Collusion in Auctions with Constrained Bids:

Theory and Evidence from Public Procurement*

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Abstract

We study the mechanics of cartel enforcement and its interaction with bidding constraints in the context of repeated procurement auctions. Under collusion, bidding constraints weaken cartels by limiting the scope for punishment. This yields a test of repeated collusive behavior exploiting the counter-intuitive prediction that introducing minimum prices can lower the distribution of winning bids. The model's predictions are borne out in procurement data from Japan, where we find considerable evidence that collusion is weakened by the introduction of minimum prices. A robust design insight is that setting minimum price constraints at the bottom of the observed distribution of winning bids necessarily improves over setting no minimum prices.

Keywords: collusion, cartel enforcement, minimum prices, entry deterrence, procurement.

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1 Introduction

This paper studies the mechanics of cartel enforcement and its interaction with bidding constraints in the context of repeated procurement auctions. Minimum prices, which place a lower bound on the price at which procurement contracts can be awarded, are frequently used in public procurement. Because minimum prices make price wars less effective, they can also make cartel enforcement more difficult. This leads to the counter-intuitive prediction that the introduction of minimum prices may lead to a first-order stochastic dominance drop in the right tail of winning bids. Because this prediction does not arise in competitive environments, it provides a joint test of collusion and of the specific channel we outline: enforcement constraints are binding, and they can be affected by institution design. The model's predictions are borne out in procurement data from Japan, showing that this cartel enforcement channel is empirically relevant.

From a policy perspective, our findings show that in the presence of colluding bidders, attempts at surplus extraction may foster collusion and reduce the auctioneer's surplus. Inversely, providing minimum surplus guarantees can limit collusion and improve the auctioneer's surplus. A robust take-away from our analysis is that introducing a minimum price at the bottom of the distribution of observed bids always dominates setting no minimum price. If there is no collusion, it does not affect the distribution of bids, and if there is collusion it can only reduce the distribution of bids.

We model firms as repeatedly playing a first-price procurement auction with i.i.d. production costs. We assume that costs are commonly observed among cartel members, and that firms are able to make transfers. In this environment, cartel behavior is limited by self-enforcement constraints: firms must be willing to follow bidding recommendations, as well as make equilibrium transfers. We provide an explicit characterization of optimal cartel behavior: first, contract allocation is efficient, provided that price constraints are not binding; second, cartel members implement the highest possible winning bid for which the sum of deviation temptations is less than the cartel's total pledgeable surplus. This simple characterization lets us delineate distinctive predictions of the model in a transparent manner.

Our main predictions relate the introduction of minimum prices and changes in the distribution of winning bids. In our repeated game environment, minimum prices may weaken cartel discipline by limiting the impact of price wars. When this is the case, sustaining collusive bids above the minimum price becomes more difficult, causing a first-order stochastic dominance drop in the distribution of winning bids to the right of the minimum price. A key observation is that minimum prices have the opposite impact in environments without collusion. Under competition, regardless of whether firms have complete or asymmetric information about costs, minimum prices lead to a (weak) first-order stochastic dominance increase in the right tail of winning bids. This provides a joint test of collusion and of the mechanics of cartel enforcement.

Allowing for entry lets us extend this test and generate new predictions. Under our model, minimum prices reduce the right tail of winning bids conditional on a cartel member winning. However, minimum prices have no effect on the right tail of winning bids conditional on an entrant winning. The reason for this is that cartel members seek to dissuade entry by pinning entrants' winning bids to their production costs. As a result the right tail of entrant winning bids does not depend on minimum prices. In contrast, minimum prices still affect the highest sustainable winning bid among potential cartel winners. This differential impact of minimum prices on cartel and entrant winners allows us to distinguish our model from a competitive model in which the introduction of minimum prices broadly increases entry. In such a model, the winning bids of both cartel and entrant winners should be affected by minimum prices.

We explore the impact of enforcement constraints on cartel behavior by using data from public procurement auctions taking place in Japanese cities of the Ibaraki prefecture between 2007 and 2015. The introduction of minimum prices in one city in 2009 lets us use the change-in-changes framework of Athey and Imbens (2006) to recover the counterfactual distribution of winning bids after the policy change. The data exhibits a large and significant drop in the distribution of winning bids to the right of the minimum price, implying that: (i) there is collusion; (ii) enforcement constraints limit the scope of collusion; (iii) minimum prices successfully weaken cartel discipline.

Richer data available from the treatment city lets us break down more finely the channels through which the distribution of winning bids is affected by minimum prices. Using a single difference approach, we show that the effect of minimum prices is equally mediated by weakened entry deterrence, and weakened enforcement among cartel members. Motivated by the fact that 25% of bidders make up 80% of the (auction, bidder) pairs, we treat the top quartile of most active bidders as cartel members. Consistent with our theory, the effect of minimum prices is entirely concentrated on cartel members.

Our paper lies at the intersection of different strands of the literature on collusion in auctions. The seminal work of Graham and Marshall (1987) and McAfee and McMillan (1992) studies static collusion in environments where bidders are able to contract. A key take-away from their analysis is that the optimal response from the auctioneer should involve setting more constraining reserve prices. In a procurement setting this means reducing the maximum price that the auctioneer is willing to pay. We argue, theoretically and empirically, that when bidders cannot contract and must enforce collusion through repeated game play, minimum price guarantees can weaken cartel enforcement.

An important observation from McAfee and McMillan (1992) is that in the absence of cash transfers, the cartel's ability to collude is severly limited even when commitment is available. A recent strand of work takes seriously the idea that in repeated games, continuation values may successfully replace transfers. Aoyagi (2003) studies bid rotation schemes and allows for communication. Skrzypacz and Hopenhayn (2004) (see also Blume and Heidhues, 2008) study collusion in environments without communication and show that while cartel members may still be able to collude, they will remain bounded away from efficient collusion. Athey et al. (2004) study collusion in a model of repeated Bertrand competition and emphasize that information revelation costs will push cartel members towards rigid pricing schemes. Because we focus on obedience rather than information revelation constraints, our model simplifies away the strategic issues emphasized in this body of work: we assume complete information among cartel members and transferable utility.[1] This yields a simple characterization of optimal collusion closely related to that obtained in the relational contracting literature (Bull, 1987, Baker et al., 1994, 2002, Levin, 2003), and provides a transparent framework in which to study the effect of price constraints on winning bids.

Several recent papers study the impact of the auction format on collusion. Fabra (2003) compares the scope for tacit collusion in uniform and discriminatory auctions. Marshall and Marx (2007) study the role of bidder registration and information revelation procedures in facilitating collusion. Pavlov (2008) and Che and Kim (2009) consider settings in which cartel members can commit to mechanisms and argue that appropriate auction design can successfully limit collusion provided participants have deep pockets and can make ex ante payments. Abdulkadiroglu and Chung (2003) make a similar point when bidders are patient.

More closely related to our work, Lee and Sabourian (2011) as well as Mezzetti and Renou (2012) study full implementation in repeated environments using dynamic mechanisms. They show that implementation in all equilibria can be achieved by restricting the set of continuation values available to players to support repeated game strategies. The incomplete contracts literature (see for instance Bernheim and Whinston, 1998, Baker et al., 2002) has suggested that the same mechanism, used in the opposite direction, provides foundations for optimally incomplete contracts. Specifically, it may be optimal to keep contracts more incomplete than needed, in order to maintain the range of continuation equilibria needed to enforce efficient behavior. We provide empirical evidence that this theoretical mechanism plays a significant role in practice, and can be meaningfully used to affect collusion between

[1] Note that we allow for incomplete information when we study the impact of minimum prices under competition. This ensures that our test of collusion is not driven by stark modeling assumptions.

firms.

On the empirical side, an important set of papers develops empirical methods to detect collusion (see Harrington (2008) for a detailed survey of prominent empirical strategies and their theoretical underpinnings). Porter and Zona (1993, 1999) contrast the behavior of suspected cartel members with that of non-cartel members, controlling for observables. Bajari and Ye (2003) use excess correlation in bids as a marker of collusion. Porter (1983), along with Ellison (1994) (see also Ishii, 2008) use patterns of price wars of the sort predicted by repeated game models of oligopoly behavior (Green and Porter, 1984, Rotemberg and Saloner, 1986) to identify collusion. In a multi-stage auction context, Kawai and Nakabayashi (2014) argue that excess switching of second and third bidder across bidding rounds, compared to first and second bidders, is a smoking gun for collusion. We propose a test of collusion exploiting changes in the cartel's ability to implement effective punishments.

The paper is structured as follows. Section 2 sets up our benchmark model of cartels and characterizes optimal cartel behavior. Section 3 derives empirical predictions from this model that distinguish it from competitive behavior. Section 4 briefly extends these results in a setting with entry. Section 5 takes the model to data. Section 6 discusses the robustness of our findings, as well as policy-design issues. Appendix A presents robustness checks for our empirical analysis. Proofs are collected in Appendix B.

2 Self-Enforcing Cartels

Modeling strategy. McAfee and McMillan (1992)'s classic model of cartel behavior focuses on the constraints imposed by information revelation among asymmetrically informed cartel members. Instead, we are interested in the enforcement of cartel recommendations through repeated play. Viewed from the mechanism design perspective of Myerson (1986), McAfee and McMillan (1992) focus on truthful revelation, while we focus on obedience. The implications of the two frictions turn out to be different: interpreted in a procurement context, McAfee and McMillan (1992) show that collusion makes lower maximum prices desirable; we argue that higher minimum prices may help weaken cartels.

This different emphasis is reflected in our modeling choices. We have three main goals:

(i) we want to provide transparent intuition on how bidding constraints, here minimum prices, affect cartel behavior and the distribution of bids;

(ii) we want to convincingly assess whether enforcement constraints are a significant determinant of cartel behavior;

(iii) we want to exploit this understanding of cartel behavior to derive a test of collusion.

Given those goals, we use a tractable complete information model of collusion when fleshing out implications of our H_1 hypothesis ("there is collusion and enforcement constraints are binding"). To ensure that our test is not dependent on this simplification, we allow for more general informational environments when we characterize behavior under our H_0 hypothesis ("there is no collusion"). This results in a transparent but powerful test.

2.1 The model

Players and payoffs. Each period $t \in \mathbb{N}$, a buyer procures a single unit of a good through a first-price auction described below. A set $N = \{1, ..., n\}$ of long-lived firms is present in the market. In each period t, a subset $\widehat{N}_t \subset N$ of firms is able to participate in the auction. Participant set $\widehat{N}_t$ is exogenous, i.i.d. over time, and cartel members are exchangeable. In other terms, for all subsets $J \subset N$ of cartel members, and all permutations $\alpha : N \to N$ of cartel member identities, we have that

$$\text{prob}(\widehat{N}_t = J) = \text{prob}(\widehat{N}_t = \alpha(J)).$$

We think of this set of participating firms as those potentially able to produce in the current period.[2] In period t, each participating firm $i \in \widehat{N}_t$ can deliver the good at a cost $c_{i,t}$. Cost $c_{i,t}$ is drawn i.i.d. across participants and time periods from a c.d.f. F with support $[\underline{c}, \overline{c}]$ and density f with $f(c) > 0$ for all $c \in [\underline{c}, \overline{c}]$.

Firms are able to send transfers to each other, regardless of whether or not they participate in the auction. We denote by $T_{i,t}$ the net transfer received or sent by firm i. Let $x_{i,t} \in \{0, 1\}$ denote whether firm i wins the procurement contract in period t. Let $b_{i,t}$ denote her bid. We assume that firms have quasi-linear preferences, so that firm i's overall stage game payoff is

$$\pi_{i,t} = x_{i,t}(b_{i,t} - c_{i,t}) + T_{i,t}.$$

Firms value future payoffs using a common discount factor $\delta < 1$.

The stage game. The procurement contract is allocated according to a first price auction with constrained bids. Specifically, each participant must submit a bid b_i in the range $[p, r]$ where r is a maximum (or reserve) price, and $p < r$ is a minimum price. Bids outside of this range are discarded. The winner is the lowest bidder. The winner then delivers the good at the price she bid. For simplicity, we assume that $r \geq \overline{c}$.[3]

To keep the model tractable and to focus on how enforcement constraints affect bidding behavior, we assume that all firms belong to the cartel, and firms in the cartel observe one another's production costs. In addition, we assume that payoffs are transferable.[4] The timing of information and decisions within period t is as follows.

1. The set of participating firms $\widehat{N}_t$ is drawn and observed by all cartel members.
2. The production costs $\mathbf{c}_t = (c_{i,t})_{i \in \widehat{N}_t}$ of participating firms are publicly observed by cartel members.

[2] We consider the endogenous participation of entrants in Section 4.

[3] This assumption is largely verified in our data since 99.7% of auctions have a winner.

[4] The assumption that firms can transfer money is not unrealistic. Indeed, many known cartels used monetary transfers; see for instance Pesendorfer (2000), Asker (2010) and Harrington and Skrzypacz (2011). In practice these transfers can be made in ways that make it difficult for authorities to detect them, like sub-contracting between cartel members or, in the case of cartels for intermediate goods, between-firms sales.

3. Participating firms $i \in \widehat{N}_t$ submit public bids $\mathbf{b}_t = (b_{i,t})_{i \in \widehat{N}_t}$. This yields allocation $\mathbf{x}_t = (x_{i,t})_{i \in \widehat{N}_t} \in [0,1]^{\widehat{N}_t}$ such that: if $b_{j,t} > b_{i,t}$ for all $j \in \widehat{N}_t \backslash \{i\}$ then $x_{i,t} = 1$; if there exists $j \in \widehat{N}_t \backslash \{i\}$ with $b_{j,t} < b_{i,t}$ then $x_{i,t} = 0$.

   In the case of ties, we follow Athey and Bagwell (2001) and let the bidders jointly determine the allocation. Specifically, bidders simultaneously pick numbers $\gamma_t = (\gamma_{i,t})_{i \in \widehat{N}_t}$ with $\gamma_{i,t} \in [0,1]$ for all i, t. When lowest bids are tied, the allocation to a lowest bidder i is

$$x_{i,t} = \frac{\gamma_{i,t}}{\sum_{\{j \in \widehat{N} \text{ s.t. } b_{j,t} = \min_k b_{k,t}\}} \gamma_{j,t}}.$$

4. Firms make transfers $T_{i,t}$.

   Positive transfers are always accepted and only negative transfers will be subject to an incentive compatibility condition. We require exact budget balance within each period at the overall cartel level, i.e. $\sum_{i \in N} T_i = 0$.

Our model is intended to capture commonly observed features of public construction procurement. Governments need to procure construction services on an ongoing basis. They face a limited and stable set of firms that can potentially perform the work, a subset of which participates regularly. Legislation frequently requires participants to register, and governments make bids and outcomes public after each auction is completed. The repeated and public nature of the interaction makes collusion a realistic concern.

Note that procurement auctions with minimum acceptable bids are frequently used in practice. For instance, auctions with minimum bids are used for procurement of public works in several countries in the European Union and by local governments in Japan. The common rationale for introducing minimum bids in the auction is to limit the incidence of strategic default by non-performing contractors.[5]

[5] Such firms can be viewed as entrants with zero costs, producing a worthless good. Since our model and predictions focus exclusively on the bidders' side of the market, our predictions regarding bid distributions hold regardless of whether such non-performing firms are included in the model. However, the presence of non-performing firms would affect aggregate welfare assessments.

The repeated game. Interaction is repeated and firms can use the promise of continued collusion to enforce obedient bidding and transfers. Formally, bids and transfers need to be part of a subgame perfect equilibrium of the repeated game among firms.

The history among cartel members at the beginning of time t is

$$h^t = \{\mathbf{c}_s, \mathbf{b}_s, \gamma_s, \mathbf{x}_s, \mathbf{T}_s\}_{s=0}^{t-1}.$$

Let $\mathcal{H}^t$ denote the set of period t public histories and $\mathcal{H} = \bigcup_{t \geq 0} \mathcal{H}^t$ denote the set of all histories. Our solution concept is subgame perfect equilibrium (SPE), with strategies

$$\sigma_i : h_t \mapsto (b_{i,t}(\mathbf{c}_t), \gamma_{i,t}(\mathbf{c}_t), T_{i,t}(\mathbf{c}_t, \mathbf{b}_t, \gamma_t, \mathbf{x}_t))$$

such that bids $(b_{i,t}(\mathbf{c}_t), \gamma_{i,t}(\mathbf{c}_t))$ and transfers $T_{i,t}(\mathbf{c}_t, \mathbf{b}_t, \gamma_t, \mathbf{x}_t)$ can depend on all public data available at the time of decision-making.

We say that a strategy σ_i is non-collusive whenever bids at history h_t depend only on the costs of participating bidders at history h_t, but not their identities: $\sigma_i(h_t) = \hat{\sigma}_i \left(c_{i,t}, \{c_{j,t}\}_{j \in \widehat{N}_t \setminus i} \right)$ for all histories h_t. Since there is no persistent state in this game, non-collusive strategies coincide with Markov perfect strategies.

Definition 1 (collusive and competitive environments)**.** *We say that we are in a collusive environment if firms play a Pareto efficient SPE.*

We say that we are in a competitive environment if firms play a SPE in non-collusive strategies that is Pareto efficient among non-collusive equilibria.

Under complete information, the unique competitive equilibrium outcome is such that the winning bid is equal to the maximum between the second lowest cost and the minimum price. The contract is allocated to the bidder with the lowest cost whenever the winning bid is above the minimum price, and is allocated randomly among all bidders with cost below the minimum price when the winning bid is equal to the minimum price.

2.2 Optimal collusion

Denote by Σ the set of SPE in the repeated stage game. Let

$$V(\sigma, h_t) = \mathbb{E}_\sigma \left[\sum_{s \geq 0} \delta^s \sum_{i \in \widehat{N}_{t+s}} x_{i,t+s}(b_{i,t+s} - c_{i,t+s}) \middle| h_t \right]$$

denote the total surplus generated under equilibrium σ conditional on history h_t. We denote by

$$\overline{V}_p \equiv \max_{\sigma \in \Sigma} V(\sigma, h_0)$$

the highest equilibrium surplus sustainable in equilibrium.[6] We emphasize that this highest equilibrium value depends on minimum price p.

Given a history h_t and a strategy profile σ, we denote by $\beta(\mathbf{c}_t | h_t, \sigma)$ the bidding profile induced by strategy profile σ at history h_t as a function of realized costs $\mathbf{c}_t$.

Lemma 1 (stationarity)**.** *Consider a subgame perfect equilibrium σ that attains $\overline{V}_p$. Equilibrium σ delivers surplus $V(\sigma, h_t) = \overline{V}_p$ after all on-path histories h_t.*

There exists a fixed bidding profile β^* such that, in a Pareto efficient equilibrium, firms bid $\beta(\mathbf{c}_t | h_t, \sigma) = \beta^*(\mathbf{c}_t)$ after all on-path histories h_t.

For any $i \in N$ and any $\sigma \in \Sigma$, let

$$V_i(\sigma, h_t) = \mathbb{E}_\sigma \left[\sum_{s \geq 0} \delta^s (x_{i,t+s}(b_{i,t+s} - c_{i,t+s}) + T_{i,t+s}) \middle| h_t \right]$$

denote the expected discounted payoff that firm i gets in equilibrium σ conditional on history h_t. Let

$$\underline{V}_p \equiv \min_{\sigma \in \Sigma} V_i(\sigma, h_0)$$

denote the lowest possible equilibrium payoff for a given firm.

[6]The existence of surplus maximizing and surplus minimizing equilibria follows from Proposition 2.5.2 in Mailath and Samuelson (2006).

