Unconventional Monetary Policy, Leverage & Default Dynamics

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Abstract

The objective of this paper to investigate the effectiveness of credit easing policy in mitigating the economic fallout from a financial recession using a model that can account for the observed default and leverage dynamics during the financial crisis of 2007. A general equilibrium model is developed with a financial sector and endogenous asset defaults able to account for the observed default and leverage dynamics. Following an adverse aggregate shock, banks deleverage through two channels: *(i)* higher non-performing loans provisions, and *(ii)* lower the marginal return of assets. Credit policy is modelled as an expansion of the central bank's balance sheet countering the disruption in private financial intermediation. Unconventional monetary policy, namely credit easing policy, is shown to be ineffective in mitigating the effects of a financial crisis due to its crowding out effect on the private asset market. Other non-monetary policy tools such as credit subsidies and their efficacy considered.

Keywords: unconventional monetary policy, credit easing, credit subsidies, financial frictions, default, leverage, financial sector.

JEL codes: E20, E32, E44, E52, E58

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1 Introduction

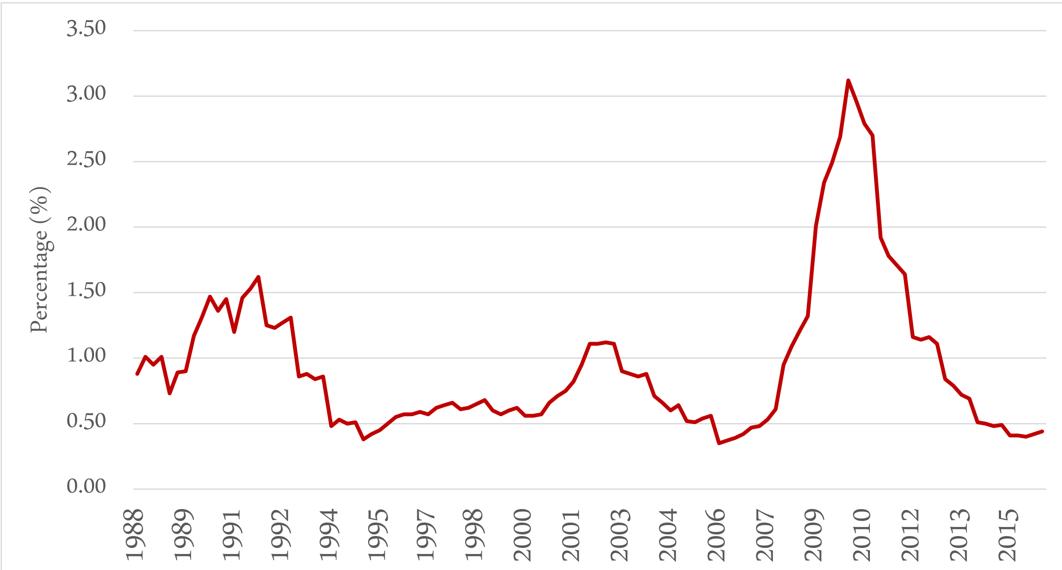
The financial crisis of 2007 saw default rates soar to unprecedented levels and in response, as financial asset losses rose, the financial sector deleveraged and restricted credit supply to the economy, leading to the biggest recession since the U.S. Great Depression. In the last decade, monetary policy in the U.S. has undertaken unconventional measures in order to mitigate the fallout from the 2007 financial crisis given that conventional measures had exhausted their effectiveness, namely interest rate policy. I investigate the effectiveness of credit easing policy in mitigating the economic fallout from a financial recession. In this endeavour, I develop a model with a financial sector that can account for the default and leverage dynamics observed during the U.S. Great Recession.

The financial crisis of 2007, which culminated with the onset of the Great Recession, has highlighted two distinguishing features of the banking sectors. Firstly, following the financial market turmoil in 2007, bank asset default rates rose to unprecedented levels as depicted in Figure 1. During the Great Recession the percentage of non-performing loans held by U.S. banks increased from a pre-Great Recession average of around 1.0% to over 3.0%. Non-performing loans are defined as the amount of loans that are not repaid out of the total proportion of loans supplied by banks. Secondly, accompanying record levels of asset default rates, the banking sector as a whole experienced one of the worse deleveraging episodes in recent times. The magnitude of this deleveraging episode is highlighted in Figure 2 where the investment bank sector halved its leverage ratio on average, whilst commercial banks decreased it by a third. Leverage in the data used by Kalemli-Ozcan et al. (2012) in Figure 2 and throughout this paper is defined as the banks' assets over banks' equity.

The sharp contraction in leverage by the financial sector restricted credit supply to the economy and caused the biggest recession since the Great Depression of 1929, with output falling by 5% and investment by 30%. The resulting recession meant that the Federal Reserve's first response was to aggressively cut interest rates until it was no longer possible to do so when they hit the Zero Lower Bound. At this point, the nature and scale of the recession meant that the Federal Reserve resorted to unconventional measures in an attempt to stabilise the financial system. They injected more than 4 billion dollars into the U.S. economy from 2007 to 2016. As shown Figure 3, the scale of the Federal Reserve's intervention was unparalleled as demonstrated by the decomposition of such intervention, the U.S. central bank started to directly purchase non-government financial securities. In this context, this paper investigates whether such credit easing policy can be effective in mitigating the fall in aggregate economic variables, such as output and investment, in a financial recession. The paper develops using a dynamic stochastic general equilibrium model with a financial sector and endogenous asset default. Crucially, the model, unlike previous literature, is able to replicate a financial crisis episode featuring both a spike in defaults rate and a sharp fall in banks' leverage.

Before proceeding, I intend to clarify two concepts. Firstly, by a financial recession, I mean any adverse shock to the economy which leads to a disruption in financial intermediation due to a direct fall in the value of the banks' assets. This differs from a typical recession which

Figure 1: Banks' Loan Default Rates during the U.S. Great Recession



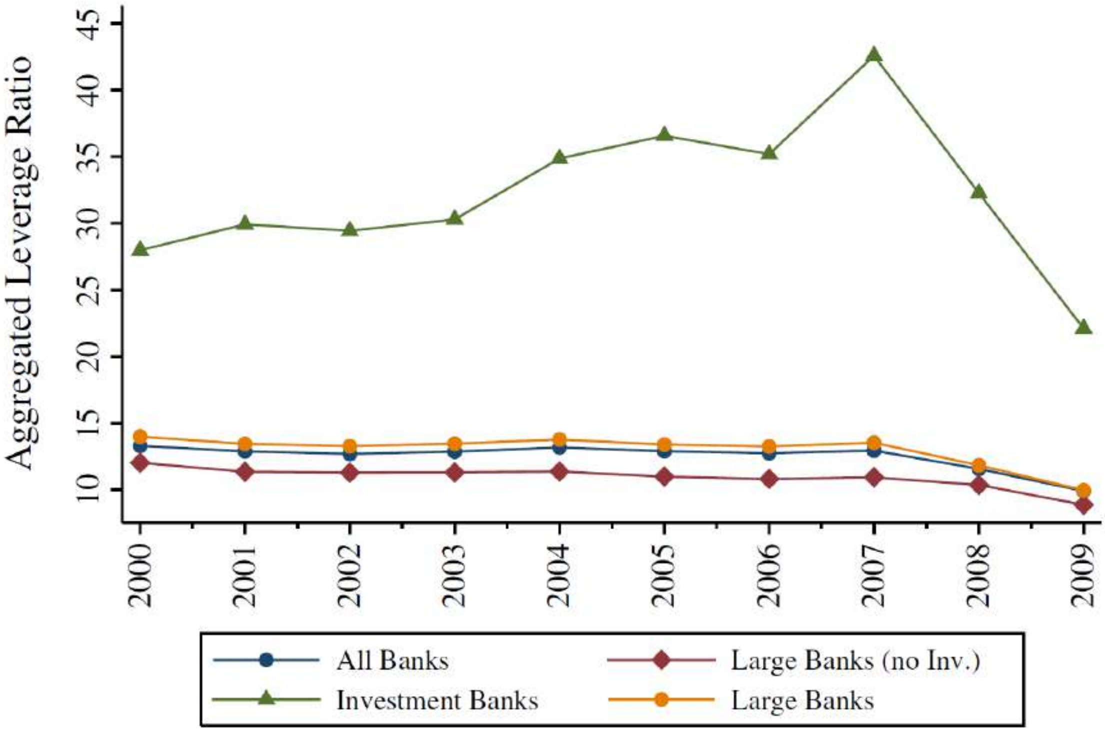
Source: *Total Net Loan Charge-offs to Total Loans for U.S. Banks [NCOTOT] from Federal Reserve Bank of St. Louis accessed via* `https://fred.stlouisfed.org/series/NCOTOT`.
Notes: *Charge-offs are measured on a net basis-loans charged off as losses minus recoveries on loans preciously charged off. The percentage of loans charged off as losses each quarter (net of recoveries on loans previously charged off as losses) is calculated by summing net charge-off for all banks in the size group and dividing by the sum of their total loans. Data are annualised for a quarterly frequency.*

is assumed to be driven by exogenous shocks to the total productivity of the economy, which may induce a decrease in the value of the banks' assets, but only indirectly. Secondly, when this paper talks about unconventional monetary policy, it refers to the *credit easing policies* used by the Federal Reserve during the Great Recession. Credit easing, used to increase asset prices and lower credit spreads, is defined as the purchase of non-government, asset-backed securities from private credit institutions[1].

