{q}$.

Therefore, we have shown that there is a set of equilibrium prices r_{ij} that makes the outcomes described above stable to breaching.

Given our findings, we can conclude the stability. Deviation by any bank to $q_{ij} = 0$ with keeping q_{ij}^b the same makes both parties worse off. If in the equilibrium $q_{ij}^b > 0$, deviation to $q_{ij} = 0$ and $q_{ij}^b = 0$ makes at least of bank worse off. If the deviating coalition is better off, the losing party will be willing to renegotiate the terms of contract at $t = 1$, such that both counterparties benefit from the cooperation. Therefore, there is a path back to the stable set. At the same time, coalition (i, j) cannot improve on the pairwise payoff, because ϕ_j and r_j^m are pairwise optimal. We also eliminated the incentives of shadow bank j to change rate r_j^m to exposure which is not pairwise efficient. Therefore, internal stability follows.

External stability can be proved in two steps. First consider an outcome outside of the equilibrium set with no liquidity support being provided, but which qualifies for the equilibrium support. Then it is clear that there exist a pairwise deviation which improves both payoffs.

Second, consider an outcome outside of the equilibrium set, such that the liquidity support is provided, and r_j^m and ϕ_j are not pairwise optimal. We assume that this outcome provides higher payoffs than in the non-cooperative case. If this is not the case, we come back to the previous case. Then the payoffs can be increased by (i, j) with an adjustment to the pairwise optimal (r_j^m, ϕ_j) , change of r_{ij}^b to either r^{cb} or $r^{cb}/\pi^{s,c}$ or zero, and a shift of interest payments between periods $t = 1$ and $t = 2$.

Proof of Proposition 4

We show the intuition with two traditional banks $i, k \in B$ and one shadow bank $j \in B^s$ and then extend the result for many traditional banks and one shadow bank. We consider a network when bank k lends amounts $q_{ki} \geq 0$ and $q_{kj} \geq 0$ to banks $i \in B$ and $j \in B^s$ correspondingly, and provides a liquidity support in the amount of q_{kj}^b at rate

r_{kj}^b . Bank $i \in B$ lends to $j \in B^s$ amount $q_{ij} = q_{ki} + \Delta q_{ij}$. This network is in fact the most general form of the stable network with two identical traditional banks and one shadow banks.

First, we determine the network that delivers the highest total utility to the three banks. I reduce the notation burden for the reader by saying that the total utility of the banks is maximized when the traditional banks get exposed to the shadow bank completely at $t = 1$: $q_{ij} = q_{ki} + \bar{q}$ and $q_{ki} + q_{kj} = \bar{q}$. This result is proved in the same way as in Proposition 3 based on the convex properties of the expected utilities.

We would initially like to know what is the most optimal bail out scheme (q_{ij}^b, q_{kj}^b) for the coalition of three banks when the exposure to the money market is fixed at some level $\phi_j = q_{ij}^b + q_{kj}^b$. We consider the case $v - r^{dep}\bar{q} \geq 0$. Having everything else fixed, we choose the scheme that generates the highest total payoff in the state of "no success, crash".

- When both banks stay solvent following the crash, the banks get the following utility in the state of "no success, crash":

$$2(v - r^{dep}\bar{q}) - r^{cb}\phi_j \quad (18)$$

- When bank j defaults, bank i defaults as well, since both banks generate the same revenue and bank i incurs larger cost. In case of the default of both banks, zero utility is being generated.
- When bank j stays solvent and bank i defaults, under assumption $v - r^{dep}\bar{q} \geq 0$, the total utility turns to

$$\max_{q_{kj}} v - r^{dep}\bar{q} - r^{cb}q_{kj}^b + p_{ki}(q_{kj}^b) = v - r^{dep}\bar{q} + p_{ki} \quad (19)$$

where $p_{ki} = p_{ki}(q_{kj}^b)$ is the payment made by defaulting bank i to bank j . Defaulting bank i also makes payment to the central bank in the amount of $p_{cb,i} < r^{cb}q_{ki}^b$. The payments p_{ki} and $p_{cb,i}$ are proportional to the loan sizes and do not exceed the debt sizes: $p_{cb,i} < r^{cb}q_{ij}^b$, $p_{ki} < r_{ki}q_{ki}$, $p_{ki} = v - r^{dep}\bar{q} - p_{cb,i}$. With simple first order conditions it can be shown that the utility of both banks increases with a decrease of q_{kj}^b , so that it is beneficial for the coalition when bank k does not provide liquidity support to the

shadow bank: $q_{kj}^b = 0$. Comparison of (18) and (19) also indicates that the total utility is maximized when only one bank defaults as a result of "no success, crash" and $q_{kj}^b = 0$:

$$v - r^{dep}\bar{q} + p_{ki} \geq 2(v - r^{dep}\bar{q}) - r^{cb}\phi_j.$$

In the same way as in Proposition 3, we find optimal money market exposure $\phi = \phi^{large}$ and money market rate $r^m = r^{m,large}$.