Given a bidding profile (β, γ), let us denote by $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ the induced winning bid and allocation profile for realized costs $\mathbf{c}$. For each firm i, we define

$$\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) \equiv \mathbf{1}_{\beta^W(\mathbf{c})>p} + \frac{\mathbf{1}_{\beta^W(\mathbf{c})=p}}{1 + \sum_{j \in \widehat{N} \setminus \{i\}: x_j(\mathbf{c})>0} \gamma_j(\mathbf{c})}.$$

Term $\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$ corresponds to a deviator's highest possible chance of winning the contract by attempting to undercut the equilibrium winning bid.

Lemma 2 (enforceable bidding)**.** *A winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ are sustainable in SPE if and only if for all $\mathbf{c}$,*

$$\sum_{i \in \widehat{N}} (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \leq \delta(\overline{V}_p - n\underline{V}_p). \qquad (1)$$

As in Levin (2003), a bidding profile can be implemented in SPE if and only if the sum of deviation temptations (both from bidders abstaining to bid above their cost, and bidders having to bid below their cost) is less than or equal to the total pledgeable surplus $\delta(\overline{V}_p - n\underline{V}_p)$, i.e. the difference between the highest possible continuation surplus, and the sum of minimal continuation surpluses guaranteed to each player in equilibrium.

For each cost realization $\mathbf{c}$, let $\mathbf{x}^*(\mathbf{c})$ denote the efficient allocation. It allocates the procurement contract to the participating firm with the lowest cost (ties are broken randomly). We define

$$b_p^*(\mathbf{c}) \equiv \sup \left\{ b \leq r : \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c})) \left[b - c_i\right]^+ \leq \delta(\overline{V}_p - n\underline{V}_p) \right\}.$$

For values of $\mathbf{c}$ such that $b_p^*(\mathbf{c}) > p$, this value is the highest enforceable winning bid when the cartel allocates the good efficiently. Note that $b_p^*(\mathbf{c})$ is always weakly greater than the second lowest cost.

Proposition 1. *On the equilibrium path, any efficient equilibrium bidding strategy sets winning bid $\beta_p^*(\mathbf{c}) = \max\{b_p^*(\mathbf{c}), p\}$ in every period. Moreover, the allocation is conditionally* *efficient: whenever $\beta_p^*(\mathbf{c}) > p$, the contract is allocated to the bidder with the lowest procurement cost.*

This result follows from obedience constraint (1). Bid $\beta_p^*(\mathbf{c})$ is the highest enforceable bid. Furthermore, allocating the good efficiently increases the surplus accruing to the cartel while also relaxing (1). Indeed, the lowest cost bidder has the largest incentives to undercut other bidders.

The firm's behavior in a competitive environment with complete information is an immediate corollary: it coincides with collusive behavior in a game with discount factor $\delta = 0$. For any profile of cost realizations $\mathbf{c}$, let $c_{(2)}$ denote the second lowest cost.

Corollary 1 (behavior under competition)**.** *In a competitive environment, the winning bid is $\beta_p^{\mathsf{comp}}(\mathbf{c}) = \max\{p, c_{(2)}\}$.*

We now clarify how minimum prices affect the set of payoffs that firms can sustain in SPE. We denote by $\beta_0^*(\underline{c})$ the lowest equilibrium bid in auctions with no minimum price. If we are in a collusive environment, $\beta_0^*(\underline{c})$ is observable from data: it is the lowest equilibrium winning bid.

Lemma 3 (worst case punishment)**.** *(i) $\underline{V}_0 = 0$, and $\underline{V}_p > 0$ whenever $p > \underline{c}$;*

(ii) there exists $\eta > 0$ such that for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $\overline{V}_p - n\underline{V}_p < \overline{V}_0 - n\underline{V}_0$.

Lemma 3(i) shows that with no minimum price, the cartel can force a firm's payoff down to a minmax value of 0, but that minmax values are bounded away from zero when the minimum price is within the support of procurement costs. Lemma 3(ii) establishes that the pledgeable surplus $\overline{V}_p - n\underline{V}_p$ that the cartel can use to provide incentives decreases after introducing a minimum price. The reason for this is that a minimum price p in the neighborhood of $\beta_0^*(\underline{c})$ increases the lowest equilibrium value $\underline{V}_p$ by an amount bounded away from 0, even for $\eta > 0$ small. This tightens enforcement constraint (1) and reduces the bids that the cartel can sustain in equilibrium.

3 Empirical implications

The effect of minimum prices on the distribution of bids. We now delineate several empirical implications of our model. Specifically, we contrast the effect that a minimum price has on the distribution of winning bids under competition and under collusion.

Proposition 2 (the effect of minimum prices on bids). *Under collusion, minimum prices can induce a first-order stochastic dominance drop in the right tail of winning bids. The opposite holds under competition. Formally:*

(i) there exists $\eta > 0$ such that, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $q > p$,

$$prob(\beta_p^* \geq q | \beta_p^* \geq p) \leq prob(\beta_0^* \geq q | \beta_0^* \geq p),$$

the inequality being strict for some $q > p$ whenever $prob(\beta_0^* < r) > 0$.

(ii) for all $p > 0$ and all $q > p$,

$$prob(\beta_p^{\mathsf{comp}} \geq q | \beta_p^{\mathsf{comp}} > p) = prob(\beta_0^{\mathsf{comp}} \geq q | \beta_0^{\mathsf{comp}} > p).^7$$

Consider now equilibrium bidding data from auctions without minimum price. Bidders may be either collusive or competitive. Let $\underline{\beta}_0^{\mathsf{obs}}$ denote the lowest observed winning bid. Since competitive bids are not affected when the minimum price is below the observed distribution of winning bids, we obtain the following corollary.

Corollary 2 (robust policy take-away). *Regardless of whether or not there is collusion, setting a minimum price $p \leq \underline{\beta}_0^{\mathsf{obs}}$ can only cause a first-order dominance drop in procurement costs.*

[7] Conditioning on a strict inequality is meaningful because the distribution of winning bids may have mass points at the minimum price, which we need to correctly take care of. When the mass of bids at the minimum price is small, the conditioning events in Propostion 2 (*i*) and (*ii*) coincide. In our data 1.2% of auctions with a minimum price have a winning bid equal to the minimum price.

Proposition 2 provides a joint test of collusion and of the fact that cartel enforcement constraints are binding. Consider the introduction of a minimum price close to the minimum observed winning bid. Under collusion, the introduction of such a minimum price will lead to a first-order stochastic dominance drop in the distribution of winning bids to the right of the minimum price. Under competition, the introduction of minimum prices will lead to a (weak) first order stochastic dominance increase in the distribution of winning bids.

Proposition 2(ii) makes a clear prediction that allows us to test the H_0 hypothesis that there is competition. Proposition 2(i) serves to clarify likely comparative statics under the H_1 hypothesis that there is collusion. This increases the power of our test, and in principle, would allow us report p-values from one-sided tests. However, the data is sufficiently clear that we lose no significance from reporting p-values from two-sided tests, and do so in our empirical analysis.[8]

We strengthen this test by showing that Proposition 2(ii) extends to asymmetric information settings.

Competitive comparative statics under asymmetric information. We assume now that firms are privately informed about their own procurement cost. Let $b_0^{AI} : [\underline{c}, \overline{c}] \rightarrow \mathbb{R}_+$ denote the equilibrium bidding function in the unique symmetric equilibrium of the first-price procurement auction with reserve price r and *no minimum price*.

Proposition 3. *Under private information, a first-price auction with reserve price r and minimum price $p < \min\{r, \overline{c}\}$ has a unique symmetric equilibrium with bidding function b_p^{AI}.*

If $b_0^{AI}(\underline{c}) \geq p$, then $b_p^{AI}(c) = b_0^{AI}(c)$ for all $c \in [\underline{c}, \overline{c}]$;

[8] Proposition 2(i) shows how minimum prices affect the *conditional* distribution of winning bids under collusion (i.e., winning bids above the minimum price). We stress that, under collusion, minimum prices also produce a drop in the right tail of the *unconditional* distribution of winning bids. Indeed, Proposition 1 and Lemma 3 imply that, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $q > p$, $\text{prob}(\beta_p^* \geq q) \leq \text{prob}(\beta_0^* \geq q)$.

If $b_0^{AI}(\underline{c}) < p$, there exists a cutoff $\hat{c} \in (\underline{c}, \overline{c})$ with $b_0^{AI}(\hat{c}) > p$ such that

$$b_p^{AI}(c) = \begin{cases} b_0^{AI}(c) & \text{if } c \geq \hat{c}, \\ p & \text{if } c < \hat{c}. \end{cases}$$

An immediate corollary of Proposition 3 is that minimum prices can only yield a first order stochastic dominance increase in the right tail of winning bids. Let $\beta_p^{AI}(\mathbf{c}) \equiv \min_i b_p^{AI}(c_i)$ denote winning bids.

Corollary 3. *For all $p > 0$ and all $q > p$,*

$$prob(\beta_p^{AI} \geq q \mid \beta_p^{AI} > p) \geq prob(\beta_0^{AI} \geq q \mid \beta_0^{AI} > p).$$

This strengthens the test of collusion provided in Proposition 2. A first-order stochastic dominance drop in the right tail of winning bids cannot be explained away by a competitive model with incomplete information.

We end this section by noting that the predictions of Proposition 2(i) cannot be explained by competitive models with incomplete information and asymmetric bidders or interdependent in costs. Indeed, by arguments similar to the ones used in the proofs of Proposition 3 and Corollary 3, under either of these settings a binding minimum price will lead to (i) a mass of bids at the minimum price, and (ii) a gap in the support of the winning bid distribution just above the minimum price. As a result, in these competitive environments a minimum price cannot generate a first-order stochastic dominance drop in the right tail of the winning bid distribution.

4 Entry

We now extend the model of Section 2 to allow for entry. The goal of this extension is twofold. First, we want to show that the testable predictions in Proposition 2 continue to hold when non-cartel members can participate. Second, this extension allows us to derive additional predictions on the differential effect of minimum prices on cartel members and entrants. These additional predictions are important since they let us distinguish our model from one in which the introduction of minimum prices broadly increases entry.

We assume that in each period t, a short-lived firm may bid in the auction along with participating cartel members $\widehat{N}_t$. To participate, the short-lived firm has to pay an entry cost k_t drawn i.i.d. over time from a distribution F_k with support $[0, \overline{k}]$. The distribution of entry costs may have a point mass at 0. We let $E_t \in \{0, 1\}$ denote the entry decision of the short-lived firm in period t, with $E_t = 1$ denoting entry.

Upon paying the entry cost, the short-lived firm learns its cost $c_{e,t}$ for delivering the good, which is drawn i.i.d. from a c.d.f. F_e with support $[\underline{c}, \overline{c}]$ and density f_e. We assume that the short-lived firm's entry decision and her procurement cost upon entry $c_{e,t}$ are publicly observed.

The timing of information and decisions within each period t is as follows:

1. The short-lived firm's entry cost k_t is drawn and privately observed. The short-lived firm makes entry decision E_t, which is observed by cartel members.

2. The set of participating cartel members $\widehat{N}_t$ is drawn and observed by both cartel members and the short-lived firm.

3. The production costs $\mathbf{c}_t$ of participating firms are drawn and publicly observed by all firms.

4. Participating firms submit public bids $\mathbf{b}_t = (b_{i,t})$ and numbers $\gamma = (\gamma_{i,t})$ with $\gamma_{i,t} \in [0, 1]$, resulting in allocation $\mathbf{x}_t = (x_{i,t})$.[9]

5. Cartel members make transfers $T_{i,t}$ to one another.

The public history at the beginning of time t is now $h^t = \{E_s, \mathbf{c}_s, \mathbf{b}_s, \gamma_s, \mathbf{x}_s, \mathbf{T}_s\}_{s=0}^{t-1}$, and is observed by both cartel members and entrants. Let $\mathcal{H}^t$ denote the set of period t public

[9]The allocation is determined in the same way as in Section 2.

histories and $\mathcal{H} = \bigcup_{t\geq 0} \mathcal{H}^t$ denote the set of all histories. Our solution concept is subgame perfect public equilibrium, with strategies

$$\sigma_i : h_t \mapsto (b_{i,t}(E_t, \mathbf{c}_t), \gamma_{i,t}(E_t, \mathbf{c}_t), T_{i,t}(E_t, \mathbf{c}_t, \mathbf{b}_t, \gamma_t, \mathbf{x}_t))$$

for cartel members and strategies

$$\sigma_e : h_t \mapsto (E_t(k_t), b_{e,t}(k_t, \mathbf{c}_t), \gamma_{e,t}(k_t, \mathbf{c}_t))$$

for the short-lived firms.

The analysis of this model is essentially identical to that of the model of Section 2 except that now the cartel will deter entry in addition to enforcing collusive bidding. Given that procurement costs are observed after entry, entry depends only on cost k_t and takes a threshold-form. Entrants with entry costs above a certain level are deterred from entering, while entrants with an entry cost below this threshold participate in the auction.

For concision, we focus on extending the main empirical predictions of our model. Appendix B provides further details on optimal cartel behavior.

Proposition 4 (the effect of minimum prices on bids). *(i) Under collusion, there exists $\eta > 0$ such that for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $q > p$, and $E \in \{0, 1\}$,*

$$prob(\beta_p^* \geq q | \beta_p^* \geq p, E) \leq prob(\beta_0^* \geq q | \beta_0^* \geq p, E).$$

(ii) Under competition, for all $p > 0$, $q > p$, and $E \in \{0, 1\}$,

$$prob(\beta_p^{\mathsf{comp}} \geq q | \beta_p^{\mathsf{comp}} > p, E) = prob(\beta_0^{\mathsf{comp}} \geq q | \beta_0^{\mathsf{comp}} > p, E).$$

In other words, the contrasting comparative statics of Proposition 2 continue to hold conditional on the entrant's entry decision.

A notable new prediction is that under collusion minimum prices have different impacts on cartel and entrant winners.

Proposition 5 (differential effect of minimum prices on bids)**.** *Under collusion, there exists* $\eta > 0$ *such that, for all* $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ *and all* $q > p$*:*

(i) $prob(\beta_p^* \geq q | \beta_p^* \geq p, E, \textit{cartel wins}) \leq prob(\beta_0^* \geq q | \beta_0^* \geq p, E, \textit{cartel wins})$ *for* $E = 0, 1$*;*

(ii) $prob(\beta_p^* \geq q | \beta_p^* > p, \textit{entrant wins}) = prob(\beta_0^* \geq q | \beta_0^* > p, \textit{entrant wins})$*.*

In words, minimum prices should only affect the right tail of winning bids when the winners are cartel members. The intuition behind this stark prediction is straightforward. Since costs are complete information, under optimal entry deterrence entrants either win at the minimum price, or at their production cost. As a result, the right tail of winning bids conditional on an entrant being the winner is independent of the cartel's continuation values, and independent of the minimum price. The prediction holds approximately if cartel members only get a noisy but precise signal of the entrant's production cost.

Note that if minimum prices were associated with greater entry (for any of many plausible reasons), then minimum prices would reduce the distribution of winning bids even in a competitive environment. However, such entry would affect the bids of both entrants and cartel members. Proposition 5 allows us to distinguish our model from this alternative theory. Under collusion, minimum prices should have a differential impact of cartel and entrant winners.

5 Empirical Analysis

Sections 2, 3 and 4 lay out a theoretical mechanism through which minimum prices can affect the distribution of winning bids, and clarify its implications for data. This empirical section aims to assess the relevance of this mechanism in a real life context and answer the following questions: are enforcement constraints binding? are they affected by minimum prices? how is within-cartel discipline affected? how is entry affected?

We provide empirical answers to these questions using auction data from Japanese cities located in the Ibaraki prefecture. The analysis focuses on the three cities among the ten largest for which data was available: Tsuchiura, Tsukuba and Ushiku. The data covers public work projects auctioned off between May 2007 and March 2015, corresponding to 3103 auctions, 1565 of which are from the treatment city (Tsuchiura).[10]

Throughout the period, all cities use first-price auctions. On October 28^{th} 2009, the city of Tsuchiura implemented a policy change, moving from a zero minimum price to a strictly positive minimum price ranging between 70% and 85% of the reserve price. The remaining cities use first-price auctions with no minimum price throughout the period.[11] This lets us explore the effect of minimum prices on bidder behavior using a differences-in-differences approach.

5.1 Some facts about the data

Sample selection. The sample of cities was selected as follows. In a study of paving auctions, Ishii (2008) notes the use of minimum prices in Japanese procurement auctions. The author was able to point us to our treatment city. We then proceeded to search for all publicly available data from the 10 most populous cities in the prefecture. We kept all cities that had public data available covering the relevant policy-change period. This left us with the three cities included in the study. The cities are broadly comparable: their population ranges from 82K to 215K, with Tsuchiura at 143K.[12] They are located within 15km of one another, and within 75km of Tokyo. We treat these three cities as distinct markets, and present supporting evidence that this is indeed the case in Section 6.

[10] Auction data is publicly available from the cities' websites.

[11] Minimum prices are calculated according to an explicit formula that applies pre-specified discount rates to engineering estimates of different components of the project.

[12] Notable trivia: Tsuchiura is a sister city of Palo Alto, CA.

Policy change. The minimum prices used in our treatment city are chosen by a formal rule and should not be interpreted as having any signalling content. Minimum prices range between 70% to 85% of the reserve price, with the 25th, 50th and 75th quantiles respectively at 80%, 82% and 84%. There is no evidence that the policy change was triggered by city specific factors also affecting the distribution of bids. Publicly available policy documents, as well as exchanges with city officials confirm that minimum prices were introduced to avoid excessively low bids that could only be executed at the expense of quality.[13] Throughout the analysis we include city specific time-trends and control for Japanese GDP.

Descriptive statistics. Some facts about our sample of auctions are worth noting. The first is that although all auctions include a reserve price, these reserve prices are not set along the lines of Myerson (1981) or Riley and Samuelson (1981) to extract greater surplus for the city. Rather, consistent with recorded practice, reserve prices are engineering estimates (Ohashi, 2009, Tanno and Hirai, 2012, Kawai and Nakabayashi, 2014) that provide an upper-bound to the range of possible costs for the project. This is corroborated by the fact that 99.7% of auctions have a winner. This lets us treat reserve prices as an exogenous scaling parameter and use it to normalize the distribution of bids to $[0, 1]$.[14] Normalized winning bids are defined as follows:

$$norm\_winning\_bid = \frac{winning\_bid}{reserve\_price}.$$

This normalization lets us take the comparative statics of Propositions 2, 3 and 4 to the data, even though there is heterogeneity in minimum prices. Indeed, our first-order stochastic dominance comparative statics hold for normalized-bids conditional on reserve prices. Furthermore the ratio of minimum price to reserve price is very homogeneous (the 25^{th}

[13] See `http://www.city.tsuchiura.lg.jp/data/doc/1394785303_doc_10_0.pdf` for policy documents.

[14] Appendix A replicates the analysis using log winning bids rather than normalized bid, and using log reserve prices as a control. Qualitative results are unaffected, and the coefficient on log reserve prices is precisely estimated as 1.

quantile being at .8 and the 75^{th} quantile at .84). Provided that the distribution of reserve prices is unchanged around the policy change, this implies (by integrating over reserve prices) that the comparative statics of Propositions 2, 3 and 4, hold for normalized-bids, conditional on normalized bids being above the minimum-price-to-reserve-price ratio. We provide evidence that reserve prices are unaffected by treatment in Appendix A (see Tables A.2 and A.3). As a robustness test, we also study the distribution of log-winning-bids using reserve prices as a control variable (see Table A.4 and Figure A.1). Our findings are unchanged.

The distribution of winning bids is closely concentrated near reserve prices. Indeed, the aggregate cost savings from running an auction rather than using reserve prices as a take-it-or-leave-it offer are equal to 4.9%. This could be because reserve prices are obtained through very precise engineering estimates, but this provides justifiable concern that collusion may be present. It is also worth observing that the 10^{th} quantile of the distribution of normalized winning bids is equal to 83% of the reserve price. This means that minimum prices (set within 70% and 85% of reserve prices) are in the lower tail of the distribution of winning bids (the median minimum price is in the first decile of the distribution of winning bids), plausibly satisfying the premise of Propositions 2(i), 3(ii) and 4(iii).