To study the effects of credit easing policy, I develop a dynamic stochastic general equilibrium model with a financial sector that can replicate both the leverage dynamics and asset default rate dynamics during the U.S. financial crisis of 2007. To achieve such an objective, an otherwise standard real business cycle model is augmented in five key ways. Firstly, the financial sector is introduced where banks intermediate funds between final good firms and

[1]The policy also involved non-banking financial institutions. During the financial crisis of 2007, non-bank institutions have also benefited from the U.S. Federal Reserve's credit easing policy. A stark example was the intervention on insurance companies, such as the American International Group (AIG).

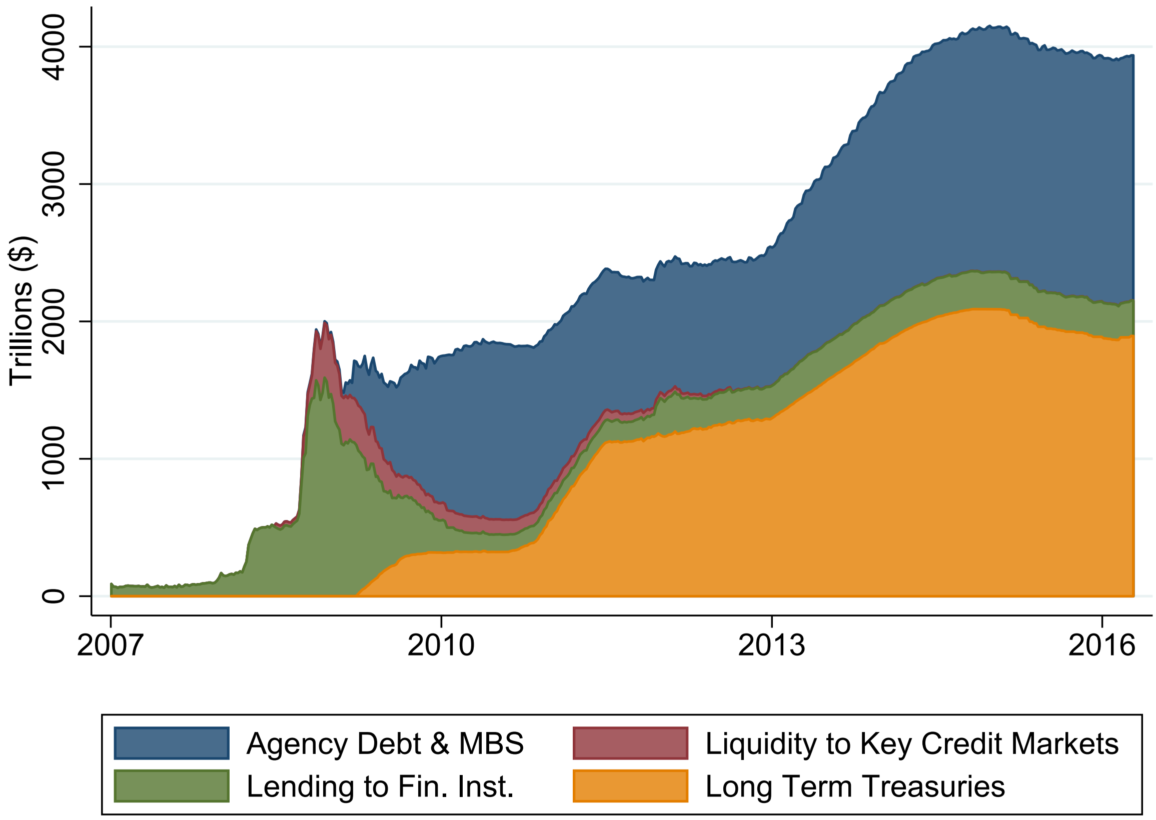
Figure 2: U.S. Banks' Leverage Ratio during the U.S. Great Recession



Source: *Kalemli-Ozcan et al. (2012)* using BankScope micro-level data.
Notes: *The figure shows the average leverage ratio for different U.S. financial institutions. The blue circled line depicts "All Banks", the green line with triangles only "Investment Banks", the orange circled line the "Large Banks", and the red line with diamonds "Large Banks excluding Investment Banks". Bank-level assets and equity are aggregated for different groups of banks and the ratio is taken of aggregated assets to aggregated equity. A "Large Bank" is defined as a bank with more than one billion dollars' worth of assets in 2000.*

households. Secondly, final good firms, who issue non-enforceable debt obligations to purchase capital from capital producers, are allowed to default if they receive sufficiently adverse idiosyncratic shocks. Thirdly, capital-producing firms are added to the model to provide the impetus for asset price dynamics, vital to the transmission of any financial shock. These three initial elements are central to the endogenous asset default mechanism and give rise to the financial friction in the model. Given that deposits are assumed to be risk-free and assets are subject to defaults, banks are constrained in their lending as any asset losses cannot exceed the banks' equity. Fourthly, to study the effectiveness of credit easing policy, central bank and government are introduced. The monetary authority is not a balance sheet constrained institution like the private financial intermediaries. Its role is to provide funds directly to final good firms when the private intermediation market is disrupted by an adverse shock. Fifth and finally, a financial crisis is modelled as a capital/asset quality shock. More specifically, it is assumed that when a financial shock hits the economy, the value of the asset decreases endogenously. This setup consists of the minimum set of assumptions needed to generate the default rate and leverage dynamics observed in the data. Moreover, the model gives credit easing policy a chance to intervene in the private asset market and potentially mitigate the economic fallout

Figure 3: Federal Reserve's Balance Sheet during the U.S. Great Recession



Source: *Federal Reserve Bank of Cleveland accessed via* `https://www.clevelandfed.org/our-research/indicators-and-data/credit-easing.aspx`.
Notes: *The figure groups the assets on the U.S. Federal Reserve balance sheet to show the evolution of different types of assets from* 2007 *to* 2016*. The blue area represents the holdings of agency debt and mortgage-backed securities (MBS), the green area shows the direct lending to financial institutions, the red area depicts securities held that provided liquidity to key credit markets, and the yellow area shows the holding of long-term government securities. The total area shows the size of the Federal Reserve intervention through unconventional monetary policy.*

from a financial crisis.

The first contribution of this study lies in the ability of the model to replicate the leverage and default rate dynamics episode which characterised the 2007 financial crisis. In this model, the pro-cyclical financial leverage and the counter-cyclical default dynamics derive from two key elements of the model: the banks' solvency constraint and the endogenous default of debt contracts. Once a financial shock hits the economy, the lower productiveness of capital renders final good firms more susceptible to defaults because they now need higher idiosyncratic productivity shocks to repay their loans. The endogenous default mechanism of the model, therefore, introduces counter-cyclical dynamics in the default rates. Furthermore, as a consequence of the higher default rates of the banks' assets, banks will deleverage through two channels: *i)* the *balance sheet channel*; and *ii)* the *bank lending channel*. In the former, banks have to increase equity to withstand the higher losses from asset defaults, and in the latter, banks reduce lending because the assets' expected marginal return falls as a result of the higher default rates. Since leverage is defined as the ratio of banks' assets over equity, the model produces pro-cyclical leverage dynamics as a financial shock increases in the banks' equity (the denominator), as well as a fall in the banks' assets (the numerator).