When there is "no success, no crash", banks generate a payoff of $2(v - r^{dep}\bar{q})$ if both banks stay solvent and the same payoff when bank i defaults and passes all the profit $v - r^{dep}\bar{q}$ to repay the debt to k . Therefore, if it is required to increase exposure from k to i , it will be done by the coalition with no downside costs.

For the case of three banks we showed that when traditional banks transfer sufficient liquidity to shadow bank and provide liquidity support, it is payoff improving for the coalition of three banks to reallocate the liquidity flows so that a core bank emerge. This core bank will be the only bank providing liquidity to $j \in B^s$.

This result can be extended to many traditional banks. We will do a proof by contradiction: assume there are at least two banks $i, k \in B$ that provide financial support to $j \in B^s$ at $t = 2$. Suppose bank i defaults in state of "no success, crash". Then the total utility of all banks can be increased when ϕ_{kj} is reallocated to bank i in the way similar to the example with three banks.

Suppose bank i does not default in state of "no success, crash". Then with the sufficient number of banks that lend to $j \in B^s$ directly or indirectly at $t = 1$, there exists a deviation where the banks reallocate the liquidity to bank i and delegate a bail out function to bank i . When the banks lending to j provide sufficient liquidity to j directly or indirectly, it is always possible to increase the leverage of i in such way that the limited liability condition will be in place:

$$v - r^{dep}\bar{q} - r^{cb}\phi^{large} < \sum_k r_{ki}\bar{q}. \quad (20)$$

In fact, when there is a sufficient number of traditional banks, but only few lend to j directly or indirectly, it is still possible to attract these banks to i , when they do not provide liquidity support themselves. It will be beneficial because, in our model, all

supported shadow banks provide the same maximum rate of return on investments.

In the way similar to the previous propositions, it can be shown that this network is in fact farsightedly stable since it is always possible to transfer an additional payoff to the bank without breaking the stability. The stable set is characterized by the presence of core banks, such that each core bank supports and lends to one shadow bank on behalf of other investors and itself. The core bank defaults in the state of crash and unsuccessful asset payoff, and possibly in the state of “no crash, no success”, depending on the parameters. the core banks have a lending rate that exceeds r^s . The non-core banks do not provide liquidity support, as a result they do not default in case of “crash, no success”, and have a lending rate below r^s .

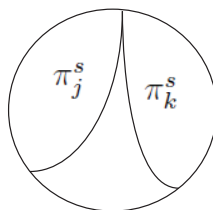
In order to prove that this network is stable, we prove that it is both internally and externally stable. We first consider if there exist any direct and farsighted coalitional deviations outside of the stable set that are beneficial and cannot be blocked. Suppose condition (20) is satisfied for all core banks. First notice that a core bank never deviates to the contracts with multiple shadow banks, because it decreases its profit. It can be seen from the fact a minimum revenue of

$$Cost = v - r^{dep}\bar{q} - r^{dep}\phi^{large}$$

is required for the traditional bank to cover its expenses in case of crash. If the revenue is below this level, traditional bank defaults and gets zero. When a traditional bank extends the portfolio to two banks, the probability measure of states where bank pays non-random cost $Cost$ increases (according to Assumption 2), while the expected revenue stays the same. Therefore, due to the fixed cost analogy, bank i prefers lending to one bank. To imagine these result, consider an example with the events depicted in Figure 7, where joint probability of default is zero (it satisfies Assumption 2) and bank i is highly leveraged. If bank lends to one shadow bank, it gets a positive payoff $r^s q_{ij} - Cost \geq 0$ in case of success. If bank allocates the funds to k and j equally, it defaults in both states because $r^s \frac{q_{ij}}{2} - Cost < 0$ and expects the payoff of zero.

Now consider deviations that break the core-periphery structure. If as a result of deviation, bank j loses support, but the number of traditional banks is sufficiently large, there will be a counter deviation by j and traditional banks that will lead to the stable

Figure 7: Event space and corresponding probabilities



set. In the same way any deviation that leads to the multiple support providers will be blocked by more efficient network and lead the path to the stable set.

Because the lending rate of core bank exceeds r^s , an alternative supported shadow bank will not attract their liquidity by offering a better rate. If an alternating (supported or non-supported) shadow bank will try to attract direct or indirect investors of j , there will be a blocking deviation that will lead to the stable set with the slightly increase lending rates for non-core banks lending to j directly or indirectly. It finishes the prove of internal stability.

External stability follows directly from the fact that the equilibrium network maximizes the total payoff of all banks investing in the same bank. Therefore, there is a sequence of deviations which lead to this network structure with the redistribution of benefits through the contracts at $t = 1$. Moreover, since shadow banks will try to attract as many traditional banks as possible, the interest rates will finally go up along some level-K farsighted paths from any outcome outside of the stable set. At some point, the rates for core banks will exceed r^s . Shadow banks will be willing to offer interbank rate r^s or slightly higher, since they gain a significant amount of cash in case of “success, crash” and lose nothing due to limited liability. Therefore, I proved that the stable set is reachable from any profile outside of the set, and proved external stability.