This implies clear predictions. If there is no collusion the introduction of a low minimum price should not change the right tail of wining bids. In fact, in a competitive environment, introducing such a low minimum price should have a very limited effect on bidding behavior. In contrast, if there is collusion, we anticipate a drop in the right tail of winning bids.

5.2 The impact of minimum prices on the distribution of winning bids

Figure 1 plots distributions of normalized winning bids in treatment and control cities before and after the policy change (which occurred on October 28^{th} 2009). The data appears well suited to a difference-in-differences approach. The distribution of normalized winning bids in the control cities seems essentially unchanged, while the distribution of normalized winning bids in the treatment city experiences a significant change.

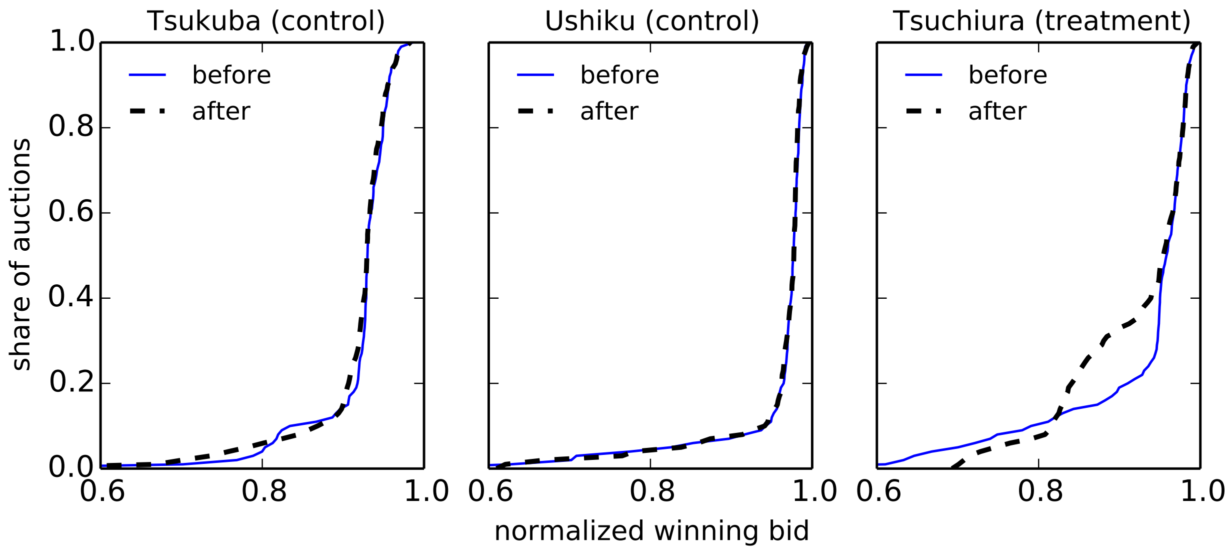


Figure 1: Distribution of winning bids, before and after treatment: 2007-2009, 2009-2011.

We take a first pass at the data using a simple difference-in-differences approach. We define variables

$$window = \mathbf{1}_{date \in \{\text{October } 28^{th} \text{ 2009} \pm 24 \text{ months}\}},$$

$$policy\_change = window \times \mathbf{1}_{date \geq \text{October } 28^{th} \text{ 2009}}$$

and perform both OLS and quantile regressions of the linear model with city fixed-effects, year fixed-effects and city-specific trends:

$$\begin{aligned} norm\_winning\_bid \sim \beta_0 window + \beta_1 policy\_change + \beta_2 \log GDP \\ + year\_fe + city\_fe + city\_trends. \end{aligned} \tag{2}$$

To match the theoretical predictions of Proposition 2, we perform regressions on the subsample of auctions whose normalized winning bid is above .8, corresponding to the sample of auctions whose winning bids are (or would have been) above the minimum price. For completeness, we also report mean effects for the unconditional sample of auctions.[15] Throughout this section we refer to the sample as *conditional*, when normalized winning bids are constrained to be above .8, and as *unconditional* when normalized winning bids are unconstrained.

The outcome of regression (2), summarized in Table 1, strongly vindicates the mechanism we analyze in Sections 2, 3 and 4. The introduction of minimum prices is associated with a first-order stochastic dominance drop in the right tail of winning bids. The implication is not only that there is collusion, but that cartel enforcement constraints are binding, and that the sustainability of collusion is limited by price constraints.

Note that the mean effect of minimum prices on the average winning bid is meaningful. Given that running an auction yields roughly a 5% drop in procurement costs relative to using reserve prices as take-it-or-leave-it offers, a 1.2% drop in winning bids corresponds to a 24% increase in the effectiveness of auctions.

	unconditional sample	sample s.t. $norm\_winning\_bid > .8$				
norm_winning_bid	mean effect	$q = .1$	$q = .2$	$q = .4$	$q = .6$	$q = .8$
window	-0.006	-0.008	-0.002	-0.003	-0.004*	-0.003*
	(0.006)	(0.012)	(0.006)	(0.004)	(0.002)	(0.002)
policy_change	-0.012**	-0.080***	-0.079***	-0.014***	-0.000	.004**
	(0.005)	(0.011)	(0.005)	(0.003)	(.002)	(.002)
ln_gdp	0.145***	0.098	-0.016	-0.046	-0.041*	-0.045**
	(0.056)	(0.118)	(0.056)	(.035)	(.023)	(.018)

***, ** and * respectively denote effects significant at the .1, .05 and .01 level.

Table 1: Difference-in-differences analysis of the effect of minimum prices on normalized winning bids. OLS estimates for unconditional sample and quantile regression estimates for conditional sample; N = 3070 (unconditional), 2873 (conditional).

We compare more accurately the effect of minimum prices on the quantiles of winning

[15] Figure A.1 shows that the findings are robust to using different thresholds for minimum prices.

bids using the framework of Athey and Imbens (2006).

Change-in-changes. The framework of Athey and Imbens (2006) allows us to compute counterfactual estimates of the distribution of normalized winning bids in our treatment city, absent minimum prices. The actual and counterfactual quantiles of normalized winning bids, conditional on prices being above 80% of the reserve price are given in Table 2 and Figure 2.[16] We use both Tsukuba and Ushiku as a controls.[17]

quantile of conditional dist	0.1	0.25	0.5	0.75	0.9
actual – counterfactual	-0.051***	-0.054***	0.003	0.004	0.004*
std error	(0.018)	(0.02)	(0.005)	(0.003)	(0.003)
actual	0.833	0.881	0.959	0.977	0.984
counterfactual	0.884	0.935	0.956	0.973	0.98

Table 2: quantiles of the actual and counterfactual conditional distributions of normalized winning bids ($> .8$)

We emphasize that the quantiles reported here are those of the distribution of normalized winning bids, conditional on the winning bid being above 80% of the reserve price.

The data is clear: there is a significant first-order stochastic dominance drop in the conditional distribution of winning bids. Propositions 2, 3 and 4 provide an unambiguous interpretation for this finding: (i) there is collusion; (ii) the cartel is constrained by enforcement constraints; (iii) these enforcement constraints are worsened by the introduction of minimum prices.

Single city regression. Our analysis going forward focuses on obtaining a better understanding of the channels through which price constraints affect the distribution of winning

[16] The results are unchanged if we consider the distribution of normalized winning bids conditional on prices being above .75, .82, or .85 of the reserve price, or if we use raw winning bids. See Appendix A for details.

[17] We do not merge the control data. This would bias results since the relative sample size of the pre and post period is different across control cities. Instead we separately run the algorithm of Athey and Imbens (2006) for each control city, and then average the corresponding counterfactual estimates. We report bootstrapped standard errors for our aggregated estimates.

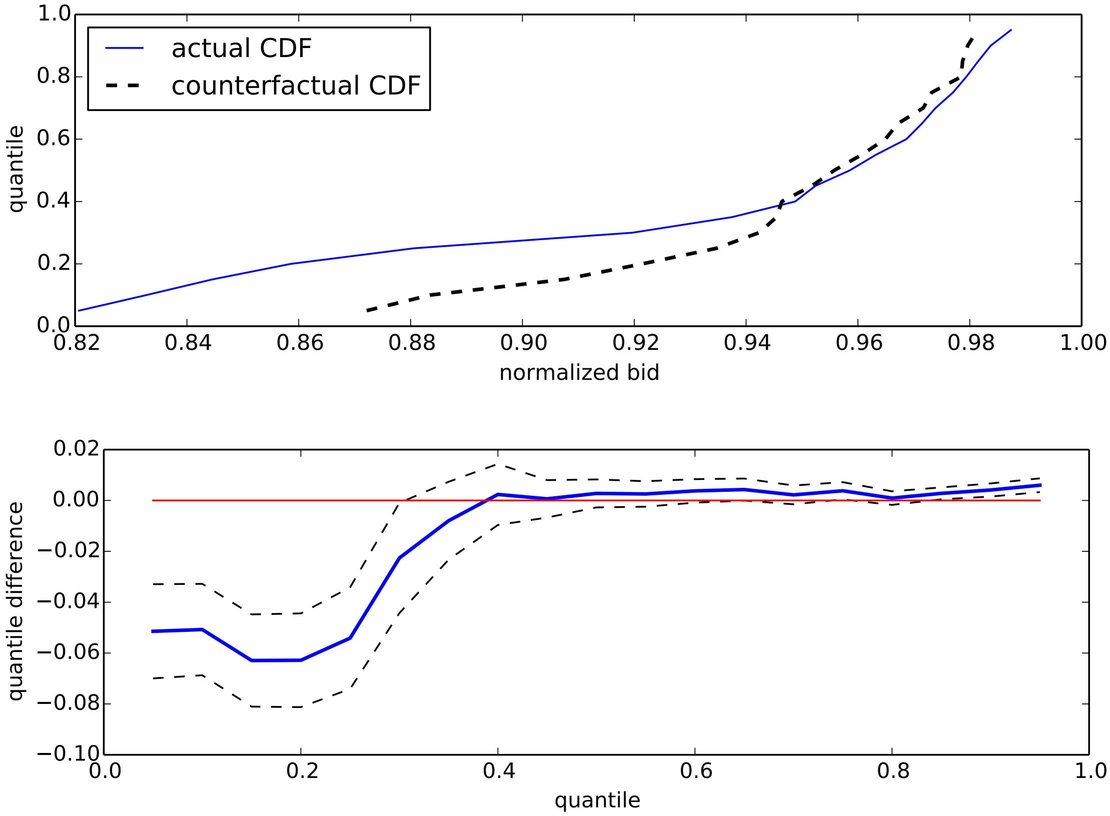


Figure 2: Actual and counterfactual conditional distributions of normalized winning bids; quantile differences; ($norm\_winning\_bid > .8$).

bids. For this purpose, we must rely on data from our treatment city alone. This is due to data restrictions: public data available from our treatment city provides detailed information about individual auctions, including the names of bidders and their bids. No such data is available in our control cities.

We use a before/after design and begin by replicating the results from our differences-in-differences framework.[18] Recall the definition of variables

$$window = \mathbf{1}_{date \in \{\text{October } 28^{th} \text{ } 2009 \pm 24 \text{ months}\}},$$

$$policy\_change = window \times \mathbf{1}_{date \geq \text{October } 28^{th} \text{ } 2009}.$$

We perform both OLS and quantile regressions of the linear model

$$norm\_winning\_bid \sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta controls \tag{3}$$

where *controls* (used throughout the analysis) include Japanese $\log GDP$ as well as a time trend. We report effects for the subsample of auctions such that the normalized winning bid is above .8, as well as the mean effect for the unconditional sample. Table 3 reports the outcome of regression (3).

	unconditional sample	sample s.t. $norm\_winning\_bid > .8$				
norm_winning_bid	mean effect	$q = .1$	$q = .2$	$q = .4$	$q = .6$	$q = .8$
window	0.001	-0.016	-0.001	-0.000	0.002	0.000
	(0.007)	(0.019)	(0.009)	(0.005)	(0.026)	(0.002)
policy_change	-0.021***	-0.070***	-0.081***	-0.011***	-0.006***	-0.003*
	(0.006)	(0.015)	(0.008)	(0.004)	(0.002)	(0.002)
ln_gdp	0.434***	0.367**	0.214**	0.074	0.063***	0.022
	(0.068)	(0.188)	(0.093)	(0.052)	(0.025)	(0.019)
year	0.005***	0.000	0.001	0.003	0.002***	0.001***
	(0.001)	(0.003)	(0.001)	(0.001)	(0.001)	(0.000)

Table 3: The effect of minimum prices on winning bids. OLS estimates for unconditional sample and quantile regression estimates for conditional sample; N=1539 (unconditional), 1457 (conditional).

While the results are not precisely identical, these magnitudes match those of our difference-in-differences design (Table 1), which gives us some confidence that our controls are sufficient

[18]This analysis implicitly assumes that pre-change behavior wasn't affected by expectations of change, and that post-change behavior adjusted immediately to its new environment. Appendix A shows that our results are robust to excluding auctions occuring in the 6 months before and after the policy change.

to make a single-city analysis not-implausible.

5.3 The Impact of minimum prices on entry and cartel behavior

We wish to better understand the channels through which minimum prices affect the distribution of winning bids. Our model makes specific predictions: we anticipate that minimum prices should affect the distribution of winning bids for cartel winners, but not for entrant winners (Proposition 5). In addition, we are interested in understanding how the effect of minimum prices decomposes into greater entry, and worse collusion among cartel members keeping entry constant.

Consistent with the theory, we define cartel members and entrants according to the frequency with which they participate in auctions. Our treatment city exhibits considerable heterogeneity in the degree of bidder activity over the seven years spanned by our data. The median number of auctions a bidder participates in is 4, whereas the average is at 22. The 25% most active bidders make up 80% of the auction$\times$bidder data. Accordingly, we define as cartel members the 25% most active bidders (58 out of 234 bidders). We define entrants as non-cartel-members.

Greater entry vs. worse collusion. We assess the relative importance of greater entry and worse within-cartel enforcement by first assessing the impact of minimum prices on entry, and second, by assessing the impact of minimum prices on winning bids, controlling for entry. We report regressions using both the number of entrants, and the total number of bidders to measure broader participation by cartel members. The data suggests that cartel participation itself is affected by minimum prices, which is not captured in our model.

As expected, minimum prices increase both entry and participation. Table 4 reports the results from OLS estimation of the following linear models:

$$num\_entrants \sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta controls \quad (4)$$

$$num\_bidders \sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta controls \quad (5)$$

$$\sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta_3 num\_entrants + \beta controls \quad (6)$$

	num_entrants	num_bidders	num_bidders	num_bidders
window	-0.495***	-0.781***	-.552***	-.521***
	(0.134)	(0.176)	(.166)	(.158)
policy_change	0.434***	0.873***	.673***	.711***
	(0.107)	(0.141)	(.133)	(.127)
ln_gdp	-5.031***	-2.18	.144	1.11
	(1.321)	(1.733)	(1.631)	(1.56)
year	-0.042*	-0.394***	-.375***	-.41***
	(0.022)	(0.029)	(.027)	(.026)
num_entrants			.462***	.503***
			(.031)	(.03)
ln_reserve_price				.426***
				(.026)

Table 4: The effect of minimum prices on entry and participation; $N = 1539$.

The introduction of minimum prices has a significant effect on both entry and participation by cartel members, adding on average .43 entrants and .87 bidders to auctions. These numbers are large given that the mean and median number of participants per auction are respectively 3.8 and 4. Note that participation increases even controlling for new entrants, suggesting that participation by cartel members is an endogenous decision. The results are unchanged when controlling for the auction's reserve price.

Next, we examine the effect of minimum prices on winning bids controlling for participation, using the linear model

$$norm\_winning\_bid \sim \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta_3 num\_bidders + \beta controls. \quad (7)$$

whose estimates are reported in Table 5.

norm_winning_bid	unconditional sample mean effect	sample s.t. *norm_winning_bid* $> .8$ $q = .1$	$q = .2$	$q = .4$	$q = .6$	$q = .8$
window	-0.008	-0.021	-0.008	-0.005	-0.004	-0.001
	(0.007)	(0.02)	(0.009)	(0.005)	(0.003)	(0.002)
policy_change	-0.01*	-0.054***	-0.057***	-0.015***	-0.004**	-0.002
	(0.005)	(0.016)	(0.007)	(0.004)	(0.002)	(0.002)
lngdp	0.408***	0.448**	0.266***	0.071	0.041*	0.02
	(0.065)	(0.198)	(0.085)	(0.046)	(0.025)	(0.018)
num_bidders	-0.012***	-0.008***	-0.01***	-0.01***	-0.008***	-0.005***
	(0.001)	(0.003)	(0.001)	(0.001)	(0.0)	(0.0)
year	0.0	-0.005	-0.003*	-0.001	-0.001	-0.0
	(0.001)	(0.004)	(0.002)	(0.001)	(0.0)	(0.0)

Table 5: The effect of minimum prices on winning bids, controlling for participation. OLS estimates for unconditional sample and quantile regression estimates for conditional sample; N= 1539 (unconditional), 1457 (conditional).

Regression (7) assigns similar shares of the drop in mean normalized winning bids (-2.1%, Table 3) to the "greater-entry" channel ($-1.2\% \times .87 = 1.04\%$) and the "worse within-cartel collusion" channel (-1%). The "worse within-cartel collusion" channel continues to come out significantly in the conditional distribution of winning bids.

We emphasize that the findings of Table 5 do not arise naturally from a model of competitive bidding: controlling for the number of bidders, minimum prices should not cause a first-order stochastic dominance drop in the right tail of winning bids under competition (Proposition 4).

Who does the policy change affect? Proposition 5 offers another test of the mechanism analyzed in Sections 2, 3 and 4. Under collusion, our theory predicts that the price paid by winning cartel members should go down, but not the price paid by winning entrants. The opposite should hold under competition. Conditional and unconditional OLS estimates of the linear model

$$norm\_winning\_bid \sim \beta_0 + \beta_1 window + \beta_2 cartel\_winner + \beta_3 policy\_change + \beta_4 cartel\_winner \times policy\_change + \beta controls \tag{8}$$

are reported in Table 6.

normalized_winning_bid	uncond. mean		cond. mean (> .8)	
cartel_winner	0.027***	0.028***	0.01***	0.012***
	(0.005)	(0.005)	(0.004)	(0.003)
cartel_winner × policy_change	-0.027***	-0.022**	-0.024***	-0.02***
	(0.009)	(0.009)	(0.006)	(0.006)
window	0.0	-0.009	-0.001	-0.007
	(0.007)	(0.007)	(0.005)	(0.004)
policy_change	0.002	0.009	-0.004	-0.001
	(0.009)	(0.009)	(0.006)	(0.006)
ln_gdp	0.428***	0.399***	0.146***	0.135***
	(0.068)	(0.065)	(0.046)	(0.043)
year	0.005***	0.0	0.002**	-0.002***
	(0.001)	(0.001)	(0.001)	(0.001)
num_bidders		-0.012***		-0.01***
		(0.001)		(0.001)

Table 6: The effect of minimum prices on cartel members and entrants. OLS estimates for unconditional sample and conditional sample; N=1539 (unconditional), 1457 (conditional).

The findings are entirely consistent with the predictions of our model under collusion: absent minimum prices, cartel winners obtain contracts at higher prices; the introduction of minimum prices reduces winning bids only for cartel winners and not for entrant winners.

This rules out competitive models in which minimum prices would increase potentially unobserved entry. In such models the bids of both cartel and entrant winners would be affected by the introduction of minimum prices.