The transmission mechanism that amplifies and propagates the financial shock to the wider economy generates a fall in output of around 1%, a decline in investment of over 5%, a slow decline in consumption of 0.25% and a 0.5% fall in employment. As default rates rise and banks deleverage, the final good firms reduce their production, investments and labour demand. As banks deleverage, the canonical *financial accelerator* channel amplifies and propagates the initial shock. As the credit supply contracts, it pushes up credit spreads and borrowing costs, which further hinders the ability of the final good firms to invest, hire and produce, resulting in an increase in defaults. The feedback mechanism pushes the banks to deleverage even more, higher borrowing costs and defaults rates, and so on and so forth.

The second contribution of the paper relies on investigating the efficacy of a credit easing policy in mitigating a fall in output and investment following an adverse financial shock to the economy. By increasing the economy's credit supply, the central bank attempts to reduce the macroeconomic effects of the adverse shocks through lower borrowing costs for the firms and higher asset prices for the banks. However, in the model, the policy is ineffective in mitigating a recession. Specifically, when the central bank purchases assets in the privately intermediated asset market, it lowers the marginal return on assets, provoking a decline in private credit supply to the same proportion as the central bank's intervention. This *crowding out* effect in the asset market means that the overall credit supply is unchanged, and as such the policy cannot reduce borrowing costs, nor raise asset prices. The root of the policy's ineffectiveness derives from its inability to reduce defaults, which are the key driver of the banks' deleveraging process. Indeed, the solvency constraint of the model, which dictates that the banks' equity needs to cover all banks' asset losses, remains unaltered by the credit easing policy. Overall, investment and output remain depressed following the central bank's intervention.

A third and final contribution of the paper relies on studying alternative policies that can help authorities in mitigating the effects of a financial crisis. A novel non-monetary policy tool has been considered in tackling a financial crisis: *credit policies*. Instead of having the central bank lending to final good firms, the government can subsidise these firms in an effort to directly reduce default rates. Credit subsidies, unlike the credit easing policies, would therefor remove the need for banks to deleverage. The policy by reducing asset defaults, it directly mitigates the effects of the financial recession by muting the response of the financial accelerator mechanism.

***Related Literature* —** Firstly, the paper speaks to the literature that studies the financial accelerator mechanism. More specifically, it examines how exogenous shocks in the economy are amplified and propagated through the role of asset intermediation of the financial sector. The seminal paper by Bernanke et al. (1999) has paved the way for studying the effects of financial friction and has served as the benchmark for modelling imperfect financial markets. The financial frictions used in this literature generate counter-cyclical financial leverage ratios, as banks can only attract deposits for a fixed fraction of its assets' value. Such constraint causes financial leverage to increase mechanically in downturns, as equity decreases proportionally more than assets[2].

In reality, however, the counter-cyclical financial leverage ratio is not validated by empirical evidence. Adrian and Shin (2010) and Kalemli-Ozcan et al. (2012) have provided empirical evidence in favour of the positive co-movement of financial leverage and economic activity. Adrian and Shin (2010) using aggregated data from the Flows of Funds Accounts from the Federal Reserve and Kalemli-Ozcan et al. (2012) using bank-level data using the BankScope dataset, show that financial leverage is in fact pro-cyclical. Such evidence is juxtaposed with the financial accelerator literature.

This paper contributes to this literature by modifying the conventional financial accelerator models to include endogenous asset default. The introduction of defaults generates a different financial friction based on the banks' necessity to cover the asset losses. Unlike previous literature, specifically Gertler and Karadi (2011), the model developed in this paper can replicate the pro-cyclical leverage and counter-cyclical default dynamics.

The paper is also closely related to the literature on models of unconventional monetary policy. This literature has been pioneered by the seminal paper by Gertler and Karadi (2011), who have offered the canonical framework for which to evaluate credit easing policy. Although the subject has been explored widely, there is no clear consensus on the effectiveness of unconventional monetary policy. On the one hand, there have been substantial papers that have highlighted the effectiveness of the policy in contrasting the fall in asset prices and the rise in credit spreads (see Gertler and Karadi (2013), Gertler and Kiyotaki (2010), Benmelech and Bergman (2012), Gertler et al. (2012), Gertler et al. (2016), Del Negro et al. (2017)). On the other hand, Benmelech and Bergman (2012) find that the central bank's unconventional measures can be ineffective as banks may find it optimal to hoard the liquidity injected by the central bank and not lend it out. As a result, Benmelech and Bergman (2012) find that credit policies may create *credit traps* where investment, asset prices and liquidity remain depressed. Bebchuk and Goldstein (2011) come to a similar conclusion using a slightly different framework based on expectations.

Within this debate, this paper brings supportive evidence to the literature which finds unconventional monetary policy ineffective. Indeed, I find that there is a crowding-out effect which eliminates the credit easing policy's effectiveness despite the objectives of the policy being to reduce borrowing costs and increase asset prices. By taking into account counter-cyclical default and pro-cyclical leverage dynamics, this paper acknowledges that the credit easing policies are unable to impact the default decisions of firms and so they are ineffective in their objective.

Finally, the paper relates to the literature that finds a role for alternative non-monetary policy instruments in tackling the effects of a financial recession. Correia et al. (2018) find that instead of using credit easing policies, a more effective policy would be to use subsidies to di-

[2] Adrian and Shin (2010) provide an extensive discussion on the leverage dynamics in the literature and contrast them with empirical evidence.

rectly reduce borrowing costs for firms. Similarly to Correia et al. (2018), I find that using these subsidies are effective. However, the framework provided in this paper works in a different way. Credit subsidies directly reduce defaults and manage to minimise the recessionary effects of a financial crisis and so overcome the central problem faced by credit easing policies.

The model is presented and described in Section 2. The model is calibrated to the pre-2007 U.S. economy in Section 3. In Section 4, a quantitative analysis of the model is conducted. In this section, I evaluate the effects of a typical recession and a financial recession. I elucidate on workings of the endogenous growth mechanism and the financial accelerator mechanism addressing how these amplify and propagate the initial shock. In Section 5, I then turn to the investigation of the effectiveness of the central bank's credit easing policy, whilst in Section 6 I discuss how alternative non-monetary policy tools can deal with the macroeconomic effects of a financial crisis. The conclusion closes the paper in Section 7. Appendix 7 contains additional details on the model and the quantitative analysis.

2 The Model

The model is a real business cycle model with five additional elements: *i)* a balance sheet constrained financial sector; *ii)* final good firms subject to endogenous default; *iii)* capital-producing firms providing capital to final good firms; *iv)* a government with a monetary authority; and *v)* an exogenous shock to the value of capital/assets able to reproduce a financial crisis.

The model developed in this paper is a version of the Gertler and Karadi (2011) framework where financial intermediaries operating in an imperfect capital market intermediate funds between households and final good firms[3]. Further, akin to Gertler and Karadi (2011), a financial crisis is modelled as an exogenous shock to the value of assets. The central innovation is that this framework includes an endogenous default mechanism using a setup similar to Hopenhayn (1992). In the model, final good firms can shut down its operations and default on its debt obligations issued to financial intermediaries. There may be different approaches to model the default of debt contracts, however, Hopenhayn (1992) offers a simple framework adaptable to the question at hand without any loss of generality. Furthermore, this modelling choice means that there is no need track the distribution of firms, nor the entry and exit dynamics of firms.

2.1 Environment

Time is discrete and the horizon is infinite. There are four types of agents in the economy: households, final good firms, capital producers and banks. Households, modelled as a representative entity, form consumption and saving decisions, as well as providing labour to firms in exchange for a wage. Savings are deposited in banks and yield a risk-free rate. Final good firms represent ongoing projects owned by entrepreneurs and live for two periods. Firms are heterogeneous with respect to their idiosyncratic productivity. They use capital and labour to produce a homogeneous final output good to be consumed. In the first period, firms invest by issuing financial claims to banks in exchange for funds to be used for capital acquisitions, supplied by capital producers. In the second period, firms observe their idiosyncratic productivity and take default and production decisions. Firms can default on their debt obligation because these debt obligations are not enforceable. If a firm decides to default, it will halt production and will not honour its financial commitments. At the end of the firm's life, whether it produces or not, the firm dies and all capital is sold back to the capital producers. Banks have the function of asset intermediation by purchasing financial claims from final good firms using deposits from households and their equity. The financial friction in the model arises from the fact deposits are non-state contingent contracts and the banks' assets are subject to default. As a result, banks must hold enough equity to cover any losses generated from asset defaults. It is assumed that households cannot engage in asset intermediation directly with firms like Bernanke et al. (1999), due to high monitoring and servicing costs.

[3] A key difference between this model and Gertler and Karadi (2011) is that here, for reasons of simplicity, I do not cast the model in a New-Keynesian setup as the paper does not deal with conventional monetary policy.

Finally, to study the effects of credit easing policy, the model also features a government with a central bank. The credit easing policy framework is taken from Gertler and Karadi (2011). In times of normality, the government uses lump-sum taxes on households to fund government expenditure. When private asset intermediation is disrupted, the central bank uses the credit easing policy to try to attenuate the impact of the crisis. The policy entails the issuance of government debt to households in exchange for funds which are used to purchase financial claims directly from non-financial firms. The purpose of credit easing policy is to intervene when the decentralised financial market is disrupted and banks no longer will, or are able to supply credit to financial firms.

2.2 Household

There is a continuum of identical households of measure unity who live infinitely. Households are assumed to pool all their resources and have access to a complete set of state-contingent claims. These are not modelled here since there is no heterogeneity in households and these assets are in zero net supply in equilibrium. As a result of these standard assumptions, the households' problem can be simplified by using a representative household framework.

The household likes consumption and dislikes labour and the household's utility can be represented by the following functional form:

$$u(c, c_{-1}, l) = log(c - hc_{-1}) - \frac{\chi}{1+\varphi} l^{1+\varphi}. \tag{1}$$

The household's utility function features a consumption habit-formation parameter $h \in (0, 1)$. The habit parameter tells us that consumers dislike sudden changes in consumption, such that today's consumption is a function of yesterday's consumption. The habit formation parameter helps replicate the consumption dynamics of the U.S. economy. The parameter $\chi > 0$ measures the relative utility weight of labour, and $\varphi > 0$ represents the inverse Frisch elasticity of labour supply.

Every period, the household's deposits b_{-1} yield a real gross risk-free rate of return r from the bank. The household will supply labour l in a competitive market in exchange for a real wage w. Additionally, the household will receive the profits of the firms, π, as a lump-sum transfer [4]. The household's expenditures consist of resources spent on the consumption good c, the lump-sum taxes to be paid to the government T, and the deposits b that the household holds in the bank as a means of saving. Therefore, the household budget constraint can be characterised as follows:

$$rb + wl + \pi = c + T + b. \tag{2}$$

As a result of this setup, the household's optimisation problem is to maximise the expected lifetime utility, subject to its budget constraint, by choosing the level of consumption, savings

[4] The household owns the financial intermediaries, final good firms and capital producers. As such it is entitled to their profits.

and labour:

$$\begin{aligned} H(b_{-1}, c_{-1}) = \quad & \max_{c,b,l} \quad log(c - hc_{-1}) - \frac{\chi}{1+\varphi} l^{1+\varphi} + \beta \mathbb{E} H(b, c) \\ & \text{s.t.} \quad rb_{-1} + wl + \pi = c + T + b. \end{aligned} \tag{3}$$

The household discounts utility at rate $\beta \in (0, 1)$. Note that the notation in the household's problem abstracts from the aggregate states of the economy for simplicity.

The solution to the household's problem yields the consumption, savings and labour optimality conditions. The first-order conditions with respect to consumption and savings yield the standard Euler equation, with the exception that it also features consumption habit formation. I can be formulated as

$$\beta \mathbb{E} \frac{\varrho'}{\varrho} r' = 1, \quad \text{where} \quad \varrho = (c - hc_{-1})^{-1} - \mathbb{E} \beta h (c' - hc)^{-1}, \tag{4}$$

where ϱ is the marginal utility of consumption today.

The first-order condition with respect to labour generates the standard labour supply equation, which equalises the real wage in consumption goods to the marginal utility cost of labour:

$$\varrho w = \chi l^{\varphi}. \tag{5}$$

2.3 Final Good Firms

There is a continuum of perfectly competitive final good firms of measure unity $i \in (0, 1]$. Firms are all identical *ex-ante*, but differ *ex-post* due to their realisations of idiosyncratic productivity, a_i. The stochastic process of the idiosyncratic productivity is assumed to be such that a_i individually and identically distributed along a log-normal distribution with mean μ_a and variance σ_a^2, that is $a_i \sim log\mathcal{N}(\mu_a, \sigma_a^2)$.

Firms live for two periods and are constantly replaced. In the first period, firms invest in physical capital by issuing financial claims to banks. In the second period, they observe their idiosyncratic productivity, as well as the aggregate states of the economy, and choose whether to produce and repay their debt obligations or default. After the second period, firms die and are fully replaced by another unity measure of new firms. This modelling choice is driven by the requirement to keep the model as simple as possible and ensures that there is no need to keep track of the distribution of firms.

Every firm i produces a final good output y_i using physical capital k_i and hiring labour l_i from households, using a Cobb-Douglass technology featuring decreasing returns to scale

$$y_i(k_i, a_i, z, \xi) = z a_i \left[(\xi k_i)^{\alpha} l_i^{1-\alpha} \right]^{\zeta}. \tag{6}$$

Aggregate total factor productivity is denoted by z and follows an AR(1) process: $logz = \rho^z logz_{-1} + \epsilon^z$. The disturbances ϵ^z are individually and identically distributed with a normal distribution with mean μ_z and variance σ_z^2, or equivalently $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$. Note that ζ is the span of control parameter that controls the concavity of the technology function. Concavity is necessary to ensure firms earn positive profits and therefore generate the endogenous default mechanism.

To model a financial recession, there is a need for a shock that alters the value and productivity only of capital and assets. Denote ξ as an aggregate capital quality shock which represents economic obsolescence. The capital quality shock also follows an AR(1) process: $log\xi = \rho^{\xi} log\xi_{-1} + \epsilon^{\xi}$, where the disturbances ϵ^{ξ} are individually and identically distributed with a normal distribution of mean zero and variance σ_{ξ}^2, that is $\xi \sim \mathcal{N}(\mu_{\xi}, \sigma_{\xi}^2)$.

The firm has to pay a fixed cost c^f if it wishes to produce and sell its final output good. The firm also has to pay the labour input at the competitive real wage w. Moreover, it has to honour the financial claims it issued in the previous period. The repayment cost is equal to $r^k q_{-1} k_i$, where q_{-1} is the price of capital paid in the previous period and r^k is the non-stochastic gross rate of return. Capital depreciates at rate δ. At the end of the project the firm dies and sells all the capital back to the capital producers. It follows that the profits conditional on producing are equal to:

$$\pi(k_i, a_i, z, \xi) = y_i(k_i, a_i, z, \xi) - wl_i - r^k q_{-1} k_i - \delta k_i \xi + q k_i \xi - c^f. \tag{7}$$

Otherwise, if the firm decides not to produce and default on their financial claims, then profits are equal to zero:

$$\pi(k_i, a_i, z, \xi) = 0. \tag{8}$$

2.3.1 Investment Decision

Investment takes place one period ahead, before firms observe their idiosyncratic productivity and the aggregate shocks to the economy. As a result, the optimal choice of capital for a firm at period $t-1$ derives from the maximisation of the expected present discounted value of the next period's profits:

$$\max_{k_i} \quad \beta \mathbb{E}_{-1} \Lambda \max\{0, \pi(k_i, a_i, z, \xi)\}. \tag{9}$$

Or, equivalently:

$$\max_{k_i} \quad \beta \mathbb{E}_{-1} \Lambda (1-\gamma) \pi(k_i, a_i, z, \xi). \tag{10}$$

Note that firms discount profits at the same rate as the household. The firm also has to discount for the possibility of receiving a low idiosyncratic productivity level and possibly not being able to produce. Consequently, it will only receive the profits with probability $(1-\gamma)$. It will be shown that the default probability γ is homogeneous across firms because all firms face the same problem and the idiosyncratic productivity shock is individually and identically distributed.

The first-order condition with respect to capital yields the following capital demand, which equalises the expected marginal productivity of capital tomorrow to its net expected marginal cost. Further, it can be re-arranged to show that capital demand is equal to:

$$k_i = \mathbb{E}_{-1}\left[\frac{\alpha y_i(k_i, a_i, z, \xi)}{r^k q_{-1} + \delta\xi - q\xi}\right]. \tag{11}$$

As a result of the homogeneity of the investment decision, all firms will choose the same k_i, thus aggregate capital at the end of period t is equal to $k = k_i$ as there is a measure unity of firms. This result is driven by the modelling choice and simplifies the model, as there is no need to keep track of the different choices of capital by different firms.