6 Discussion

6.1 Summary

This paper provides a tractable framework to analyze the effect of price constraints on repeated collusion. Our model delivers a simple intuition: price constraints limit the range of continuation equilibrium payoffs, making cartel enforcement and entry deterrence more difficult. Our model yields a number of transparent empirical predictions that allow us to test whether there is collusion, whether cartel-enforcement constraints are binding, and whether they are affected by minimum prices.

We take those predictions to procurement data from Japan, and confirm that the channel emphasized in this paper (and more broadly in the relational contracting literature) is indeed relevant. There is collusion, cartel enforcement constraints are binding, and price constraints can weaken enforcement.

In the remainder of this section we discuss alternative models of entry, and provide additional empirical support for the main qualitative features of our model and for our interpretation of the data. We also discuss design issues related to the use of minimum prices as well as modeling challenges for future work.

6.2 Alternative models of entry

Our model of entry assumes random participation by cartel members and that bidders know the set of participants before placing their bids. Under these assumptions, Proposition 4 shows that the predictions of our baseline model in Proposition 2 continue to hold conditional on entry.

Other models of entry would yield different results. If participation is unknown at the bidding stage, minimum prices may increase the expected number of competitors and therefore induce firms to place lower bids. Similarly, if bidders are partially informed about their procurement costs prior to entry and face heterogenous entry costs, minimum prices could change the composition of participants in favor of more efficient firms and also lead to lower equilibrium bids.

These competitive models offer alternative explanations of how minimum prices may lead to lower winning bids even under competition. As we highlight throughout the paper, these alternative models of entry do not predict that the effect of minimum prices should be concentrated in auctions won by cartel members (i.e. firms that participate often (Table 6). Indeed, in a model in which the set of participants is unknown, all firms would place lower bids after the introduction of a minimum price if they expect more competitors. Similarly, if minimum prices affect the composition of participants and lead to a lower winning bid distribution, all participating firms would respond by bidding more aggressively.[19] This vindicates our modeling choices.

6.3 Further empirical investigation

Our model and our interpretation of the data relies on several assumptions which can be motivated from data. We briefly summarize our findings below, and present more detailed results in Appendix A.

Smooth equilibrium adjustments. Propositions 2, 3 and 4 provide a test of collusion by contrasting the comparative statics of the distribution of winning bids following the introduction of minimum prices, depending on whether we are in a collusive or competitive environment. These comparative statics presume that bidders are in equilibrium given the existing policy, which is necessarily an approximation. Indeed, although communication with city officials suggest that the move to a minimum price format was unexpected, it is still

[19]Moreover, as we discuss at the end of Section 3, in competitive models with asymmetric information the introduction of a binding minimum price will typically lead to a mass of bids at the minimum price and a gap in the distribution of winning bids just above the minimum price. As a result, in these competitive models the introduction of a minimum price cannot generate a drop in the right-tail of the distribution of winning bids.

possible that the anticipation of the change may have affected behavior before the change, or that behavior after the change did not immediately move to the equilibrium corresponding to the new policy.

A priori, smooth equilibrium adjustment would bias estimates against our findings. And indeed, replicating our analysis excluding auctions occurring in six months period before and after the policy change strengthens our results (see Table A.1).

Separate markets. Our difference-in-differences analysis presumes that control cities are not affected by the policy change. One concern with this assumption is that the cities we study are very close from one another. This improves the quality of our control since the cities are presumably hit by the same local shocks. However, it is plausible that some of the cartel members active in our treatment city may also be active in our control cities. If that is the case, the introduction of minimum bids in Tsuchiura may also cause a shift in the distribution of bids in control cities.

If present, this effect should lead to an attenuation bias: part of the treatment effect would be interpreted as a common shock. Still, we believe that the assumption of separate markets is correct and argue so (see Appendix A) by showing that the bulk of cartel members active in the treatment city are geographically much closer to the treatment city than to control cities, whereas they should be more or less uniformly distributed if the cities were an integrated market.

Changes in the distribution of reserve prices. The analysis of Section 5 describes the behavior of normalized winning bids following the introduction of minimum prices. This means that we include reserve prices as a left-hand side variable rather than a control. Under the assumption that the distribution of reserve prices is unchanged, this allows us to implement the tests of first-order stochastic dominance described in Propositions 2, 3 and 4 with pooled data exhibiting heterogenous minimum prices.

The data broadly supports the assumption that reserve prices are unchanged. A Kolmogorov-Smirnov test does not reject the H_0 hypothesis that the distribution of reserve prices is the same before and after the change with an exact p-value of .49. The result also holds for the distributions of residuals controlling for trends and GDP (see Tables A.2 and A.3).

Appendix A further complements this argument by replicating the analysis of Section 5 using log bids on the left-hand side, and controlling for reserve prices on the right-hand side. The qualitative findings are unchanged.

Observable participation. Our model assumes that participation is observed at the bidding stage. As we already pointed out, unobserved participation may lead to different predictions under competition. However, the assumption that participation is observed can be motivated from data. We test this hypothesis by estimating the effect of entrant participation and cartel participation on realized bids (winning or not). Table A.5 summarizes the results: even controlling for auction size through reserve prices, both entrant and cartel participation have a significant effect on bids. This suggests that participants must have information about the set of bidders and vindicates our modeling choice.

6.4 Design and Modeling Challenges

The findings of this paper suggest that minimum prices can significantly weaken collusion. The policy implication of this finding is straightforward: mechanisms designed to extract surplus in static settings can backfire and increase collusion in repeated environments. As a result, giving bidders a surplus guarantee (here in the form of minimum prices) can reduce collusion and increase the auctioneer's surplus.

How much surplus to guarantee the bidders is less obvious: minimum prices can lower the right tail of winning bids, but they also increase the left tail of winning bids. Still, Corollary 2 suggests an unambiguous improvement over no minimum prices: introducing non-binding minimum prices, i.e. minimum prices below the distribution of winning bids (conditional on observables). In this case, minimum prices necessarily lower the cost of procurement.[20]

Evaluating the effect of introducing minimum prices in the interior of the support of winning bids requires us to identify of the distribution of firms' costs under the assumption of collusion. Such a quantitative interpretation stretches the limits of our simple model of collusion (although procurement costs are in fact identified from winning bids in our model). Assumptions such as transferability, complete information or observed participation make the derivation of predictions under our H_1 hypothesis transparent. However, from a quantitative perspective, these assumptions lead us to overestimate the bidders' ability to collude and therefore underestimate the distribution of costs. A credible structural model would need to take repeated game frictions seriously.

Beyond the need to take repeated game frictions more seriously, some empirical findings suggest modeling challenges that our work does not address. The first is that participation of cartel members is affected by the introduction of minimum prices, meaning that it is endogenous (see Table 4; the number of bidders increase following the introduction of minimum prices, even controlling for the number of entrants). Endogenously limiting participation by cartel members could make sense in a model where firms learn their cost of production over the course of the procurement auction itself.

A second challenging finding is that cartel members avoid bids that may appear collusive. In particular, the distribution of winning bids in our data has density up to the reserve price, but has no point mass at the reserve price. This is an unlikely outcome under our model of collusion: there should either be no density at the reserve price, or mass at the reserve price. One interpretation is that bidders are playing under the threat of investigation and that bidding at the reserve price is suspicious behavior. The impact of such threats on collusive behavior deserves a proper theoretical treatment.

[20] One design subtlety worth emphasizing is that the minimum prices studied in this paper are not indexed on bids. In some settings (e.g. Italy) minimum prices are set as an increasing function of submitted bids (e.g. a quantile of submitted bids, Conley and Decarolis (2011), Decarolis (2013)). We expect such minimum price policies to be less effective than fixed minimum prices in deterring collusion: by coordinating on low bids, cartel bidders can still bring minimum prices down, blunting the effect that the policy has on punishments.

Appendix

A Further Empirical Exploration

Smooth equilibrium transition. A potential concern with the analysis in Section 5 is that it implicitly assumes that firms' bidding behavior prior to the introduction of the minimum price was not affected by expectations of change, and that their behavior after the introduction of minimum prices adjusted immediately to the new environment. We have argued that this should bias results against our findings.

We further address these concerns by performing OLS and quantile regressions on the linear model (3), excluding the data on auctions that were conducted within six months before or after the policy change. Table A.1 reports the results. Our results are strengthened.

	unconditional sample	sample s.t. $norm\_winning\_bid > .8$				
norm_winning_bid	mean effect	$q = .1$	$q = .2$	$q = .4$	$q = .6$	$q = .8$
window	0.015**	-0.006	0.002	0.002	0.003	0.001
	(0.006)	(0.023)	(0.008)	(0.005)	(0.002)	(0.002)
policy_change	-0.037***	-0.076***	-0.087***	-0.014***	-0.007***	-0.005***
	(0.006)	(0.021)	(0.007)	(0.005)	(0.002)	(0.002)
lngdp	0.246***	0.294	0.148*	0.043	0.051*	0.009
	(0.068)	(0.246)	(0.088)	(0.056)	(0.026)	(0.022)
year	0.006***	-0.0	0.002	0.003***	0.002***	0.002***
	(0.001)	(0.004)	(0.001)	(0.001)	(0.0)	(0.0)

Table A.1: The effect of minimum prices on winning bids, excluding auctions occurring around the policy change. OLS estimates for unconditional sample and quantile regression estimates for conditional sample; N=1342 (unconditional), 1291 (conditional).

Separate Markets. We now provide support for the assumption that markets are separate. The argument is geographical and uses the fact that bidder names are publicly available in our treatment city. This allows us to geolocate all cartel bidders, and compute their (straight line) distance to treatment and control cities. We then compute two measures of proximity indicating that the three markets are not integrated.

The first metric is the proportion of cartel bidders whose closest city is Tsuchiura (treatment) rather than Tsukuba or Ushiku (controls). If the three markets were integrated, given that the population of Tsuchiura is bracketed by that of the control cities, we should expect roughly 1/3 of cartel bidders to have Tsuchiura as their closest location. Instead the number in our data is 87%.

Our second metric compares the share of bidders within a fixed radius from each city. Given a quantile Q, we compute the Q^{th} quantile radius for Tsuchiura, i.e. the distance d_Q such that a proportion Q of cartel bidders are within distance d_Q of Tsuchiura. We then compute the proportion of cartel bidders within distance d of either control cities. Since the distance between control cities is roughly equal to the distance between Tsuchiura and each control city, if the markets were integrated, we would expect that a proportion Q of cartel bidders would be within distance d_Q of each control city. This is not the case: for $Q = .5$, the proportion of cartel bidders within distance d_Q of control cities is exactly equal to 0; for $Q = .75$, it is 13%. This suggests that markets are indeed separate.

Changes in the distribution of reserve prices. As we discussed in Section 6 it is important to clarify that changes in the distribution of normalized bids are not driven by changes in the distribution of reserve prices. We test whether reserve prices are affected by the policy change by running a Kolmogorov-Smirnov test on the raw distribution of log reserve prices before and after the policy change. We also run a Kolmogorov-Smirnov test on the distributions of residuals obtained from running the regression

$$ln\_reserve\_price \sim \beta_0 + \beta_1 window + \beta_2 ln\_gdp + \beta_3 year.$$

	raw distribution (p-value)	residual distribution
before < after	0.646	.791
after < before	0.261	.075
combined	.490	.139

Table A.2: Exact Kolmogorov-Smirnov tests; $N = 1539$.

We further run OLS and quantile regressions of the linear model

$$log\_reserve\_price \sim \beta_0 + \beta_1 window + \beta_3 policy\_change + \beta controls. \tag{9}$$

The results are summarized in Tables A.2 and A.3. Both Kolmogorov-Smirnov tests do not reject equality. Five out of the six moments reported in Table A.3 are insignificantly affected by the policy change. The effect, if any, seems to be a reduction in reserve prices following the policy change. This would tend to bias estimators against our results: if reserve prices tend to be lower after the policy change, keeping winning bids constant, this should mechanically increase normalized bid rather than diminish them.

ln_reserve_price	mean	q = .1	q = .2	q = .4	q = .6	q = .8
window	-0.025	-0.018	0.001	0.151	-0.034	-0.081
	(0.116)	(0.154)	(0.164)	(0.177)	(0.148)	(0.164)
window_policy	-0.132	0.033	-0.083	-0.264*	-0.14	-0.163
	(0.092)	(0.123)	(0.131)	(0.142)	(0.118)	(0.131)
lngdp	-1.778	-0.126	-0.091	2.491	-0.302	-5.335***
	(1.136)	(1.512)	(1.611)	(1.744)	(1.453)	(1.617)
year	0.086***	0.023	0.051*	0.103***	0.072***	0.079***
	(0.019)	(0.026)	(0.027)	(0.029)	(0.025)	(0.027)

Table A.3: The impact of treatment on reserve prices. OLS and quantile regression estimates; N = 1539 (unconditional).

We also ensure that our results continue to hold if we use log bids on the left hand side and control for reserve prices on the right-hand side. Table A.4 reports conditional OLS estimates of the linear model binning auctions according to large ($> 75^{th}$ quantile) and small

ln_winning_bid	large cond. mean	small cond. mean
cartel_member	-.002	0.013**
	(.007)	(0.005)
cartel_policy_change	-.024*	-.026***
	(0.012)	(0.008)
window	-0.006	0.002
	(0.009)	(0.006)
policy_change	-.022*	-0.001
	(.013)	(0.008)
ln_reserve	1.006***	1.006***
	(0.003)	(0.002)
lngdp	-.074	0.239***
	(.086)	(0.061)
year	0.004	0.001
	(0.001)	(0.001)

Table A.4: The effect of mininmum prices on log winning bids for large and small reserve prices; N= 381 (large), 1076 (small).

($< 75^{th}$ quantile) reserve prices.

Observability of participation. To assess whether the assumption of observable participants is plausible, we compute OLS estimates of linear models

$$\begin{aligned} norm\_bid \sim & \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta_3 num\_entrants \\ & + \beta_4 num\_cartel\_participants + \beta controls \\ ln\_bid \sim & \beta_0 + \beta_1 window + \beta_2 policy\_change + \beta_3 num\_entrants \\ & + \beta_4 num\_cartel\_participants + \beta_5 ln\_reserve + \beta controls \end{aligned}$$

for all (bidder, auction) pairs. The results are presented in Table A.5. For concision we do not reports coefficients for control variables (year and log GDP).

The data supports the assumption that participation is observable. Indeed, even conditional on auction size (proxied here by the reserve price), both the realized number of

	norm_bid	ln_bid
window	-0.003	-0.005
	(0.006)	(0.007)
policy_change	-0.024***	-0.023***
	(0.005)	(0.006)
num_entrants	-0.012***	-0.014***
	(0.002)	(0.003)
num_cartel	-0.011***	-0.013***
	(0.001)	(0.001)
ln_reserve		1.008***
		(0.003)

Table A.5: Bid (winning or not) as a function of realized participation; $N = 5898$, clustered by auction id.

entrants and the realized number of participating cartel members have a significant effect on bids.

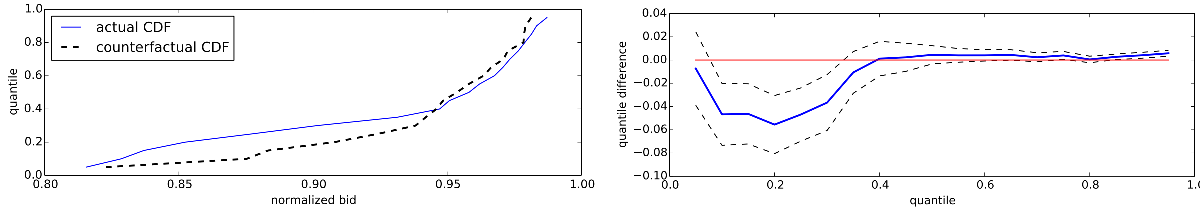
Sensitivity analysis. The analysis of Section 5 is not sensitive to small departures from our main specification. We illustrate this by presenting a few robustness checks that include: varying the threshold for normalized winning bids; using log-bids rather than normalized bids; using a two year window around the policy change rather than a four year window.

We first replicate the change-in-changes estimate (corresponding to Table 2 and Figure 2) of the counterfactual conditional distribution of bids for different specification. We use normalized bid thresholds equal to .75 and .82 and estimate the effect of policy change on both normalized bids and log-bids. The results are summarized in Figure A.1.

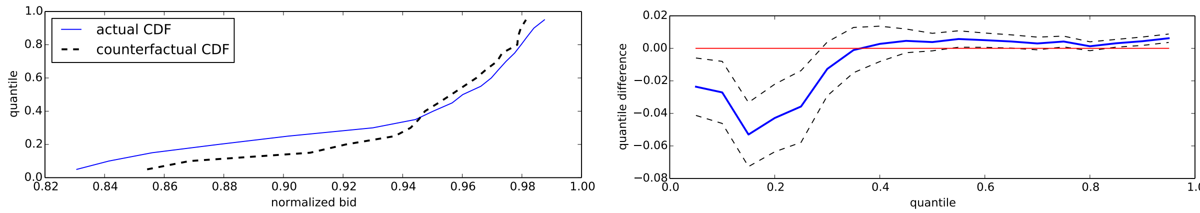
The single city analyses presented in Section 5 is also robust to the specification of the window around the policy change. Define the two year window and policy change variable

$$window_B = \mathbf{1}_{date \in \{\text{October } 28^{th} \text{ } 2009 \pm 12 \text{ months}\}},$$

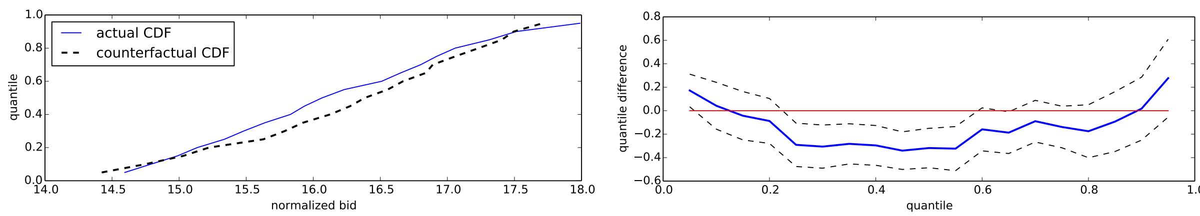
$$policy\_change_B = window_B \times \mathbf{1}_{date \geq \text{October } 28^{th} \text{ } 2009}$$



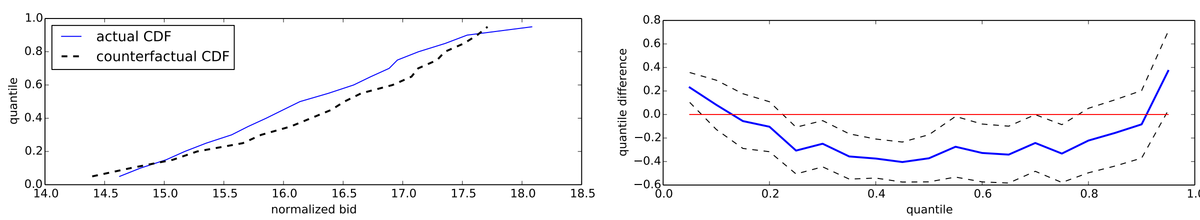
(a) c.d.f. of normalized bids and actual − counterfactual quantile difference, threshold = .75



(b) c.d.f. of normalized bids and actual − counterfactual quantile difference, threshold = .82



(c) c.d.f. of log bids and actual − counterfactual quantile difference, threshold = .75



(d) c.d.f. of log bids and actual − counterfactual quantile difference, threshold = .82

Figure A.1: actual and counterfactual conditional c.d.f. of normalized winning bids

and perform both OLS and quantile regressions on the linear model

$$norm\_winning\_bid \sim \beta_0 + \beta_1 window_B + \beta_2 policy\_change_B + \beta controls.$$

Table A.6 reports the results. The effects are very similar to those obtained using a four year window.

norm_winning_bid	unconditional sample mean effect	sample s.t. $norm\_winning\_bid > .8$ $q = .1$	$q = .2$	$q = .4$	$q = .6$	$q = .8$
window_B	-0.011	0.059**	0.012	0.004	0.001	0.001
	(0.007)	(0.024)	(0.011)	(0.006)	(0.003)	(0.002)
policy_change_B	-0.015**	-0.103***	-0.087***	-0.017***	-0.005*	-0.003
	(0.007)	(0.024)	(0.011)	(0.006)	(0.003)	(0.002)
lngdp	0.385***	0.954***	0.481***	0.128**	0.071***	0.025
	(0.075)	(0.238)	(0.113)	(0.056)	(0.027)	(0.022)
year	0.004***	0.003	0.002*	0.003***	0.002***	0.002***
	(0.001)	(0.003)	(0.001)	(0.001)	(0.0)	(0.0)

Table A.6: The effect of minimum prices on winning bids. OLS estimates for unconditional sample and quantile regression estimates for conditional sample.