2.3.2 Default Decision

In period t, once the firms have made their investment decision, and after observing aggregate productivity state (z), the idiosyncratic productivity (a_i) and the aggregate capital quality shock (ξ), the firm will choose either to default or to produce. Let's define the threshold idiosyncratic productivity $\underline{a}$ as the idiosyncratic productivity level for which any firm is indifferent between production and default $\pi(k_i, a_i, z, \xi) = 0$. The firms' default decision can be represented by the following piecewise function:

$$\pi(k_i, a_i, z, \xi) = \begin{cases} y_i(k_i, a_i, z, \xi) - wl_i - r^k q_{-1} k_i - \delta k_i \xi + q k_i \xi - c^f & \text{if} \quad a_i < \underline{a}, \\ 0 & \text{otherwise.} \end{cases}$$

More specifically, if the firm decides to go ahead with production it will receive profits $\pi(k_i, a_i, z, \xi)$. Otherwise, if the firm decides to default it will yield the outside option where $\pi(k_i, a_i, z, \xi) = 0$. A firm will default on its debt obligations if the profits from production are lower than the outside option, $\pi(k_i, a_i, z, \xi) < 0$. On the other hand, the firm will produce if profits exceed the outside option, $\pi(k_i, a_i, z, \xi) \geq 0$. As such, it is possible to define the default condition as

$$y(k, \underline{a}, z, \xi) - wl(\underline{a}, z, \xi) - r^k q_{-1} k - \delta k \xi + q k \xi - c^f = 0, \tag{12}$$

where $y(k, \underline{a}, z, \xi)$ and $l(\underline{a}, z, \xi)$ are the output function and labour demand evaluated at the threshold idiosyncratic productivity $\underline{a}$.

Since there is a measure unity of firms, the default rate of firms γ, and therefore of debt obligations, can be defined as the portion of firms which receive an idiosyncratic draw below the threshold value $\underline{a}$

$$\gamma = \int_0^{\underline{a}} f(a_i)\, da, \tag{13}$$

where $f(a)$ is the probability distribution density of idiosyncratic productivity.

Notice that the left-hand-side of the default condition in Equation 12 is monotonically increasing in aggregate productivity (z) and capital quality (ξ). Hence, a negative shock to either would increase the idiosyncratic productivity threshold $\underline{a}$, which would produce an increase in the default rates of debt obligations.

2.3.3 Production and Labour Decision

Every period there is a fraction $(1-\gamma)$ of firms which decide to produce. Since capital is chosen one period ahead, the operational firms' remaining objective is to maximise the profits by choosing the level of labour to employ:

$$\max_{l_i} \quad \pi(k_i, a_i, z, \xi) = y_i(k_i, a_i, z, \xi) - wl_i - r^k q_{-1} k_i - \delta k_i \xi + q k_i \xi - c^f. \tag{14}$$

The problem yields the standard first-order condition with respect to labour, which equalises the firm's i marginal productivity of labour to the real wage. Therefore, the firm's labour demand can be stated as:

$$l_i = \frac{(1-\alpha)y_i}{w}. \tag{15}$$

Consequentially, the aggregate labour demand for all final good firms can be expressed as:

$$l = \int_{\underline{a}}^{\infty} \frac{(1-\alpha)y_i(k_i, a_i, z, \xi)}{w} \, da, \tag{16}$$

Similarly, the aggregate output produced by the final good sector is equal to:

$$y = \int_{\underline{a}}^{\infty} y_i(k_i, a_i, z, \xi) \, da. \tag{17}$$

2.4 Banks

The role of the banks is to emulate the financial sector in an economy[5]. Every period there is a measure unity of banks $j \in (0, 1]$. A bank lends funds to final good firms by purchasing financial claims qs_j^p. Banks use liabilities from households in the form of deposits b_j and their equity n_j to fund their asset purchases. Hence, every period the balance sheet of bank j is:

$$qs_j^p = n_j + b_j. \tag{18}$$

Since the financial claims are subject to defaults, the assets acquired in the previous period will yield a stochastic real gross rate of return r^k today if the issuing firm repays its debt. Otherwise, the bank will recover a fixed fraction $(1-\tau)$ of the principal from the defaulted financial claims. The model has a one-to-one mapping between firm default and asset default, therefore every period only a fraction $(1-\gamma)$ of assets will be profitable. However, asset intermediation is not costless, in fact, it features a non-convex adjustment cost equal to $C(s_j^p) = \omega s_j^p$. These intermediation costs reflect any administrative costs that are incurred with the services of security origination[6]. Finally, every period banks must pay a non-state contingent risk-free rate r

[5]For simplicity, there is no distinction between investment and commercial banks, but this may be the subject of future research. I also omit the role of non-banking financial institutions.

[6]The parameter of the intermediation cost function ω will prove useful for the calibration of the model's steady-state credit spread.

to its depositors. Consequently, the banks' budget constraint can be expressed as follows:

$$rb_{j,-1} + qs_j^p + C(s_j^p)q = b_j + r^k(1-\gamma)q_{-1}s_{j,-1}^p + (1-\tau)\gamma q_{-1}s_{j,-1}^p. \quad (19)$$

Using the balance sheet identity in Equation 18, we can rewrite the banks' budget constraint in the following manner

$$n_j = r^k(1-\gamma)q_{-1}s_{j,-1}^p - rb_{j,-1} - C(s_j^p)q + (1-\tau)\gamma q_{-1}s_{j,-1}^p, \quad (20)$$

which state that today's equity is equal to the difference between the revenues from asset purchases plus the recoveries from defaulted claims and costs from deposits and asset intermediation.

Nonetheless, financial intermediation is not friction-less in the model. The financial fiction arises from the fact that whilst deposits are non-state contingent contracts between the household and the bank, the assets held by the bank are subject to default. Indeed, the household can withdraw deposits at the end of every period if it thinks the bank is insolvent. Therefore, every period the bank must have enough equity to withstand the losses from defaulted claims to avoid a run on its deposits. This gives rise to the banks' equity constraint:

$$n_j \geq \gamma\tau q_{-1}s_{j,-1}^p. \quad (21)$$

This equity constraint ensures that the deposits are risk-free because they are backed by non-defaulted assets at any given time. The equity constraint can be reformulated by substituting the balance sheet identity in Equation 18

$$b_j \leq qs_j^p - \gamma\tau q_{-1}s_{j,-1}^p, \quad (22)$$

that is, deposits cannot exceed the assets held by the bank minus the losses from defaulting assets net of any recoveries.

The banks' problem is to maximise the present discounted value of equity, subject to the balance sheet and equity constraint, by choosing the optimal quantity of financial claims and deposits:

$$B(s_{j-1}^p, q_{-1}, b_{j,-1}) = \max_{s_j^p, b_j} \quad n_j + \beta\mathbb{E}\Lambda' B'(s_j^p, q, b_j) \quad (23)$$

s.t.

$$n_j = r^k(1-\gamma)q_{-1}s_{j,-1}^p - rb_{j,-1} - C(s_j^p)q + (1-\tau)\gamma q_{-1}s_{j,-1}^p, \quad (24)$$

$$b_j \leq qs_j^p - \gamma\tau q_{-1}s_{j,-1}^p. \quad (25)$$

Note that the banker discounts equity at the same rate as the household. Let ν be the Lagrangian multiplier attached to the equity constraint. The first-order conditions with respect to deposits and assets yield the following optimality conditions:

$$\nu = \beta\mathbb{E}\Lambda' r', \quad (26)$$

$$C'(s_j^p) + \nu = \beta \mathbb{E} \Lambda' \left[r^{k'}(1-\gamma') + (1 - \tau + \nu'\tau)\gamma' \right]. \tag{27}$$

The first-order condition of deposits, Equation 26, states that the bank will attract deposits until the marginal benefit of relaxing the equity constraint ν equals the expected present discounted risk-free rate, which is the marginal unit cost of deposits. Substituting the household's Euler equation, Equation 4, into the first-order condition, the solvency constraint is found to be binding as $\nu = 1$. Therefore, at any given time equity is equal to the losses from the defaulted claims $n = \gamma\tau q_{-1} s_{j,-1}^p$.

The first-order condition with respect to assets, Equation 27, illustrates how banks equalise the marginal cost of purchasing an asset with their marginal revenue. By increasing the asset side of the balance sheet by one unit, the bank must raise funds through deposits and so pays ν, as well as the marginal asset intermediation costs $C'(s_j^p)$. The expected present discounted revenue from the extra unit of assets equals the weighted average between receiving the rate of return of capital from non-defaulted claims and the recoveries from defaulted claims.