A remark on cartel structure. One possible objection to our findings is that the cartel as we define it (58 firms) is rather large for collusion to be sustainable. This concern is alleviated when we consider the finer network structure of cartel interaction. Figure A.2 plots the network of cartel where nodes are cartel members and the plotting algorithm attempts to keep the distance between nodes inversely proportional to the number of auctions cartel members jointly participate in. Only edges corresponding to at least 10 interactions are plotted. The network of interactions is highly clustered: its clustering coefficient is .6,

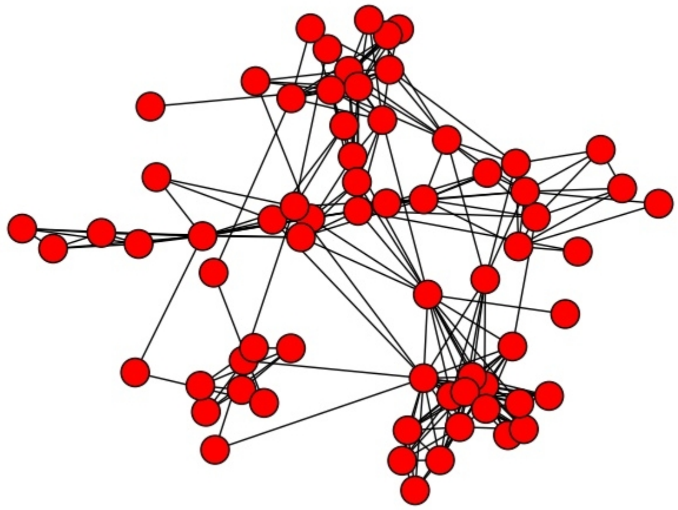


Figure A.2: the network of cartel interactions

compared to a theoretical maximum of 1 and an average value of .1 for Erdős-Renyi random graphs with the same number of nodes and the same number of edges. In addition, two bidders that interact at least once interact on average 8.7 times. Altgether, this suggests that the cartel really consists of smaller subgroups interacting frequently, thereby facilitating joint monitoring and incentive provision.

B Proofs – For online publication

B.1 Proofs for Section 2

This appendix contains the proofs of Section 2. We start with a few preliminary observations. First, since the game we are studying is a complete information game with perfect monitoring, the set of SPE payoffs is compact (Proposition 2.5.2 in Mailath and Samuelson (2006)). Hence, $\overline{V}_p$ and $\underline{V}_p$ are attained. Fix an SPE σ and a history h_t. Let $\beta(\mathbf{c})$, $\gamma(\mathbf{c})$ and $T(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ be the bidding and transfer profile that firms play in this equilibrium after history h_t. Let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ be, respectively, the winning bid and the allocation induced by bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. Let $h_{t+1} = h_t \sqcup (\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x}, \mathbf{T})$ be the concatenated history composed of h_t followed by $(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x}, \mathbf{T})$, and let $\{V(h_{t+1})\}_{i\in N}$ be the vector of continuation payoffs after history h_{t+1}. We let $h_{t+1}(\mathbf{c}) = h_t \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ denote the on-path history that follows h_t when current costs are $\mathbf{c}$. Note that the following inequalities must hold:

(i) for all $i \in \widehat{N}$ such that $c_i \leq \beta^W(\mathbf{c})$,

$$x_i(\mathbf{c})(\beta^W(\mathbf{c})-c_i)+T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))+\delta V_i(h_{t+1}(\mathbf{c})) \geq \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})(\beta^W(\mathbf{c})-c_i)+\delta \underline{V}_p. \tag{10}$$

(ii) for all $i \in \widehat{N}$ such that $c_i > \beta^W(\mathbf{c})$,

$$x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \delta \underline{V}_p. \tag{11}$$

(iii) for all $i \in N$,

$$T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_{t+1}(\mathbf{c})) \geq \delta \underline{V}_p. \tag{12}$$

The inequality in (10) must hold since a firm with cost below $\beta^W(\mathbf{c})$ can obtain a payoff at least as large as the right-hand side by undercutting the winning bid when $\beta^W(\mathbf{c}) > p$, or, by bidding p and choosing $\gamma_i = 1$ when $\beta^W(\mathbf{c}) = p$. Similarly, the inequality in (11) must hold since firms with cost larger than $\beta^W(\mathbf{c})$ can obtain a payoff at least as large as the right-hand side by bidding more than $\beta^W(\mathbf{c})$. Finally, the inequality in (12) must hold since otherwise firm i would not be willing to make the required transfer.

Conversely, suppose there exists a winning bid profile $\beta^W(\mathbf{c})$, an allocation $\mathbf{x}(\mathbf{c})$, a transfer profile $\mathbf{T}$ and equilibrium continuation payoffs $\{V_i(h_{t+1}(\mathbf{c}))\}_{i\in N}$ that satisfy inequalities (10)-(12) for some $\gamma(\mathbf{c})$ that is consistent with $\mathbf{x}(\mathbf{c})$ (i.e., $\gamma(\mathbf{c})$ is such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c})/\sum_{j:x_j(\mathbf{c})>0} \gamma_j(\mathbf{c})$ for all i with $x_i(\mathbf{c}) > 0$). Then, $(\beta^W, \mathbf{x}, \mathbf{T})$ can be supported in an SPE as follows. For all $\mathbf{c}$, all firms $i \in \widehat{N}$ bid $\beta^W(\mathbf{c})$. Firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \widehat{N}$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c})/\sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm deviates at the bidding stage, firms make transfers $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$. If no firm deviates at the transfer stage, in the next period firms play an SPE that gives payoff vector $\{V(h_{t+1}(\mathbf{c}))\}_{i\in N}$. If firm i deviates at the bidding stage, there are no transfers and the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_p$; if firm i deviates at the transfer stage, the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_p$ (deviations by more than one firm go unpunished). Since (10) holds, under this strategy profile no firm has an incentive to undercut the winning bid $\beta^W(\mathbf{c})$. Since (11) holds, no firm with $c_i > \beta^W(\mathbf{c})$ and $x_i(\mathbf{c}) > 0$ has an incentive to bid above $\beta^W(\mathbf{c})$ and lose. Upward deviations by a firm i with $c_i < \beta^W(\mathbf{c})$ who wins the auction are not profitable since the firm would lose the auction by bidding $b > \beta^W(\mathbf{c})$. Finally, since (12) holds, all firms have an incentive to make their required transfers.

Proof of Lemma 1. Let σ be an SPE that attains $\overline{V}_p$. Towards a contradiction, suppose there exists an on-path history $h_t = h_{t-1} \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ such that $\sum_i V_i(\sigma, h_t) = V(\sigma, h_t) < \overline{V}_p$. Let $\{V_i\}_{i\in N}$ be an equilibrium payoff vector with $\sum_i V_i = \overline{V}_p$.

Consider changing the continuation equilibrium at history h_t by an equilibrium that delivers payoff vector $\{V_i\}_{i\in N}$, and changing the transfers after history $h_{t-1} \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ as follows. First, for each $i \in N$, let $\hat{T}_i$ be such that $\hat{T}_i + \delta V_i = T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(\sigma, h_t)$. Note that

$$\sum_i \hat{T}_i = \sum_i \{T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta(V_i(\sigma, h_t) - V_i)\} < 0,$$

where we used $\sum_i V_i = \overline{V}_p > \sum_i V_i(\sigma, h_t)$ and $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$. For each $i \in N$, let $\tilde{T}_i = \hat{T}_i + \frac{\epsilon}{n}$, where $\epsilon > 0$ is such that $\sum_i \tilde{T}_i = \sum_i \hat{T}_i + \epsilon = 0$. Replacing transfers $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ and continuation values $V_i(\sigma, h_t)$ by transfers $\tilde{T}_i$ and values V_i relaxes constraints (10)-(12) and increases the total expected discounted surplus that the equilibrium generates. Therefore, if σ attains $\overline{V}_p$, it must be that $V(\sigma, h_t) = \overline{V}_p$ for all on-path histories.

We now prove the second statement in the Lemma. Fix an optimal equilibrium σ, and let $\{V_i\}_{i\in N}$ be the equilibrium payoff vector that this equilibrium delivers, with $\sum_i V_i = \overline{V}_p$.

For each $\mathbf{c}$, let $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$ be the bidding profile that firms use in the first period under σ, and let $\mathbf{x}(\mathbf{c})$ denote the allocation induced by bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. It follows that

$$\overline{V}_p = \mathbb{E}\left[\sum_{i \in \widehat{N}} x_i(\mathbf{c})(\beta_i(\mathbf{c}) - c_i)\right] + \delta \overline{V}_p \Longleftrightarrow \overline{V}_p = \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i \in \widehat{N}} x_i(\mathbf{c})(\beta_i(\mathbf{c}) - \mathbf{x}(\mathbf{c}))\right].$$

We show that there exists an optimal equilibrium in which firms use bidding profile $(\beta(\cdot), \gamma(\cdot))$ after all on-path histories. For any $(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$, let $T_i(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ denote the transfer that firm i makes at the end of the first period under equilibrium σ when first period costs, bids and allocation are given by $\mathbf{c}$, $\mathbf{b}$, γ and $\mathbf{x}$. Let $V_i(h_1(\mathbf{c}))$ denote firm i's continuation payoff under equilibrium σ after first period history $h_1(\mathbf{c}) = (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$. By our arguments above, $\sum_i V_i(h_1(\mathbf{c})) = \overline{V}_p$ for all $\mathbf{c}$. Since σ is an equilibrium, it must be that $\beta(\mathbf{c})$, $\gamma(\mathbf{c})$, $\mathbf{x}(\mathbf{c})$, $T_i(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ and $V_i(h_1(\mathbf{c}))$ satisfy (10)-(12).

Consider the following strategy profile. Along the equilibrium path, at each period t firms bid according to $(\beta(\cdot), \gamma(\cdot))$. For any $(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$, firm i makes transfer $\hat{T}_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ such that $\hat{T}_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i = T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta V_i(h_1(\mathbf{c}))$. Note that

$$\sum_i \hat{T}_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = \sum_i \{T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta(V_i(h_1(\mathbf{c})) - V_i)\} = 0,$$

where we used $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$ and $\sum_i V_i(h_1(\mathbf{c})) = \overline{V}_p = \sum_i V_i$. If firm i deviates at the bidding stage or transfer stage, then firms revert to an equilibrium that gives firm i a payoff of $\underline{V}_p$. Clearly, this strategy profile delivers total payoff $\overline{V}_p$. Moreover, firms have the same incentives to bid according to (β, γ) and make their required transfers than under the original equilibrium σ. Hence, no firm has an incentive to deviate at any stage and this strategy profile can be supported as an equilibrium. ■

Proof of Lemma 2. Suppose there exists an SPE σ and a history h_t at which firms bid according to a bidding profile (β, γ) that induces winning bid $\beta^W(\mathbf{c})$ and allocation $\mathbf{x}(\mathbf{c})$. Let $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$ be firm i's transfers at history h_t when costs are $\mathbf{c}$ and all firms play according to the SPE σ. Let $h_{t+1}(\mathbf{c}) = h_t \sqcup (\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ be the on-path history that follows h_t when costs are $\mathbf{c}$, and let $V_i(h_{t+1}(\mathbf{c}))$ be firm i's equilibrium payoff at history $h_{t+1}(\mathbf{c})$. Since the equilibrium must satisfy (10)-(12) for all $\mathbf{c}$,

$$\begin{aligned}
&\sum_{i \in \widehat{N}} \left\{ (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \right\} \\
&\leq \sum_{i \in N} T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) + \delta \sum_{i \in N} (V_i(h_{t+1}(\mathbf{c})) - \underline{V}_p) \leq \delta(\overline{V}_p - n\underline{V}_p),
\end{aligned}$$

where we used $\sum_i T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})) = 0$ and $\sum_i V_i(h_{t+1}(\mathbf{c})) \leq \overline{V}_p$.

Next, consider a winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ that satisfy (1) for all $\mathbf{c}$ for some $\gamma(\mathbf{c})$ consistent with $\mathbf{x}(\mathbf{c})$ (i.e., such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c}) / \sum_{j: x_j(\mathbf{c}) > 0} \gamma_j(\mathbf{c})$ for all $i \in \widehat{N}$ with $x_i(\mathbf{c}) > 0$). We now construct an SPE that supports $\beta^W(\cdot)$ and $\mathbf{x}(\cdot)$ in the first period. Let $\{V_i\}_{i \in N}$ be an equilibrium payoff vector with $\sum_i V_i = \overline{V}_p$. For each $i \in N$ and each $\mathbf{c}$, we construct transfers $T_i(\mathbf{c})$ as follows:

$$T_i(\mathbf{c}) = \begin{cases} -\delta(V_i - \underline{V}_p) + (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c}))(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \leq \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_p) - x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i > \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_p) + \epsilon(\mathbf{c}) & \text{if } i \notin \widehat{N}, \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined below. Note that, for all $\mathbf{c}$,

$$\begin{aligned}
&\sum_{i \in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) \\
&= -\delta(\overline{V}_p - n\underline{V}_p) + \sum_{i \in \widehat{N}} \left\{ (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c})) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c}) \left[\beta^W(\mathbf{c}) - c_i\right]^- \right\} \leq 0,
\end{aligned}$$

where the inequality follows since β^W and $\mathbf{x}$ satisfy (1). We set $\epsilon(\mathbf{c}) \geq 0$ such that transfers are budget balance; i.e., such that $\sum_{i \in N} T_i(\mathbf{c}) = 0$.

The SPE we construct is as follows. At $t = 0$, for each $\mathbf{c}$ all participating firms bid $\beta^W(\mathbf{c})$. Firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \widehat{N}$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \widehat{N}$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c}) / \sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm deviates at the bidding stage, firms exchange transfers $T_i(\mathbf{c})$. If no firm deviates at the transfer stage, from $t = 1$ onwards they play an SPE that supports payoff vector $\{V_i\}$. If firm $i \in N$ deviates either at the bidding stage or at the transfer stage, from $t = 1$ onwards firms play an SPE that gives firm i a payoff $\underline{V}_p$ (if more than one firm deviates, firms punish the lowest indexed firm that deviated). This strategy profile satisfies (10)-(12), and so β^W and $\mathbf{x}$ are implementable. ■

Proof of Proposition 1. By Lemma 1, there exists an optimal equilibrium in which firms use the same bidding profile (β, γ) at every on-path history. For each cost vector $\mathbf{c}$, let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ denote the winning bid and the allocation induced by this bidding profile under cost vector $\mathbf{c}$.

We first show that $\beta^W(\mathbf{c}) = b_p^*(\mathbf{c})$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) > p$. Towards a contradiction, suppose there exists $\mathbf{c}$ with $\beta^W(\mathbf{c}) \neq b_p^*(\mathbf{c}) > p$. Since $\mathbf{x}^*(\mathbf{c})$ is the efficient allocation, the procurement cost under allocation $\mathbf{x}(\mathbf{c})$ is at least as large as the procurement cost under allocation $\mathbf{x}^*(\mathbf{c})$. Since bidding profile (β, γ) is optimal, it must be that $\beta^W(\mathbf{c}) > b_p^*(\mathbf{c}) > p$. Indeed, if $\beta^W(\mathbf{c}) < b_p^*(\mathbf{c})$, then the cartel would strictly prefer to use a bidding profile that allocates the contract efficiently and has winning bid $b_p^*(\mathbf{c})$ under cost vector $\mathbf{c}$ than to use bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. By Lemma 2, $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ must satisfy

$$\begin{aligned}\delta(\overline{V}_p - n\underline{V}_p) &\geq \sum_{i \in \widehat{N}} \left\{(1 - x_i(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c})\left[\beta^W(\mathbf{c}) - c_i\right]^-\right\} \\ &\geq \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+,\end{aligned}$$

which contradicts $\beta^W(\mathbf{c}) > b_p^*(\mathbf{c}) > p$. Therefore, $\beta^W(\mathbf{c}) = b_p^*(\mathbf{c})$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) > p$.

Next, we show that $\beta^W(\mathbf{c}) = p$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) \leq p$. Towards a contradiction, suppose there exists $\mathbf{c}$ with $b_p^*(\mathbf{c}) \leq p$ and $\beta^W(\mathbf{c}) > p$. By Lemma 2, $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ satisfy

$$\begin{aligned}\delta(\overline{V}_p - n\underline{V}_p) &\geq \sum_{i \in \widehat{N}} \left\{(1 - x_i(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i(\mathbf{c})\left[\beta^W(\mathbf{c}) - c_i\right]^-\right\} \\ &\geq \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c}))\left[\beta^W(\mathbf{c}) - c_i\right]^+,\end{aligned}$$

which contradicts $\beta^W(\mathbf{c}) > p \geq b_p^*(\mathbf{c})$. Therefore, $\beta^W(\mathbf{c}) = p$ for all $\mathbf{c}$ such that $b_p^*(\mathbf{c}) \leq p$. Combining this with the arguments above, $\beta^W(\mathbf{c}) = \beta_p^*(\mathbf{c}) = \max\{p, b_p^*(\mathbf{c})\}$.

Finally, we characterize the allocation in an optimal equilibrium. Note first that under an optimal bidding profile the cartel must allocate the procurement contract efficiently whenever $\beta_p^*(\mathbf{c}) > p$. Indeed, by construction, the efficient allocation is sustainable whenever the winning bid is $\beta_p^*(\mathbf{c}) > p$. Therefore, if the allocation was not efficient for some $\mathbf{c}$ with $\beta_p^*(\mathbf{c}) > p$, the cartel could strictly improve its profits by using a bidding profile with winning bid $\beta_p^*(\mathbf{c})$ that allocates the good efficiently.

Consider next a cost vector $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$. In this case, the cartel's bidding profile in an optimal equilibrium induces the most efficient allocation (i.e., the allocation that minimizes expected procurement costs) consistent with (1) when the winning bid is p.