From the two optimality conditions, it is observed that neither of them depends on bank-specific factors. As a result, all the banks would choose the level of assets and deposits and the j subscript can be dropped. Therefore, it is possible to denote $n = n_j$ as the total sector's equity, $b = b_j$ as the overall deposits and $s^p = s_j^p$ as the total financial claim bought by the private sector.

2.4.1 Leverage Dynamics

It is now possible to examine analytically the leverage dynamics that result from this setup. I will show how the endogenous default mechanism generates an endogenous leverage ratio. Let's define leverage as equal to the total value of financial claims over the equity of the financial intermediary:

$$\phi = \frac{q s^p}{n}. \tag{28}$$

Using Equation 21 and Equation 20, it is possible to express the equation for leverage as:

$$\phi = \frac{r'}{\gamma' - [r^{k'}(1-\gamma') - r] + \omega \Delta q' \Delta s^{p'}},$$
$$\text{where} \quad \Delta s^{p'} = \frac{s^{p'}}{s^p} \quad \text{and} \quad \Delta q' = \frac{q'}{q}. \tag{29}$$

Note that $\Delta s^{p'}$ is the gross growth rate of banks' assets and $\Delta q'$ is the gross growth rate of asset prices.

The dynamics of leverage can be dissected from Equation 29. There are two channels through which banks' leverage is affected by financial claims defaults: *i)* the *balance sheet channel* and *ii)* the *bank lending channel*.

The *balance sheet channel* works through the solvency constraint. As defaults increase through an exogenous shock, the equity needed to cover the higher losses must increase for any given level of assets. This channel generates a downturn in leverage. As banks need to keep their obligations to their depositors, they need to adjust their equity such that all asset losses are covered.

The second channel is the *bank lending channel*. When an exogenous shock increases defaults, the marginal rate of return of assets falls, so banks decrease the acquisition of financial claims. Defaults make the assets less desirable as their revenue falls, hence, banks are not as eager to purchase these financial claims from the final good firms. These two key channels drive the leverage dynamics and the financial accelerator in the model.

2.5 Capital-Producing Firm

As in Gertler and Karadi (2011), a capital-producing sector is introduced to capture the variations of the price of capital throughout the business cycle. We assume perfectly competitive capital producers build new capital and repair depreciated capital. They sell capital to final good firms and then purchase un-depreciated capital back at the end of production. The cost of repairing worn out capital is unity and it is not subject to any adjustment costs. However, building new capital incurs flow adjustment costs represented by the function $f(\cdot)$. The value of a unit of capital is q. Therefore, the capital producer's problem is to choose net investment i^n in order to maximise the present discounted value of profits

$$P(i^n_{-1}, i^{ss}) = \max_{i^n} \quad (q-1)i^n - f\left(\frac{i^n + i^{ss}}{i^n_{-1} + i^{ss}}\right)(i^n + i^{ss}) + \beta \mathbb{E}\Lambda' P'(i^n, i^{ss}), \tag{30}$$

where net investment i^n can be defined as the difference between gross investment and required investment $i^n = i - \delta k$. Denote i^{ss} as the steady-state level of gross investment. The flow adjustment cost function is a quadratic cost function such that: $f(1) = f'(1) = 0$ and $f''(1) > 0$. Capital producers can earn profits outside the steady-state due to the flow adjustments costs. These profits are transferred back to the household as a lump-sum, as ultimately they are the owners of these firms.

Since all the capital producers face the same problem, they will all choose the same net investment rate. Solving the capital producer maximisation problem, the Q-relation for net investment is obtained:

$$\begin{aligned} q =& 1 + f\left(\frac{i^n + i^{ss}}{i^n_{-1} + i^{ss}}\right) + \left(\frac{i^n + i^{ss}}{i^n_{-1} + i^{ss}}\right) f'\left(\frac{i^n + i^{ss}}{i^n_{-1} + i^{ss}}\right) \\ &- E_t \beta \Lambda_{t+1} \left(\frac{i^{n\prime} + i^{ss}}{i^n + i^{ss}}\right)^2 f'\left(\frac{i^{n\prime} + i^{ss}}{i^n + i^{ss}}\right). \end{aligned} \tag{31}$$

It can be observed that capital producers would choose net investment such that the marginal revenue from selling one unit of net investment, equals its marginal cost characterised by the unity cost of refurbishing capital and the marginal flow adjustment costs.

2.6 Credit Easing Policy

So far the model has abstracted from the intervention by the central bank in terms of credit easing policy. I have described is how the assets are privately intermediated. The modelling of the credit easing policy is taken directly from Gertler and Karadi (2011).

Assuming qs are the total assets intermediated in the economy, they can be decomposed in qs^g and qs^p, which are the assets intermediated in the public and private sector respectively. Hence, the following equation represents the division of assets in an economy:

$$qs = qs^g + qs^p. \tag{32}$$

In times of crisis, such as during the financial crisis of 2007, central bankers used credit easing policy to attenuate the financial crisis and limit its impact to the wider economy. The policy consists of issuing government debt to the household by paying a risk-free rate r so as to sustain the economy by lending funds to non-financial firms at the market rate r^k. Since it assumed that the central bank is less efficient than the private sector in raising funds and choosing the best investments, I impose a cost v per asset publicly intermediated. However, seen as the central bank is not subject to any financial friction because the government always honours its debt, credit easing policy is not balance sheet constrained[7]. We can suppose that the central bank is willing to supply a fraction ψ of total assets in an economy:

$$qs^g = \psi qs. \tag{33}$$

The central bank can influence the total amount of financial assets in an economy to counter a crisis, for example when the banking sector is unwilling or unable to supply credit due to a negative shock to their balance sheets. The choice of ψ can be described by the following credit easing policy rule

$$\psi = \kappa \mathbb{E}\left\{ \left[log(r^{k'}) - log(r') \right] - \left[log(r^{k*}) - log(r^*) \right] \right\}, \tag{34}$$

where $\left[log(r^{k'}) - log(r') \right]$ is the credit spread, $\left[log(r^{k*}) - log(r^*) \right]$ is the steady-state credit spread and κ can be used to control the intensity of the credit easing policy, which allows the investigation of different sized interventions during a financial crisis.

This policy rule reflects the actions of the Federal Reserve during the U.S. Great Recession. It uses the credit spread as a measure of the health of financial markets and intervenes according to the disparity between the current credit spread and its steady-state level. In simpler terms, the central bank would increase the purchase of financial obligations the worse the financial crisis.

By aggregating the public and banking sector's supply of credit, the total assets in an econ-

[7] This assumption is equivalent to assuming that depositors do not doubt the solvency of the monetary authority, due to the ability to print money and the ability of the government to raise taxes. I abstract from cases in which depositors no longer wish to lend money to the central bank and the government.

omy is defined as

$$qs = \phi n + \psi qs = \phi^c n \quad \text{where} \quad \phi^c = \frac{\phi}{1-\psi}, \tag{35}$$

where ϕ^c is the economy's total leverage ratio of intermediated assets over total equity. Equation 35, shows how the credit easing policy operates. By controlling ψ with the credit easing policy rule, the central bank can influence the economy's leverage directly, so as to stimulate the economy by increasing the level of intermediated assets.

2.7 Market Clearing and Government

To close the model, the capital, the goods and the labour market have to clear. As for the capital market, the total supply of financial claims issued by final good firms must equal the total capital expenditure. As such, the following no-arbitrage condition between financial and physical capital must hold:

$$qs = qk'. \tag{36}$$

The labour market clears when the labour supplied by the household exactly equals the labour demand by firms:

$$l = \left[\frac{\varrho w}{\chi}\right]^{\frac{1}{\varphi}} = \int_{\underline{a}}^{\infty} \frac{(1-\alpha)y_i(k_i, a_i, z, \xi)}{w}\, da. \tag{37}$$

The market-clearing condition in the final good market, derived from the household's budget constraint, means that total output must be equal to: total consumption, gross investment, government expenditure, asset intermediation costs, physical capital flow investments costs and the fixed costs of production. The economy's aggregate constraint is:

$$y = c + i + g + C(s^p) + f\left(\frac{i^n + i^{ss}}{i^n_{-1} + i^{ss}}\right)(i^n + i^{ss}) + (1-\gamma)c^f. \tag{38}$$

Finally, the government budget constraint dictates that the government expenditure g and the cost of the central bank asset intermediation must be equal to the revenue from lump-sum taxes imposed on households and the revenue from credit easing policy

$$g + vb^g = T + \left[r^k(1-\gamma) - r\right] b^g_{-1} \tag{39}$$

where government bonds issued today, b^g, are used to finance the public asset intermediation, qs^g.