■

Proof of Corollary 1. Note that, for $\delta = 0$, $b_p^*(\mathbf{c}) = c_{(2)}$ for all $\mathbf{c}$. By Proposition 1, when $\delta = 0$ the winning bid under the best equilibrium for the cartel is equal to $\beta^{\mathsf{comp}}(\mathbf{c}) = \max\{c_{(2)}, p\}$, which is the winning bid under competition. ■

Fix a minimum price p. For every value $V \geq n\underline{V}_p$ and every $\mathbf{c}$, let

$$b_p(\mathbf{c}; V) \equiv \sup \left\{ b \leq r : \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c}))[b - c_i]^+ \leq \delta(V - n\underline{V}_p) \right\},$$

and let $\beta_p(\mathbf{c}; V) = \max\{b_p(\mathbf{c}; V), p\}$. Note that $\beta_p(\mathbf{c}; V)$ would be the winning bid in an optimal equilibrium if the cartel's total surplus was equal to V. Let $\mathbf{x}^p(\mathbf{c}; V)$ be the allocation under an optimal equilibrium when the cartel's total surplus is V. For every $V \geq n\underline{V}_p$, let

$$U_p(V) \equiv \frac{1}{1-\delta} \mathbb{E} \left[\sum_{i \in \widehat{N}} x_i^p(\mathbf{c}; V)(\beta_p(\mathbf{c}, V) - c_i) \right],$$

be the total surplus generated under a bidding profile that induces winning bid $\beta_p(\mathbf{c}; V)$ and allocation $\mathbf{x}^p(\mathbf{c}; V)$. The winning bid and allocation in an optimal equilibrium are $\beta_p^*(\mathbf{c}) = \beta_p(\mathbf{c}; \overline{V}_p)$ and $\mathbf{x}^p(\mathbf{c}; \overline{V}_p)$, and so $\overline{V}_p = U_p(\overline{V}_p)$. Let

$$\overline{U}_p \equiv \sup\{V \geq n\underline{V}_p : V \leq U_p(V)\}.$$

Lemma B.1. $\overline{V}_p = \overline{U}_p$.

Proof. Since $\overline{V}_p = U_p(\overline{V}_p)$, it follows that $\overline{U}_p \geq \overline{V}_p$. We now show that $\overline{U}_p \leq \overline{V}_p$. Towards a contradiction, suppose that $\overline{U}_p > \overline{V}_p$. Hence, there exists $\tilde{V}$ such that $U_p(\tilde{V}) \geq \tilde{V} > \overline{V}_p$. Let $V = \frac{U_p(\tilde{V})}{n}$, and consider the following strategy profile. For all on-path histories, cartel members use a bidding profile (β, γ) inducing winning bid $\beta_p(\mathbf{c}; \tilde{V})$ and allocation $\mathbf{x}^p(\mathbf{c}; \tilde{V})$. If firm i deviates at the bidding stage, there are no transfers and in the next period firms play an equilibrium that gives firm i a payoff of $\underline{V}_p$ (if more than one firm deviates, firms play an equilibrium that gives $\underline{V}_p$ to the lowest indexed firm that deviated). If no firm deviates at the bidding stage, firms make transfers $T_i(\mathbf{c})$ given by

$$T_i(\mathbf{c}) = \begin{cases} -\delta(V - \underline{V}_p) + (\rho_i(\beta_p, \gamma, \mathbf{x}^p)(\mathbf{c}) - x_i^p(\mathbf{c}; \tilde{V}))(\beta_p(\mathbf{c}; \tilde{V}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \leq \beta_p(\mathbf{c}; \tilde{V}), \\ -\delta(V - \underline{V}_p) + \epsilon(\mathbf{c}) & \text{otherwise,} \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined.[21] Note that

$$\sum_{i \in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) = -\delta(U_p(\tilde{V}) - n\underline{V}_p) + \sum_{i \in \widehat{N}} ((\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i^p(\mathbf{c}; \tilde{V}))[\beta_p(\mathbf{c}; \tilde{V}) - c_i]^+ \leq 0,$$

where the inequality follows since $\beta_p(\mathbf{c}; \tilde{V})$ and $x_i^p(\mathbf{c}; \tilde{V})$ are the winning bid and the allocation under an optimal equilibrium when the cartel's total surplus is $\tilde{V} \leq U_p(\tilde{V})$. We set $\epsilon(\mathbf{c}) \geq 0$ such that $\sum_i T_i(\mathbf{c}) = 0$. If firm i deviates at the transfer stage, in the next period firms play an equilibrium that gives firm i a payoff of $\underline{V}_p$ (if more than one firm deviates, firms play an equilibrium that gives $\underline{V}_p$ to the lowest indexed firm that deviated). Otherwise, in the next period firms continue playing the same strategy as above. This strategy profile generates total surplus $U_p(\tilde{V}) \geq \tilde{V} > \overline{V}_p$ to the cartel. Since firms play symmetric strategies, it gives a payoff $V = \frac{U_p(\tilde{V})}{n}$ to each cartel member. One can check that no firm has an incentive to deviate at any stage, and so this strategy profile constitutes an equilibrium. This contradicts $U_p(\tilde{V}) > \overline{V}_p$, so it must be that $\overline{U}_p \leq \overline{V}_p$. ■

Proof of Lemma 3. We first establish part (i). Suppose that $p \leq \underline{c}$ and fix equilibrium payoffs $\{V_i\}_{i \in N}$. Fix $j \in N$ and consider the following strategy profile. At $t = 0$, firms' behavior depends on whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. If $j \in \widehat{N}$, all firms $i \in \widehat{N}$ bid $\min\{c_j, c_{(2)}\}$ (where $c_{(2)}$ is the second lowest procurement cost). Firm $i \in \widehat{N}$ chooses $\gamma_i = 1$ if $c_i = \min_{k \in \widehat{N}} c_k$ and chooses $\gamma_i = 0$ otherwise. Note that this bidding profile constitutes an

[21] Recall that $\mathbf{x}^p(\mathbf{c}; \tilde{V})$ is the allocation under an optimal equilibrium when continuation payoff is $\tilde{V}$. Therefore, $\mathbf{x}^p(\mathbf{c}; \tilde{V})$ is such that $x_i^p(\mathbf{c}; \tilde{V}) = 0$ for all i with $c_i > \beta_p(\mathbf{c}; \tilde{V})$.

equilibrium of the stage game. If $j \notin \widehat{N}$, at $t = 0$ participating firms play according to some equilibrium of the stage game. If all firms bid according to this profile, firm j's transfer is $T_j = -\delta V_j$ at the end of the period regardless of whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. The transfer of firm $i \neq j$ is $T_i = \frac{1}{n-1}\delta V_j$ at the end of the period, so $\sum_i T_i = 0$. If no firm deviates at the bidding or transfer stage, at $t = 1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}$. If firm i deviates at the bidding stage, there are no transfers and at $t = 1$ firms play the strategy just described with i in place of j. If no firm deviates at the bidding stage and firm i deviates at the transfer stage, at $t = 1$ firms play the strategy just described with i in place of j (if more than one firm deviates at the bidding or transfer stage, from $t = 1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}_{i \in N}$). Note that this strategy profile gives player j a payoff of 0. Moreover, no firm has an incentive to deviate at $t = 0$, and so $\underline{V}_p = 0$ for all $p \leq \underline{c}$.

Suppose next that $p > \underline{c}$, and note that

$$\underline{V}_p \geq \underline{v}_p \equiv \frac{1}{1-\delta}\text{prob}(i \in \widehat{N})\mathbb{E}\left[\frac{1}{\widehat{N}}\mathbf{1}_{c_i \leq p}(p - c_i)|i \in \widehat{N}\right] > 0,$$

where the first inequality follows since $\underline{v}_p$ is the minimax payoff for a firm in an auction with minimum price p. This establishes part (i).

We now turn to part (ii). Note that $\beta_0^*(\underline{c}) = \inf_{\mathbf{c}} \beta_0^*(\mathbf{c}) = \underline{c} + \frac{\delta \overline{V}_0}{n-1} > \underline{c}$.[22] We now show that there exists $\eta > 0$ such that $\overline{V}_p - n\underline{V}_p < \overline{V}_0$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$. Fix $\eta > 0$ and $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$. For every $V \geq n\underline{V}_p$ and every $\mathbf{c}$, let $\tilde{\beta}_p(\mathbf{c}; V) \equiv \max\{b_0(\mathbf{c}; V), p\}$. Since $\underline{V}_p > 0$ for all $p > \beta_0^*(\underline{c})$, it follows that $b_0(\mathbf{c}; V) \geq b_p(\mathbf{c}; V)$ for all $\mathbf{c}$ and all $V \geq n\underline{V}_p$, and so $\tilde{\beta}_p(\mathbf{c}; V) \geq \beta_p(\mathbf{c}; V) = \max\{b_p(\mathbf{c}; V), p\}$ for all $\mathbf{c}$ and all $V \geq n\underline{V}_p$. Define

$$\tilde{U}_p(V) \equiv \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i \in \widehat{N}} x_i^*(\mathbf{c})(\tilde{\beta}_p(\mathbf{c}; V) - c_i)\right],$$

[22]Term $\beta_0^*(\mathbf{c})$ attains its lowest value when all cartel members participate in the auction and costs are $\mathbf{c} = (\underline{c})_{i \in N}$ (i.e., all firms have cost $\underline{c}$). For this cost vector, $\beta_0^*(\mathbf{c}) = \underline{c} + \frac{\delta \overline{V}_0}{n-1}$.

and note that $\tilde{U}_p(V) \geq U_p(V)$ for all $V \geq n\underline{V}_p$. Let $\tilde{V}_p \equiv \sup\{V \geq n\underline{V}_p : \tilde{U}_p(V) \geq V\}$, and note that $\tilde{V}_p \geq \overline{V}_p$. Recall that, for all V, $U_0(V) = \frac{1}{1-\delta}\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^*(\mathbf{c})(b_0(\mathbf{c};V) - c_i)\right]$.[23] Therefore, for all V,

$$\tilde{U}_p(V) - U_0(V) = \frac{1}{1-\delta}\mathbb{E}\left[(p - b_0(\mathbf{c};V))\mathbf{1}_{\{\mathbf{c}:b_0(\mathbf{c};V)<p\}}\right] > 0.$$

Note that for all V and all $\mathbf{c}$, $b_0(\mathbf{c};V) \geq \underline{c} + \frac{\delta V}{n-1}$. Let $\hat{V} > 0$ be such that $\underline{c} + \frac{\delta\hat{V}}{n-1} = \beta_0^*(\underline{c}) + \eta = \underline{c} + \frac{\delta\overline{V}_0}{n-1} + \eta$; that is, $\hat{V} = \overline{V}_0 + \frac{(n-1)\eta}{\delta} > \overline{V}_0$. Then, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $V \geq \hat{V}$, $b_0(\mathbf{c};V) \geq p$ for all $\mathbf{c}$, and so $\tilde{U}_p(V) = U_0(V)$. Since $\hat{V} > \overline{V}_0$ and since $\overline{V}_0 = \sup\{V \geq 0 : U_0(V) \geq V\}$, it follows that $V > U_0(V) = \tilde{U}_p(V)$ for all $V \geq \hat{V}$ and all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, and so $\hat{V} = \overline{V}_0 + \frac{(n-1)\eta}{\delta} > \tilde{V}_p \geq \overline{V}_p$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$.

Finally, let $\eta > 0$ be such that $\frac{(n-1)\eta}{\delta} = n\underline{v}_{\beta_0^*(\underline{c})} = n\frac{\text{prob}(i\in\widehat{N})}{1-\delta}\mathbb{E}\left[\frac{1}{\widehat{N}}\mathbf{1}_{c_i\leq\beta_0^*(\underline{c})}(\beta_0^*(\underline{c}) - c_i)|i \in \widehat{N}\right]$.[24] Since $\underline{V}_p \geq \underline{v}_p \geq \underline{v}_{\beta_0^*(\underline{c})}$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$,

$$\hat{V} = \overline{V}_0 + \frac{(n-1)\eta}{\delta} > \overline{V}_p \Rightarrow \overline{V}_0 > \overline{V}_p - n\underline{V}_p,$$

which completes the proof. ■

B.2 Proofs of Section 3

Proof of Proposition 2. Consider first a collusive environment. By Proposition 1 and Lemma 3, there exists $\eta > 0$ such that $\beta_p^*(\mathbf{c}) \leq \beta_0^*(\mathbf{c})$ for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c})+\eta]$ and all $\mathbf{c}$ such that $\beta_0^*(\mathbf{c}) \geq p$, with strict inequality if $\beta_0^*(\mathbf{c}) < r$. Therefore, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $\text{prob}(\beta_p^* \geq q|\beta_p^* \geq p) \leq \text{prob}(\beta_0^* \geq q|\beta_0^* \geq p)$, and the inequality is strict for some $q > p$ whenever $\text{prob}(\beta_0^* < r) > 0$. This proves part (i).

[23] Indeed, by Proposition 1, $\mathbf{x}^{p=0}(\mathbf{c};V) = \mathbf{x}^*(\mathbf{c})$ for all V.

[24] Recall that for all p, $\underline{V}_p \geq \underline{v}_p = \frac{\text{prob}(i\in\widehat{N})}{1-\delta}\mathbb{E}\left[\frac{1}{\widehat{N}}\mathbf{1}_{c_i\leq p}(p - c_i)|i \in \widehat{N}\right]$.

Under competition, for all p and all $q > p$, $\text{prob}(\beta_p^{\mathsf{comp}} \geq q | \beta_p^{\mathsf{comp}} > p) = \text{prob}(c_{(2)} \geq q | c_{(2)} > p) = \text{prob}(\beta_0^{\mathsf{comp}} \geq q | \beta_0^{\mathsf{comp}} > p)$. This proves part (ii). ■

Proof of Proposition 3. We first show that there exists a symmetric equilibrium as described in the statement of the proposition, and then we show uniqueness.

Consider first a minimum price $p \leq b_0^{AI}(\underline{c})$. Clearly, in this case all firms using the bidding function $b_0^{AI}(\cdot)$ is a symmetric equilibrium of the auction with minimum price p.

Consider next the case in which $b_0^{AI}(\underline{c}) < p$. For any $c \in [\underline{c}, \overline{c}]$, define

$$P(c) \equiv \sum_{j=0}^{\widehat{N}-1} \binom{\widehat{N}-1}{j} \frac{1}{j+1} F(c)^j (1 - F(c))^{\widehat{N}-j-1}.$$

$P(c)$ is the probability with which a firm with cost $c' \leq c$ wins the auction if all firms use a bidding function $\beta(\cdot)$ with $\beta(c') = b \geq p$ for all $c' \leq c$ and $\beta(c') > b$ for all $c' > c$.

Let $\hat{c} \in (\underline{c}, \overline{c})$ be the unique solution to $P(\hat{c})(p - \hat{c}) = (1 - F(\hat{c}))^{\widehat{N}-1}(b_0^{AI}(\hat{c}) - \hat{c})$.[25] Let $b_p^{AI}(\cdot)$ be given by

$$b_p^{AI}(c) = \begin{cases} b_0^{AI}(c) & \text{if } c \geq \hat{c}, \\ p & \text{if } c < \hat{c}. \end{cases}$$

Note that if all firms bid according to bidding function $b_p^{AI}(\cdot)$, the probability with which a firm with cost $c < \hat{c}$ wins the auction is $P(\hat{c})$. We now show that all firms bidding according to $b_p^{AI}(\cdot)$ is an equilibrium.

Suppose that all firms $j \neq i$ bid according to $b_p^{AI}(\cdot)$. Note first that it is never optimal for firm i to bid $b \in (p, b_p^{AI}(\hat{c}))$. Indeed, if $c_i < b_p^{AI}(\hat{c})$, bidding $b \in (p, b_p^{AI}(\hat{c}))$ gives firm i a strictly lower payoff than bidding $b_p^{AI}(\hat{c})$: in both cases firm i wins with probability $(1 - F(\hat{c}))^{\widehat{N}-1}$, but by bidding $b_p^{AI}(\hat{c})$ the firm gets a strictly larger payoff in case of winning.

[25] Note first that such a $\hat{c}$ always exists whenever $b^{AI}(\underline{c}) < p$. Indeed, in this case $P(\underline{c})(p - \underline{c}) = p - \underline{c} > b_0^{AI}(\underline{c}) - \underline{c}$, while $P(p)(p - p) = 0 < (1 - F(p))^{\widehat{N}-1}(b_0^{AI}(p) - \overline{c})$. By the Intermediate value Theorem, there exists $\hat{c} \in [\underline{c}, p]$ such that $P(\hat{c})(p - \hat{c}) = (1 - F(\hat{c}))^{\widehat{N}-1}(b_0^{AI}(\hat{c}) - \hat{c})$. Moreover, for all $c \leq p$, $\frac{\partial}{\partial c} P(c)(p - c) = -P(c) + P'(c)(p - c) \leq -P(c) < -(1 - F(c))^{\widehat{N}-1} = \frac{\partial}{\partial c}(1 - F(c))^{\widehat{N}-1}(b_0^{AI}(c) - c)$, so $\hat{c}$ is unique.

If $c_i > b_p^{AI}(\hat{c})$, bidding $b \in (p, b_p^{AI}(\hat{c}))$ gives firm i a strictly lower payoff than bidding $b_p^{AI}(c_i)$.

Suppose that $c_i \geq \hat{c}$. Since $b_p^{AI}(x) = b_0^{AI}(x)$ for all $x \geq \hat{c}$, firm i with cost c_i gets a larger payoff bidding $b_p^{AI}(c_i)$ than bidding $b_p^{AI}(x)$ with $x \in [\hat{c}, \overline{c}]$. If $c_i = \hat{c}$, firm i is by construction indifferent between bidding p and bidding $b_p^{AI}(\hat{c})$. Moreover, for all $c_i > \hat{c}$,

$$
\begin{aligned}
(1 - F(c_i))^{\widehat{N}-1}(b_p^{AI}(c_i) - c_i) &\geq (1 - F(\hat{c}))^{\widehat{N}-1}(b_p^{AI}(\hat{c}) - \hat{c}) + (1 - F(\hat{c}))^{\widehat{N}-1}(\hat{c} - c_i) \\
&= P(\hat{c})(p - \hat{c}) + (1 - F(\hat{c}))^{\widehat{N}-1}(\hat{c} - c_i) \\
&> P(\hat{c})(p - \hat{c}) + P(\hat{c})(\hat{c} - c_i),
\end{aligned}
$$

where the strict inequality follows since $P(\hat{c}) > (1 - F(\hat{c}))^{\widehat{N}-1}$ and $c_i > \hat{c}$. Hence, firm i strictly prefers to bid $b_p^{AI}(c_i)$ when her cost is $c_i > \hat{c}$ than to bid p. Combining all these arguments, a firm with cost $c_i \geq \hat{c}$ finds it optimal to bid $b_p^{AI}(c_i)$ when her cost is $c_i \geq \hat{c}$.

Finally, suppose that $c_i < \hat{c}$. Firm i's payoff from bidding $b_p^{AI}(c_i) = p$ is $P(\hat{c})(p - c_i)$. Note that, for all $c \geq \hat{c}$,

$$
\begin{aligned}
P(\hat{c})(p - c_i) &= P(\hat{c})(p - \hat{c}) + P(\hat{c})(\hat{c} - c_i) \\
&\geq (1 - F(c))^{\widehat{N}-1}(b_p^{AI}(c) - \hat{c}) + P(\hat{c})(\hat{c} - c_i) \\
&> (1 - F(c))^{\widehat{N}-1}(b_p^{AI}(c) - c_i),
\end{aligned}
$$

where the first inequality follows since $P(\hat{c})(p - \hat{c}) = (1 - F(\hat{c}))^{\widehat{N}-1}(b_p^{AI}(\hat{c}) - \hat{c}) \geq (1 - F(c))^{\widehat{N}-1}(b_p^{AI}(c) - \hat{c})$ for all $c \geq \hat{c}$, and the second inequality follows since $P(\hat{c}) > (1 - F(c))^{\widehat{N}-1}$ for all $c \geq \hat{c}$ and since $c_i < \hat{c}$. Therefore, firm i finds it optimal to bid $b_p^{AI}(c_i) = p$ when her cost is $c_i < \hat{c}$.

Next we establish uniqueness. We start with a few preliminary observations. Fix an auction with minimum price $p > 0$ and let b_p be the bidding function in a symmetric equilibrium. By standard arguments (see, for instance, Maskin and Riley (1984)), b_p must be weakly increasing; and it must be strictly increasing and differentiable at all points c such that $b_p(c) > p$. Lastly, b_p must be such that $b_p(\bar{c}) = \bar{c}$.[26]

Consider a bidder with cost c such that $b_p(c) > p$, and suppose all of her opponents bid according to b_p. The expected payoff that this bidder gets from bidding $b_p(\tilde{c}) > p$ is $(1 - F(\tilde{c}))^{\widehat{N}-1}(b_p(\tilde{c}) - c)$. Since bidding $b_p(c) > p$ is optimal, the first-order conditions imply that b_p solves

$$b_p'(c) = \frac{f(c)}{1 - F(c)}(\widehat{N} - 1)(b_p(c) - c),$$

with boundary condition $b_p(\bar{c}) = \bar{c}$. Note that bidding function b_0^{AI} solves the same differential equation with the same boundary condition, and so $b_p(c) = b_0^{AI}(c)$ for all c such that $b_p(c) > p$.

Consider the case in which $p < b_0^{AI}(\underline{c})$, and suppose that there exists a symmetric equilibrium $b_p \neq b_0^{AI}$. By the previous paragraph, $b_p(c) = b_0^{AI}(c)$ for all c such that $b_p(c) > p$. Therefore, if $b_p \neq b_0^{AI}$ is an equilibrium, there must exist $\tilde{c} > \underline{c}$ such that $b_p(c) = p$ for all $c < \tilde{c}$, and $b_p(c) = b_0^{AI}(c)$ for all $c \geq \tilde{c}$. For this to be an equilibrium, a bidder with cost $\tilde{c}$ must be indifferent between bidding $b_0^{AI}(\tilde{c}) = b_p(\tilde{c})$ or bidding p: $P(\tilde{c})(p - \tilde{c}) = (1 - F(\tilde{c}))^{\widehat{N}-1}(b_0^{AI}(\tilde{c}) - \tilde{c})$. But this can never happen when $p < b_0^{AI}(\underline{c})$ since $P(\underline{c})(p - \underline{c}) = p - \underline{c} < b_0^{AI}(\underline{c}) - \underline{c}$, and for all $c \in [\underline{c}, p]$, $\frac{\partial}{\partial c}P(c)(p - c) = -P(c) + P'(c)(p - c) \leq -P(c) < -(1 - F(c))^{\widehat{N}-1} = \frac{\partial}{\partial c}(1 - F(c))^{\widehat{N}-1}(b_0^{AI}(c) - c)$. Therefore, in this case the unique symmetric equilibrium is b_0^{AI}.

Consider next the case with $p > b_0^{AI}(\underline{c})$. By the arguments above, any symmetric equilibrium b_p must be such that $b_p(c) = b_0^{AI}(c)$ for all c with $b_p(c) > p$. Therefore, in any symmetric equilibrium, there exists $\tilde{c} > \underline{c}$ such that $b_p(c) = p$ for all $c < \tilde{c}$, and $b_p(c) = b_0^{AI}(c)$ for all $c \geq \tilde{c}$. Moreover, $\tilde{c}$ satisfies $P(\tilde{c})(p - \tilde{c}) = (1 - F(\tilde{c}))^{\widehat{N}-1}(b_p^{AI}(\tilde{c}) - \tilde{c})$. When $p > b_0^{AI}(\underline{c})$, there exists a unique such $\tilde{c}$ (see footnote 25). Therefore, in this case the unique symmetric equilibrium is b_p^{AI}. $\blacksquare$

Proof of Corollary 3. Suppose first that $p \leq b_0^{AI}(\underline{c})$. Then, $\text{prob}(\beta_p^{AI} \geq q | \beta_p^{AI} > p) =$

[26] This condition holds for the case in which $r \geq \bar{c}$. If $r < \bar{c}$, then b_p must be such that $b_p(r) = r$.

$\text{prob}(\beta_0^{AI} \geq q | \beta_0^{AI} > p)$ for all $q > p$.

Consider next the case in which $p > b_0^{AI}(\underline{c})$. For all $b \in [b_0^{AI}(\underline{c}), b_0^{AI}(\overline{c})]$, let $c(b)$ be such that $b_0^{AI}(c(b)) = b$. Since $\hat{c}$ is such that $b_0^{AI}(\hat{c}) > p$, it follows that $\hat{c} > c(p)$. Note then that, for all $q \geq b_0^{AI}(\hat{c})$, $\text{prob}(\beta_p^{AI} \geq q | \beta_p^{AI} > p) = \frac{(1-F(c(q)))^{\widehat{N}}}{(1-F(\hat{c}))^{\widehat{N}}} > \frac{(1-F(c(q)))^{\widehat{N}}}{(1-F(c(p)))^{\widehat{N}}} = \text{prob}(\beta_0^{AI} \geq q | \beta_0^{AI} > p)$. For $q \in (p, b_0^{AI}(\hat{c}))$, $\text{prob}(\beta_p^{AI} \geq q | \beta_p^{AI} > p) = 1 > \frac{(1-F(c(q)))^{\widehat{N}}}{(1-F(c(p)))^{\widehat{N}}} = \text{prob}(\beta_0^{AI} \geq q | \beta_0^{AI} > p)$. ■

B.3 Additional results and Proofs for Section 4

B.3.1 Collusion under threat of entry

This appendix analyzes the model with entry in Section 4. We let $\widehat{N}_e$ denote the set of all participants in the auction; i.e., $\widehat{N}_e = \widehat{N}$ when $E = 0$, and $\widehat{N}_e = \widehat{N} \cup \{e\}$ when $E = 1$. Given a history h_t and an equilibrium σ, we let $\beta(\mathbf{c}|h_t, \sigma)$ be the bidding profile of cartel members and short-lived firm induced by σ at history h_t as a function of procurement costs $\mathbf{c} = (c_i)_{i \in \widehat{N}_e}$.[27] Our first result generalizes Lemma 1 to the current setting.

Lemma B.2 (stationarity – entry)**.** *Consider a subgame perfect equilibrium σ that attains $\overline{V}_p$. Equilibrium σ delivers surplus $V(\sigma, h_t) = \overline{V}_p$ after all on-path histories h_t.*

There exists a fixed bidding profile β^* such that, in a Pareto efficient equilibrium, firms bid $\beta(\mathbf{c}_t|h_t, \sigma) = \beta^*(\mathbf{c}_t)$ after all on-path histories h_t.

Given a bidding profile (β, γ), we let $\beta^W(\mathbf{c})$ be the winning bid and $\mathbf{x}(\mathbf{c}) = (x_i(\mathbf{c}))_{i \in \widehat{N}_e}$ be the induced allocation when realized costs are $\mathbf{c} = (c_i)_{i \in \widehat{N}_e}$. As in Section 2, for all $i \in \widehat{N}_e$ we let

$$\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) \equiv \mathbf{1}_{\beta^W(\mathbf{c}) > p} + \frac{\mathbf{1}_{\beta^W(\mathbf{c}) = p}}{\sum_{j \in \widehat{N}_e \setminus \{i\} : x_j(\mathbf{c}) > 0} \gamma_j(\mathbf{c}) + 1}.$$

[27] Since the vector of costs $\mathbf{c}$ includes the cost of the short-lived firm in case of entry, the cartel's bidding profile can be different depending on whether the short-lived firm enters the auction or not.

Lemma B.3 (enforceable bidding – entry)**.** *A winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ are sustainable in SPE if and only if, for $E \in \{0, 1\}$ and for all $\mathbf{c}$,*

$$\sum_{i \in \widehat{N}} \{(\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c}))[\beta^W(\mathbf{c}) - c_i]^+ + x_i(\mathbf{c})[\beta^W(\mathbf{c}) - c_i]^-\} \leq \delta(\overline{V}_p - n\underline{V}_p). \quad (13)$$

$$E \times \{(\rho_e(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_e(\mathbf{c}))[\beta^W(\mathbf{c}) - c_e]^+ + x_e(\mathbf{c})[\beta^W(\mathbf{c}) - c_e]^-\} \leq 0. \quad (14)$$

Recall that

$$b_p^*(\mathbf{c}) = \sup \left\{ b \leq r : \sum_{i \in \widehat{N}} (1 - x_i^*(\mathbf{c}))\,[b - c_i]^+ \leq \delta(\overline{V}_p - n\underline{V}_p) \right\}.$$

Proposition B.1. *In an optimal equilibrium, the on-path bidding profile is such that:*

(i) if $E = 0$, the cartel sets winning bid $\beta_p^*(\mathbf{c}) = \max\{b_p^*(\mathbf{c}), p\}$;

(ii) if $E = 1$, the winning bid is $\beta_p^*(\mathbf{c}) = \max\{p, \min\{c_e, b_p^*(\mathbf{c})\}\}$ when a cartel wins the auction, and is $\beta_p^*(\mathbf{c}) = \max\{c_{(e)}, p\}$ when the entrant wins the auction.

Proposition B.1 characterizes bidding behavior under an optimal equilibrium. In periods in which the short-lived firm does not participate, the cartel's bidding behavior is the same as in Section 2. Entry by a short-lived firm reduces the cartels profits in two ways: (i) the cartel losses the auction whenever the entrant's procurement cost is low enough, and (ii) entry leads to weakly lower winning bids when the cartel wins the auction.

By Proposition B.1, the winning bid when the entrant wins the auction is $\beta_p^*(\mathbf{c}) = \max\{c_{(e)}, p\}$. For $p \leq \underline{c}$, the entrant earns zero payoff from participating in the auction. Therefore, for $p \leq \underline{c}$ the entrant participates in the auction if and only if its entry cost is equal to zero.[28] For $p > \underline{c}$, the entrant's payoff from participating in the auction is strictly positive. From now on we assume that the distribution of entry costs F_k has a mass point at zero, so that there is positive probability of entry for all minimum prices p.

[28]We assume that the short-lived firm participates in the auction whenever its indifferent.

Our last result in this section extends Lemma 3 to the current setting. Recall that $\beta_0^*(\underline{c})$ is the lowest bid under minimum price $p = 0$.

Lemma B.4 (worse case punishment – entry). *(i)* $\underline{V}_0 = 0$*, and* $\underline{V}_p > 0$ *whenever* $p > \underline{c}$*;*

(ii) there exists $\eta > 0$ *such that, for all* $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$*,* $\overline{V}_p - n\underline{V}_p \leq \overline{V}_0 - n\underline{V}_0$*. The inequality is strict if* $p \in (\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$*.*

B.3.2 Proofs for Section 4 and Appendix B.3.1

Fix an SPE σ and a history h_t, and suppose that the entry decision of the short-lived firm at time t is E. For each $\mathbf{c}$, let $\beta(\mathbf{c})$, $\gamma(\mathbf{c})$ and $T(\mathbf{c}, \mathbf{b}, \gamma, \mathbf{x})$ be the bidding profile of cartel members and short-lived firm and the transfer profile of cartel members in this equilibrium after history $h_t \sqcup (E, \mathbf{c})$. For each $\mathbf{c}$, let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ be winning bid and the allocation induced by bidding profile $(\beta(\mathbf{c}), \gamma(\mathbf{c}))$. For each $h_{t+1} = h_t \sqcup (E, \mathbf{c}, \mathbf{b}, \gamma, \mathbf{x}, \mathbf{T})$, let $\{V(h_{t+1})\}_{i \in N}$ be the vector of continuation payoffs of cartel members after history h_{t+1}. We let $h_{t+1}(\mathbf{c}) = h_t \sqcup (E, \mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}), \mathbf{T}(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c})))$ denote the on-path history that follows $h_t \sqcup (E, \mathbf{c})$. With this notation, the inequalities (10)-(12) must also hold in this setting. Moreover, if $E = 1$, it must also be that

$$\mathbf{x}_e(\mathbf{c})[\beta^W(\mathbf{c}) - c_e]^+ \geq \rho_e(\beta^W, \gamma, \mathbf{x})(\mathbf{c})[\beta^W(\mathbf{c}) - c_e]^+ \text{ and } x_e(\mathbf{c})[\beta^W(\mathbf{c}) - c_e]^- \leq 0. \tag{15}$$

Conversely, suppose there exists a winning bid profile $\beta^W(\mathbf{c})$, an allocation $\mathbf{x}(\mathbf{c})$, a transfer profile $\mathbf{T}$ and equilibrium continuation payoffs $\{V_i(h_{t+1}(\mathbf{c})\}_{i \in N}$ that satisfy inequalities (10)-(12) for some $\gamma(\mathbf{c})$ that is consistent with $\mathbf{x}(\mathbf{c})$ (i.e., $x_i(\mathbf{c}) = \gamma_i(\mathbf{c}) / \sum_{j: x_j(\mathbf{c}) > 0} \gamma_j(\mathbf{c})$ for all $i \in \widehat{N}_e$ with $x_i(\mathbf{c}) > 0$) and satisfy (15) if $E = 1$. Then, $(\beta^W, \mathbf{x}, \mathbf{T})$ can be supported in an SPE as follows. For all $\mathbf{c}$, all firms $i \in \widehat{N}_e$ bid $\beta^W(\mathbf{c})$. Firms $i \in \widehat{N}_e$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$ and firms $i \in \widehat{N}_e$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \widehat{N}_e$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c}) / \sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm $i \in \widehat{N}$ deviates at the bidding stage, cartel members make transfers $T_i(\mathbf{c}, \beta(\mathbf{c}), \gamma(\mathbf{c}), \mathbf{x}(\mathbf{c}))$. If no firm $i \in N$

deviates at the transfer stage, in the next period cartel members play an SPE that gives payoff vector $\{V(h_{t+1}(\mathbf{c}))\}_{i\in N}$. If firm $i \in \widehat{N}$ deviates at the bidding stage, there are no transfers and the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_p$; if firm $i \in N$ deviates at the transfer stage, the cartel reverts to an equilibrium that gives firm i a payoff of $\underline{V}_p$ (deviations by more than one firm go unpunished). Since (10) holds, under this strategy profile no firm $i \in \widehat{N}$ has an incentive to undercut the winning bid $\beta^W(\mathbf{c})$. Since (11) holds, no firm $i \in \widehat{N}$ with $c_i > \beta^W(\mathbf{c})$ and $x_i(\mathbf{c}) > 0$ has an incentive to bid above $\beta^W(\mathbf{c})$ and lose. Upward deviations by a firm $i \in \widehat{N}_e$ with $c_i < \beta^W(\mathbf{c})$ who bids $\beta^W(\mathbf{c})$ are not profitable since the firm would lose the auction by bidding $b > \beta^W(\mathbf{c})$. Since (15) holds, the short-lived firm does not have an incentive to deviate when $E = 1$. Finally, since (12) holds, all firms $i \in N$ have an incentive to make their required transfers.

Proof of Lemma B.2. The proof is identical to the proof of Lemma 1, and hence omitted. ■

Proof of Lemma B.3. The proof that (13) must hold in any equilibrium uses the same arguments used in the proof of Lemma 2, and hence we omit it. Since (15) must hold for $E = 1$, it follows that

$$E \times \{(\rho_e(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_e(\mathbf{c}))[\beta^W(\mathbf{c}) - c_e]^+ + x_e(\mathbf{c})[\beta^W(\mathbf{c}) - c_e]^-\} \leq 0.$$

Next, consider a winning bid profile $\beta^W(\mathbf{c})$ and an allocation $\mathbf{x}(\mathbf{c})$ that satisfy (13) and (14) for all $\mathbf{c}$ for some $\gamma(\mathbf{c})$ consistent with $\mathbf{x}(\mathbf{c})$ (i.e., such that $x_i(\mathbf{c}) = \gamma_i(\mathbf{c})/\sum_{j:x_j(\mathbf{c})>0}\gamma_j(\mathbf{c})$ for all i with $x_i(\mathbf{c}) > 0$). We construct an SPE that supports $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ in the first period. Let $\{V_i\}_{i\in N}$ be an equilibrium payoff vector with $\sum_i V_i = \overline{V}_p$. For each $\mathbf{c} = (c_i)_{i\in\widehat{N}_e}$

and $i \in N$, we construct transfers $T_i(\mathbf{c})$ as follows:

$$T_i(\mathbf{c}) = \begin{cases} -\delta(V_i - \underline{V}_p) + (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i(\mathbf{c}))(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \leq \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_p) - x_i(\mathbf{c})(\beta^W(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i > \beta^W(\mathbf{c}), \\ -\delta(V_i - \underline{V}_p) + \epsilon(\mathbf{c}) & \text{if } i \notin \widehat{N}, \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined below. Since $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ satisfy (13), it follows that for all $\mathbf{c}$,

$$\begin{aligned} &\sum_{i \in N} T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) \\ =& -\delta(\overline{V}_p - n\underline{V}_p) + \sum_{i \in \widehat{N}} \left\{ (\rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c}) - x_i) \left[\beta^W(\mathbf{c}) - c_i\right]^+ + x_i \left[\beta^W(\mathbf{c}) - c_i\right]^- \right\} \leq 0. \end{aligned}$$

We set $\epsilon(\mathbf{c}) \geq 0$ such that transfers are budget balance; i.e., such that $\sum_{i \in N} T_i(\mathbf{c}) = 0$.

The SPE we construct is as follows. At $t = 0$, for each $\mathbf{c} = (c_i)_{i \in \widehat{N}_e}$ all firms $i \in \widehat{N}_e$ bid $\beta^W(\mathbf{c})$. Firms $i \in \widehat{N}_e$ with $x_i(\mathbf{c}) = 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = 0$, and firms $i \in \widehat{N}_e$ with $x_i(\mathbf{c}) > 0$ choose $\tilde{\gamma}_i(\mathbf{c}) = \gamma_i(\mathbf{c})$. Note that, for all $i \in \widehat{N}_e$, $x_i(\mathbf{c}) = \tilde{\gamma}_i(\mathbf{c}) / \sum_j \tilde{\gamma}_j(\mathbf{c})$ and $\rho_i(\beta^W, \tilde{\gamma}, \mathbf{x})(\mathbf{c}) = \rho_i(\beta^W, \gamma, \mathbf{x})(\mathbf{c})$. If no firm $i \in \widehat{N}$ deviates at the bidding stage, cartel members exchange transfers $T_i(\mathbf{c})$. If no firm $i \in N$ deviates at the transfer stage, from $t = 1$ onwards firms play an SPE that supports payoff vector $\{V_i\}$. If firm $i \in N$ deviates either at the bidding stage or at the transfer stage, from $t = 1$ onwards firms play an SPE that gives firm i a payoff $\underline{V}_p$ (if more than one firm deviates, then firms punish the lowest indexed firm that deviated). One can check that this strategy profile satisfies (10)-(12) and (15). Hence, winning bid profile β^W and allocation $\mathbf{x}$ are implementable. ■

Proof of Proposition B.1. The proof of part (i) is identical to the proof of Proposition 1, and hence omitted.

We now turn to part (ii). Note first that, by Lemma B.3, entry by the short-lived firm reduces the set of sustainable bidding profiles and thus the profits that the cartel can obtain in an auction. Therefore, in an optimal equilibrium the cartel seeks to maximize its payoff and minimize the short-lived firm's payoff from entry.

Suppose $E = 1$. For any $\mathbf{c}$, let $\beta^W(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ be, respectively, the winning bid and allocation in an optimal equilibrium. We let $c_{(1)} = \min_{i \in \widehat{N}} c_i$ be the lowest cost among participating cartel members. Consider first cost realizations $\mathbf{c}$ such that $c_{(1)} > c_e \geq p$. In this case, $x_e(\mathbf{c}) = 1$ in an optimal bidding profile. Indeed, by equation (14), $\beta^W(\mathbf{c}) \leq c_e$ if $x_e(\mathbf{c}) < 1$. Hence, the cartel is better-off letting the short-lived firm win whenever $c_{(1)} > c_e \geq p$. Moreover, by setting $\beta^W(\mathbf{c}) = c_e$, the cartel guarantees that the short-lived firm earns zero payoff.[29]

Consider next $\mathbf{c}$ such that $c_{(1)} > p > c_e$. By (14), it must be that $x_e(\mathbf{c}) > 0$. In this case, in an optimal equilibrium the cartel sets winning bid equal to $\beta^W(\mathbf{c}) = p$, as this minimizes the short-lived firm's payoff from winning.