This completes the description of the model, where the system of non-linear equations which encapsulates the model is laid out in Appendix A. I now turn to the calibration of the model before the quantitative analysis.

3 Calibration

The model is calibrated to the U.S. economy for the pre-2007 Great Recession period at a quarterly frequency. Where possible, the parameters are calibrated such that the model's steady-state matches key moments of the data, otherwise, I rely on the estimates of previous literature. The model's steady-state equations are described in Appendix B.

There are fourteen parameters in total to calibrate. Six parameters relate to the standard real business cycle model $(\beta, h, \chi, \varphi, \alpha, \delta)$. Three parameters relate to the heterogeneous final good firms $(\zeta, \mu_a, \sigma_a, c^f)$. Another three parameters relate to the introduction of the financial sector and asset price intermediation (ω, τ, η_i). Finally, two more parameters relate to the government sector and credit easing policy $(v, g/y)$. The results of the calibration exercise are presented in Table 1.

The standard parameters are calibrated in the following manner. The discount rate is set to 0.99 so as to achieve a steady-state annual risk-free interest rate of 4%. The consumption habit parameter is set to 0.815, as suggested by Primiceri et al. (2006), which helps deliver the consumption dynamics observed in the U.S. economy. The relative utility weight of labour is calibrated to hit a steady-state labour supply of a third, whilst inverse Frisch elasticity of labour supply is set to 0.279 in line with macro-estimates in Primiceri et al. (2006). The output elasticity to capital is calibrated to the effective capital share of the U.S. economy, hence I set $\alpha = 0.333$. The depreciation rate δ is set to 0.025 to obtain an annual depreciation of capital of around 10%.

The heterogeneous final good firms' parameters relate to the stochastic process of idiosyncratic productivity process a, the fixed costs of production c^f and the decreasing returns to scale parameter ζ. The mean of the idiosyncratic productivity distribution μ_a is normalised to zero, whilst the standard deviation is taken from the heterogeneous firm literature, specifically Khan and Thomas (2008). The fixed costs of production c^f, in absence of data that can allow a specific calibration, is set in order to achieve a steady-state default rate of 1.0%. This matches the average bank loan net charge-off rate from 1988 until 2007 in the U.S. The decreasing returns to scale parameters is taken to be equal to 0.896, as it is in Khan and Thomas (2008).

The parameters that define the dynamics of private asset intermediation are calibrated as follows. Specifically, the banks' asset intermediation cost coefficient ω is meant to be suggestive, as it is hard to calculate accurately the costs associated with purchasing assets by banks (servicing, monitoring, screening etc.). The value for ω is chosen to be equal to 0.019 so as to achieve a steady-state credit spread of 300 basis points, matching the pre-recession levels of Moody's Seasoned Baa Corporate Bond minus Federal Funds Rate[8]. The recovery rate is estimated by Moody's Ultimate Recovery Database[9] at around 37% in its report in 2007 for all bonds, therefore the fraction assets not recovered by banks is set to $\tau = 0.64$. Regarding the capital-producing firms, the inverse elasticity of net investment to the price of capital η_i is set to 1.72 as in Gertler and Karadi (2011).

Lastly, the parameters that speak about the government's sector. The steady-state fraction

[8]Accessible via `https://fred.stlouisfed.org/series/BAAFFM`.
[9]Accessible via `https://www.moodys.com/Pages/Default-and-Recovery-Analytics.aspx`.

Table 1: Model Calibration

Parameter	*Value*	*Description*	*Source/Target*
Household			
β	0.990	Discount rate	Interest rate of 4%
h	0.815	Habit parameter	Primiceri et al. (2006)
χ	0.329	Relative utility weight of labour	Labour hours of 1/3
φ	0.279	Inverse Frisch elasticity of labour	Primiceri et al. (2006)
Banks			
ω	0.019	Asset intermediation cost	Credit spread of 2%
τ	0.640	Portion of assets not recovered	Moody's Database
Final Good Firms			
α	0.333	Output elasticity to capital	Capital share of 1/3
δ	0.025	Depreciation rate	Depreciation of 10%
ζ	0.896	DRTS parameter	Khan and Thomas (2008)
μ_a	0.000	Mean of idio. productivity	Normalisation
σ_a	0.022	St. dev. of idio. productivity	Khan and Thomas (2008)
c^f	0.292	Firms' fixed cost of production	Default rate of 1.5%
Capital-Producing Firms			
η_i	1.720	Inverse elasticity of net inv.	Gertler and Karadi (2011)
Government			
v	0.001	Govt. asset transaction costs	Gertler and Karadi (2011)
g/y	0.200	Share of govt. expenditure	Gertler and Karadi (2011)

Notes: *The table shows the parameterisation of the model. The parameters, its value, the description and their respective targets and sources are presented in order.*

of government expenditure to output is set to a fifth and the government's asset transaction cost is equal to 0.1%, both set according to Gertler and Karadi (2011).

4 Quantitative Analysis

In this section, I analyse the model's response to different shocks to evaluate its behaviour. Firstly, a typical recession is simulated using a total factor productivity shock, by which I explain the mechanism of endogenous default in the model. Secondly, a financial recession is replicated using a capital quality shock, where I elicit the effects of endogenous defaults on financial leverage dynamics, which underpins the financial accelerator mechanism in the model. Thirdly, I turn to evaluate the effectiveness of the central bank's credit easing policy in mitigating a financial crisis. Lastly, after having demonstrated the ineffectiveness of credit easing policy in dealing the economic fallout of a financial recession, I propose an alternative non-monetary policy tool, namely credit subsidies, and show their efficacy in mitigating the effects of a financial crisis.