Consider next $\mathbf{c}$ such that $c_{(1)} < c_e$ and $c_e \geq p$. Clearly, an optimal bidding profile for the cartel must be such that $x_e(\mathbf{c}) = 0$. Equation (14) then implies that $\beta^W(\mathbf{c}) \leq c_e$. We now show that, in this case, $\beta^W(\mathbf{c}) = \max\{p, \min\{c_e, b^*_p(\mathbf{c})\}\}$. There are two cases to consider: (a) $b^*_p(\mathbf{c}) > c_e$, and (b) $b^*_p(\mathbf{c}) \leq c_e$. Consider case (a), so $b^*_p(\mathbf{c}) > c_e \geq p$. It follows that

$$\sum_{i \in \widehat{N}} (1 - x^*_i(\mathbf{c}))[c_e - c_i]^+ < \sum_{i \in \widehat{N}} (1 - x^*_i(\mathbf{c}))[b^*_p(\mathbf{c}) - c_i]^+ \leq \delta(\overline{V}_p - n\underline{V}_p).$$

Therefore, a bidding profile that induces winning bid c_e and allocation $\mathbf{x}^*(\mathbf{c})$ satisfies (13) and (14). Since such a bidding profile is optimal for the cartel among all bidding profiles with winning bid lower than c_e, it must be that $\beta^W(\mathbf{c}) = c_e$.

[29] This is achieved by having all participating cartel members bidding $\beta^W(\mathbf{c}) = c_e$ and $\gamma_i(\mathbf{c}) = 0$, and having the entrant bidding $\beta^W(\mathbf{c}) = c_e$ and $\gamma_e(\mathbf{c}) = 1$.

Consider next case (b). Note that for all $b > \max\{b_p^*(\mathbf{c}), p\}$ and any allocation $\mathbf{x}(\mathbf{c})$,

$$\sum_{i\in\widehat{N}} \left\{(1-x_i(\mathbf{c}))[b-c_i]^+ + x_i(\mathbf{c})[b-c_i]^-\right\} \geq \sum_{i\in\widehat{N}} (1-x_i^*(\mathbf{c}))[b-c_i]^+ > \delta(\overline{V}_p - n\underline{V}_p),$$

so $\max\{b_p^*(\mathbf{c}), p\}$ is the largest winning bid that can be supported in an equilibrium. Therefore, in an optimal equilibrium cartel members must use a bidding profile inducing winning bid $\max\{b_p^*(\mathbf{c}), p\}$.

Finally, consider $\mathbf{c}$ such that $c_{(1)} < p$ and $c_e < p$. We now show that, in an optimal equilibrium, $\beta^W(\mathbf{c}) = p$. Indeed, by (14), a winning $\beta^W(\mathbf{c}) > p > c_e$ can only be implemented if $x_e(\mathbf{c}) = 1$. But this is clearly suboptimal for the cartel. Indeed, the cartel could make strictly positive profits by having a firm with cost $c_{(1)}$ bidding p; and doing this would also strictly reduce the short-lived firm's expected payoff from entering. Therefore, in an optimal equilibrium it must be that $\beta^W(\mathbf{c}) = p$. ■

Proof of Lemma B.4. We first establish part (i). Suppose that $p \leq \underline{c}$ and fix equilibrium payoffs $\{V_i\}_{i\in N}$. Fix $j \in N$ and consider the following strategy profile. At $t = 0$, firms' behavior depends on whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. If $j \in \widehat{N}$, all firms $i \in \widehat{N}_e$ bid $\min\{c_j, \hat{c}_{(2)}\}$ (where $\hat{c}_{(2)}$ is the second lowest procurement cost among firms in $\widehat{N}_e$). Firm $i \in \widehat{N}_e$ chooses $\gamma_i = 1$ if $c_i = \min_{k\in\widehat{N}_e} c_k$, and chooses $\gamma_i = 0$ otherwise. Note that this bidding profile constitutes an equilibrium of the stage game. If $j \notin \widehat{N}$, at $t = 0$ participating firms play according to some equilibrium of the stage game. If all firms bid according to this profile, firm j's transfer is $T_j = -\delta V_j$ at the end of the period regardless of whether $j \in \widehat{N}$ or $j \notin \widehat{N}$. The transfer of firm $i \in N\backslash\{j\}$ is $T_i = \frac{1}{n-1}\delta V_j$ at the end of the period, so $\sum_i T_i = 0$. If no firm deviates at the bidding or transfer stage, at $t = 1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}$. If firm i deviates at the bidding stage, there are no transfers and at $t = 1$ firms play the strategy just described with i in place of j. If no firm deviates at the bidding stage and firm i deviates at the transfer stage, at $t = 1$ firms play the strategy just described with i in place of j (if more than one firm deviates at the bidding or transfer stage, from $t=1$ firms play according to an equilibrium that delivers payoffs $\{V_i\}_{i\in N}$). Note that this strategy profile gives player j a payoff of 0. Moreover, no firm has an incentive to deviate at $t=0$, and so $\underline{V}_p = 0$ for all $p \leq \underline{c}$.

Suppose next that $p > \underline{c}$, and note that

$$\underline{V}_p \geq \underline{u}_p \equiv \frac{1}{1-\delta}\text{prob}(i \in \widehat{N})\mathbb{E}\left[\frac{1}{\widehat{N}+1}\mathbf{1}_{c_i\leq p}(p-c_i)|i \in \widehat{N}\right] > 0,$$

where the inequality follows since firm i can always guarantee a payoff at least as large as $\underline{u}_p$ by bidding p whenever $c_i \leq p$ and bidding $b \geq c_i$ otherwise. This establishes part (i).

We now turn to part (ii). Note that $\beta_0^*(\underline{c}) = \underline{c}$.[30] Fix $\eta > 0$ and $p \in [\underline{c}, \underline{c}+\eta]$. For $E = 0, 1$, let (β^E, γ^E) be the bidding profile that firms use on the equilibrium path at periods in which the short-lived firm's entry decision is E under an optimal equilibrium that attains $\overline{V}_p$ when the minimum price is p. Let $\beta_p^*(\mathbf{c})$ and $\mathbf{x}^p(\mathbf{c})$ denote, respectively, the winning bid and the allocation under this optimal equilibrium. The cartel's expected payoff under this optimal equilibrium satisfies

$$\begin{aligned}(1-\delta)\overline{V}_p =& \text{prob}(E=0|p)\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c})-c_i)\,|E=0\right] \\ &+ \text{prob}(E=1|p)\mathbb{E}\left[\sum_{i\in\widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c})-c_i)\,|E=1\right].\end{aligned}$$

Suppose there is no minimum price and consider the following bidding profile for cartel members. For $E = 0, 1$ and all $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) > p$, participating firms bid according to (β^E, γ^E). For $E = 0$ and all $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$, all participating cartel members bid $c_{(2)}$; firm $i \in \widehat{N}$ with $c_i = c_{(1)} = \min_{j\in\widehat{N}} c_j$ sets $\gamma_i = 1$, and firm $i \in \widehat{N}$ with $c_i > c_{(1)}$ sets $\gamma_i = 0$. For $E = 1$ and all $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$, all participating firms bid $\min\{c_{(2)}, c_e\}$;

[30]Indeed, by Proposition B.1, $\beta_0^*(\mathbf{c}) = \underline{c}$ whenever $E = 1$ and $c_e = \underline{c}$.

firm $i \in \widehat{N}_e$ sets $\gamma_i = 1$ if $c_i = \min_{k \in \widehat{N}_e} c_k$ and sets $\gamma_i = 0$ otherwise. Note that, for $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$, the bidding profile that firms use constitutes an equilibrium of the stage game when there is no minimum price. Note further that the entrant earns a lower expected payoff under this bidding profile than under the optimal equilibrium for minimum price $p \in [\underline{c}, \underline{c} + \eta]$; indeed, under this bidding profile, the entrant earns the same payoff than under the optimal equilibrium whenever $\beta_p^*(\mathbf{c}) > p$, and earns a payoff of zero whenever $\beta_p^*(\mathbf{c}) = p$. Therefore, the probability of entry under this strategy profile is lower than under the optimal equilibrium when minimum price is p. Let $\beta(\mathbf{c})$ and $\mathbf{x}(\mathbf{c})$ denote the winning bid and the allocation that this bidding profile induces. Let $\hat{V}_p$ be the cartel's total surplus under this strategy profile, and note that

$$
\begin{aligned}
(1-\delta)\hat{V}_p =& \text{prob}(E=0|\text{no min price})\mathbb{E}\left[\sum_{i \in \widehat{N}} x_i(\mathbf{c})(\beta(\mathbf{c}) - c_i)\,|E=0\right] \\
&+ \text{prob}(E=1|\text{no min price})\mathbb{E}\left[\sum_{i \in \widehat{N}} x_i(\mathbf{c})(\beta(\mathbf{c}) - c_i)\,|E=1\right] \\
\geq& \text{prob}(E=0|p)\mathbb{E}\left[\sum_{i \in \widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c}) - c_i)\mathbf{1}_{\beta_p^*(\mathbf{c})>p}\,|E=0\right] \\
&+ \text{prob}(E=1|p)\mathbb{E}\left[\sum_{i \in \widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c}) - c_i)\mathbf{1}_{\beta_p^*(\mathbf{c})>p}\,|E=1\right],
\end{aligned}
$$

where we used the fact that the $\text{prob}(E=0|p) \leq \text{prob}(E=0|\text{no min price})$ and that the cartel's payoff conditional on $E=0$ is weakly larger than its payoff conditional on $E=1$.

Note that $b_p^*(\mathbf{c}) \geq \underline{c} + \frac{\delta(\overline{V}_p - n\underline{V}_p)}{n-1} > \underline{c}$.[31] By Proposition B.1, $\beta_p^*(\mathbf{c}) = \max\{p, b_p^*(\mathbf{c})\}$ whenever $E=0$. Therefore, for $\eta > 0$ small enough and for $E=0$, $\beta_p^*(\mathbf{c}) > p$ for all $\mathbf{c}$ and all $p \in [\underline{c}, \underline{c}+\eta]$. For all such $\eta > 0$ and for all $p \in [\underline{c}, \underline{c}+\eta]$, $\text{prob}(\beta_p^*(\mathbf{c}) = p|E = 0) = 0$. Moreover, Proposition B.1 also implies that $\text{prob}(\beta_p^*(\mathbf{c}) = p|E=1) = F_e(p)$ for all

[31]Indeed, $\inf_{\mathbf{c}} b_p^*(\mathbf{c})$ is attained when all cartel members participate and they all have a cost equal to $\underline{c}$. In this case, $b_p^*(\mathbf{c}) = \underline{c} + \frac{\delta(\overline{V}_p - n\underline{V}_p)}{n-1}$.

$p \in [\underline{c}, \underline{c}+\eta]$.[32] Therefore, for $\eta > 0$ small enough and for $p \in [\underline{c}, \underline{c}+\eta]$,

$$
\begin{aligned}
(1-\delta)(\overline{V}_p - \hat{V}_p) \leq & \mathrm{prob}(E=1|p)\mathbb{E}\left[\sum_{i \in \widehat{N}} x_i^p(\mathbf{c})(\beta_p^*(\mathbf{c}) - c_i)\mathbf{1}_{\beta_p^*(\mathbf{c})=p}|E=1\right] \\
\leq & \mathrm{prob}(E=1|p)\frac{n}{n+1}F_e(p)\mathbb{E}[(p-c_{(1)})\mathbf{1}_{c_{(1)}\leq p}] \\
\leq & \mathrm{prob}(E=1|p)\frac{n}{n+1}F_e(p)\int_{\underline{c}}^{p}(p-c)n(1-F(c))^{n-1}f(c)dc
\end{aligned}
$$

where the second inequality follows since the probability with which the cartel wins the auction when the entrant's cost is below p is bounded above by $\frac{n}{n+1}$, and since the cartel's payoff from winning the auction at price p is bounded above by $(p - c_{(1)})\mathbf{1}_{c_{(1)}\leq p}$. On the other hand,

$$
(1-\delta)n\underline{V}_p \geq (1-\delta)n\underline{u}_p \geq \frac{n}{n+1}\mathrm{prob}(i \in \widehat{N})\mathbb{E}[(p-c_i)\mathbf{1}_{c_i \leq p}] = \frac{n}{n+1}\mathrm{prob}(i \in \widehat{N})\int_{\underline{c}}^{p}(p-c)f(c)dc.
$$

Note that, for $p = \underline{c}$, $\hat{V}_p \geq \overline{V}_p - n\underline{V}_p = \overline{V}_p$. Note further that

$$
\begin{aligned}
\frac{\partial}{\partial p}\Big|_{p=\underline{c}} F_e(p)\int_{\underline{c}}^{p}(p-c)n(1-F(c))^{n-1}f(c)dc = 0 \\
\frac{\partial^2}{\partial p^2}\Big|_{p=\underline{c}} F_e(p)\int_{\underline{c}}^{p}(p-c)n(1-F(c))^{n-1}f(c)dc = 0 \\
\frac{\partial}{\partial p}\Big|_{p=\underline{c}} \int_{\underline{c}}^{p}(p-c)f(c)dc = 0 \\
\frac{\partial^2}{\partial p^2}\Big|_{p=\underline{c}} \int_{\underline{c}}^{p}(p-c)f(c)dc = f(\underline{c}) > 0.
\end{aligned}
$$

Therefore, there exists $\eta > 0$ small enough such that $\hat{V}_p \geq \overline{V}_p - n\underline{V}_p$ for all $p \in [\underline{c}, \underline{c}+\eta]$, with strict inequality if $p > \underline{c}$. To establish part (ii) of the Lemma, we show that $\overline{V}_0 \geq \hat{V}_p$ for all $p \in [\underline{c}, \underline{c}+\eta]$.

[32] Indeed, $b_p^*(\mathbf{c}) > p$ for all $\mathbf{c}$ and all $p \in [\underline{c}, \underline{c}+\eta]$. Therefore, by Proposition B.1, for all $p \in [\underline{c}, \underline{c}+\eta]$ and for $E = 1$, the winning bid $\beta_p^*(\mathbf{c})$ is equal to p only when the entrant's cost is below p.

Suppose there is no minimum price, and consider the following strategy profile. Along the equilibrium path, bidders bid according to the bidding profile described above, which generates surplus $\hat{V}_p$ for the cartel. If firm $i \in \widehat{N}$ deviates at the bidding stage, there are no transfers and in the next period cartel members play an equilibrium that gives firm i a payoff of $\underline{V}_0 = 0$ (if more than one firm deviates, cartel members punish the lowest indexed firm that deviated). If no firm deviates at the bidding stage, each firm $i \in N$ makes transfer $T_i(\mathbf{c})$ to be determined below. If a firm $i \in N$ deviates at the transfer stage, in the next period firms play an equilibrium that gives firm i a payoff of $\underline{V}_0 = 0$ (if more than one firm deviates, cartel members again punish the lowest indexed firm that deviated). Otherwise, if no firm deviates at the bidding and transfer stages, in the next period firms continue playing the same strategies as above.

Let $V = \hat{V}_p/n$. The transfers $T_i(\mathbf{c})$ are determined as follows. For all $\mathbf{c}$ such that $\beta_p^*(\mathbf{c}) = p$, $T_i(\mathbf{c}) = 0$ for all $i \in N$. Otherwise,

$$T_i(\mathbf{c}) = \begin{cases} -\delta V + (1 - x_i^p(\mathbf{c}))(\beta_p^*(\mathbf{c}) - c_i) + \epsilon(\mathbf{c}) & \text{if } i \in \widehat{N}, c_i \leq \beta_p^*(\mathbf{c}), \\ -\delta V + \epsilon(\mathbf{c}) & \text{otherwise,} \end{cases}$$

where $\epsilon(\mathbf{c}) \geq 0$ is a constant to be determined.[33] Note that

$$\sum_i T_i(\mathbf{c}) - n\epsilon(\mathbf{c}) = -\delta \hat{V}_p + \sum_i (1 - x_i^p(\mathbf{c}))[\beta_p^*(\mathbf{c}) - c_i]^+ \leq 0,$$

where the inequality follows since $\beta_p^*(\mathbf{c})$ is implementable with minimum price p, and since $\hat{V}_p \geq \overline{V}_p - n\underline{V}_p$. We set $\epsilon(\mathbf{c}) \geq 0$ such that $\sum_i T_i(\mathbf{c}) = 0$. This strategy profile generates total surplus $\hat{V}_p$ for the cartel. Since firms play symmetric strategies, it gives a payoff $V = \frac{\hat{V}_p}{n}$ to each cartel member. One can check that no firm has an incentive to deviate at any stage, and so this strategy profile constitutes an equilibrium. Hence, it must be

[33] Recall that $\mathbf{x}^p(\mathbf{c})$ is the allocation under an optimal equilibrium when the minimum price is p. Therefore, $\mathbf{x}^p(\mathbf{c})$ is such that $x_i^p(\mathbf{c}) = 0$ for all i with $c_i > \beta_p^*(\mathbf{c})$.

that $\overline{V}_0 \geq \hat{V}_p \geq \overline{V}_p - n\underline{V}_p$ for all $p \in [\underline{c}, \underline{c}+\eta]$, and the second inequality is strict if $p > \underline{c}$. ■

Proof of Proposition 4. Consider first a collusive environment and suppose that $E \in \{0,1\}$. By Proposition B.1 and Lemma B.4, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $\beta_p^*(\mathbf{c}) \leq \beta_0^*(\mathbf{c})$ for all $\mathbf{c}$ such that $\beta_0^*(\mathbf{c}) \geq p$. Therefore, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $q > p$, $\text{prob}(\beta_p^* \geq q | \beta_p^* \geq p, E) \leq \text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E)$. This completes the proof of part (i).

Consider next a competitive environment. Let $\hat{c}_{(2)}$ be the second lowest cost among all participating firms (including the entrant if $E = 1$). Then, for all $p > 0$ and all $q > p$, $\text{prob}(\beta_p^{\mathsf{comp}} \geq q | \beta_p^{\mathsf{comp}} > p, E) = \text{prob}(\hat{c}_{(2)} \geq q | \hat{c}_{(2)} > p, E) = \text{prob}(\beta_0^{\mathsf{comp}} \geq q | \beta_0^{\mathsf{comp}} > p, E)$. This completes the proof of part (ii). ■

Proof of Proposition 5. We start with part (i). If $E = 0$, the result follows from Proposition 4. Suppose next that $E = 1$, and consider cost realizations $\mathbf{c}$ such that the cartel wins. By Proposition B.1 and Lemma B.4, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$, $\beta_p^*(\mathbf{c}) \leq \beta_0^*(\mathbf{c})$ whenever $\beta_0^*(\mathbf{c}) \geq p$. Therefore, for all $p \in [\beta_0^*(\underline{c}), \beta_0^*(\underline{c}) + \eta]$ and all $q > p$, $\text{prob}(\beta_p^* \geq q | \beta_p^* \geq p, E = 1, \text{cartel wins}) \leq \text{prob}(\beta_0^* \geq q | \beta_0^* \geq p, E = 1, \text{cartel wins})$. This completes the proof of part (i).

We now turn to part (ii). Consider cost realizations $\mathbf{c}$ such that the entrant wins. By Proposition B.1, $\beta_0^*(\mathbf{c}) = c_{(e)}$ and $\beta_p^*(\mathbf{c}) = \max\{c_{(e)}, p\}$. Therefore, for all $p > 0$ and all $q > p$, $\text{prob}(\beta_p^* \geq q | \beta_p^* > p, \text{entrant wins}) = \text{prob}(\beta_0^* \geq q | \beta_0^* > p, \text{entrant wins})$. This completes the proof of part (ii). ■

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