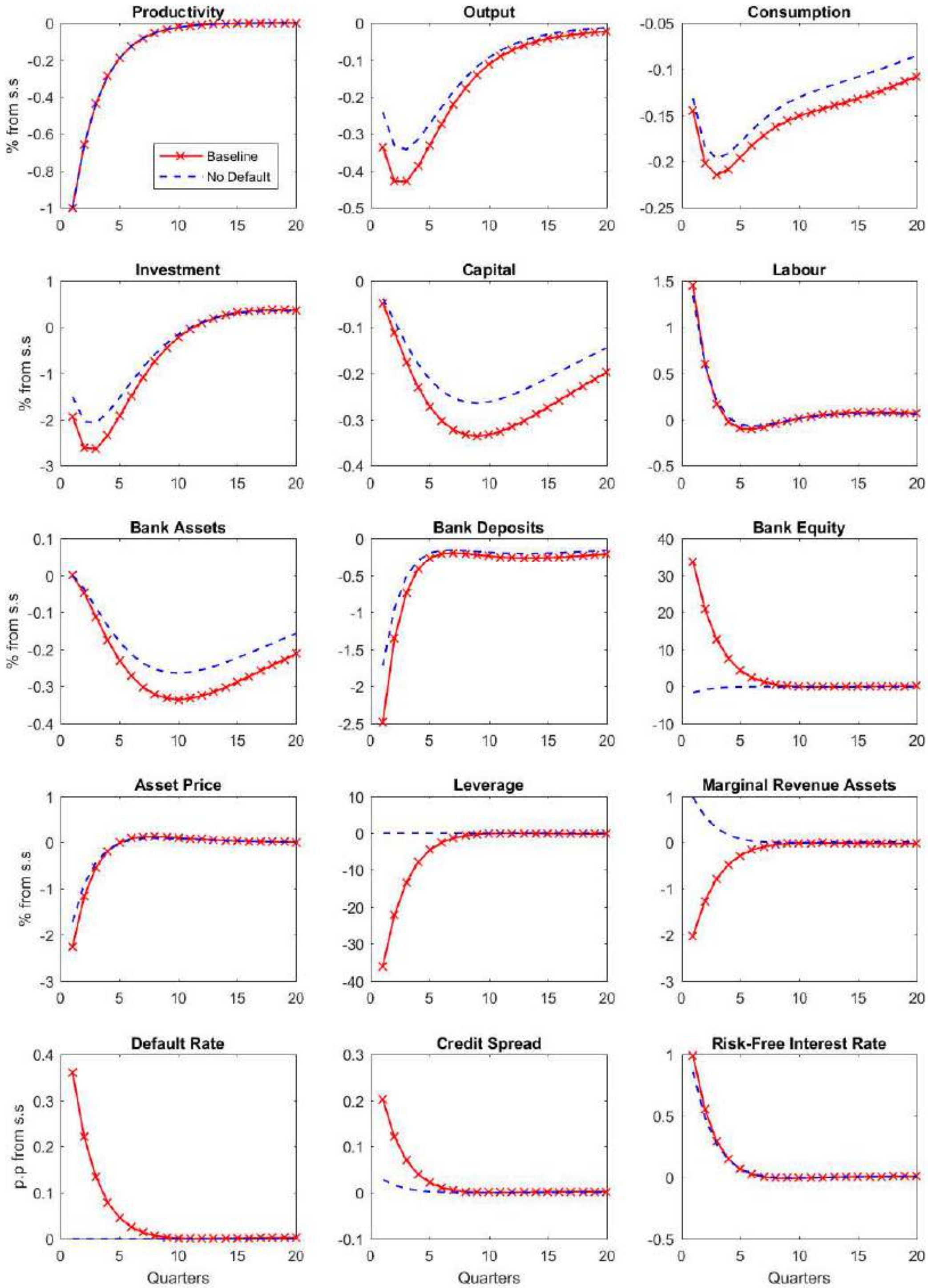
4.1 A Typical Recession

The first experiment is to simulate a typical fluctuation in the business cycle. It is assumed that the economy suffers an unanticipated one-time negative aggregate total factor productivity shock equal to one percent. Note that aggregate total factor productivity evolves following an AR(1) with an auto-correlation coefficient of 0.66. Figure 4 shows the impulse response functions following the total factor productivity shock. The red line with $\times$ symbols plots the baseline model's response without any credit easing policy. In contrast, to elicit the effects of endogenous default, the blue dashed line represents the model's response where the default mechanism is shut-off *ad-hoc*. Figure 4 highlights the different responses of the model with, and without, the endogenous default mechanism. In the baseline model, the total factor productivity shock causes a typical response in investment and output, but the key dynamics of the model operate through endogenous defaults.

Endogenous Default Mechanism — The lower aggregate productivity causes an initial decline in investment and output through a fall in the marginal productivity of factor inputs. The result is a lower demand for both capital and labour. Lower aggregate productivity of firms leads to higher defaults of financial claims. Indeed, since the firms are less productive, as demonstrated by the default condition in Equation 12, the idiosyncratic productivity realisation needed to repay the debt increases. The higher defaults of financial claims by financial good firms lead to amplification and propagation of the initial shock, whose mechanism is explained in detail in Section 4.2.

Overall, a one percent fall in total factor productivity generates a 0.45% in output, and a 2.5% fall in investment. As for the banking sector, leverage falls by around 38% as bank equity increases by almost the same amount and assets gradually fall by 0.35%. Default rates rise by 35 basis points and credit spreads by 20 basis points. Interestingly, labour rises during the recession. As the risk-free interest rate rises, the wealth effect dominates and labour increases even in a recession. Using an alternative, but more complicated, utility function such as Jaimovich

Figure 4: Typical Recession Experiment



Notes: *The figure shows the effects of the endogenous default mechanism and the leverage dynamics when the economy suffers a one percent total factor productivity shock, with a persistence of two-thirds. The red line shows the baseline model response with endogenous default. The blue dashed line displays the impulse response of the model without endogenous defaults. The unit of time is in quarters. All the plots are in percentage deviations from the steady-state, apart from the last row which is percentage point deviations from the steady-state.*

and Rebelo (2009) would solve this peculiarity, but since I am more interested in modelling a financial recession, this is outside the scope of the paper.

4.2 A Financial Recession

For the second experiment, a financial crisis is simulated using a capital quality shock adopted from Gertler and Karadi (2011). Unlike aggregate total factor productivity shocks, one should think of a capital quality shock as a rare event, such as a financial crisis. The shock represents economic obsolescence of capital and so can create an exogenous variation in the asset price and credit spreads, which helps to emulate the dynamics of the financial crisis. Consequently, it is assumed that the economy suffers an unexpected one-time negative shock to capital quality of two percent, with an auto-correlation coefficient of two thirds. The size and persistence of the shock are meant to capture broadly the leverage dynamics of the Great Recession. Figure 5 shows the impulse response functions following the capital quality shock. The red line with $\times$ symbols plots the baseline model's response without any credit easing policy. To elicit the effects of the financial accelerator mechanism, the blue dashed line represents the model's response where the default mechanism is shut-off *ad-hoc*.

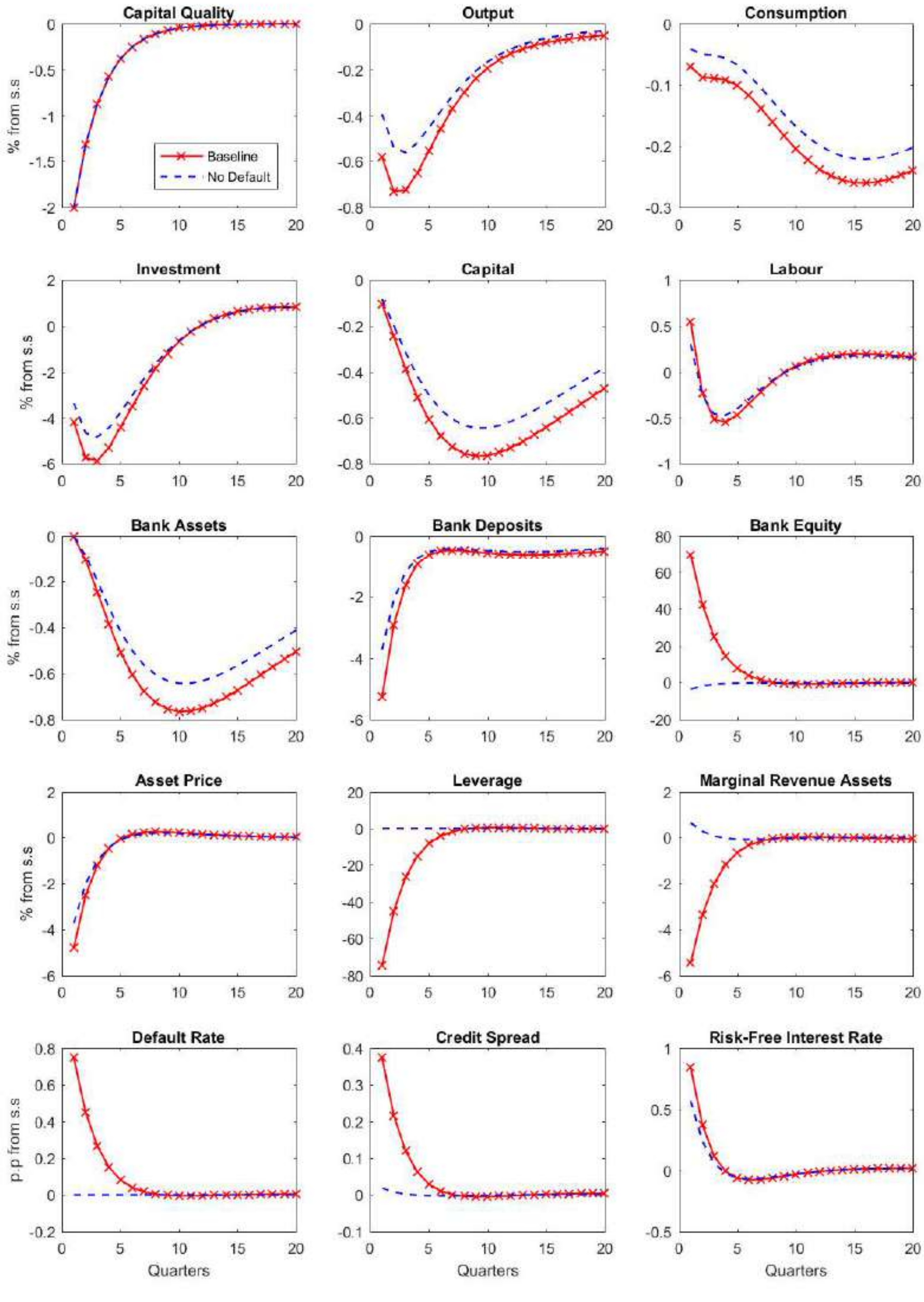
The capital quality shock, through the reduction of credit demand of firms, decreases asset prices by over 5% on impact and generates a spike in the credit spread. Moreover, the capital quality shock, by making capital less productive and increasing the repayment cost of financial claims, increases the threshold idiosyncratic productivity for which firms decide to default. The result is that default rates rise by 175 basis points. Labour, after an initial rise due to the wealth effects, also falls due to the complementarity between the two factors of input.

In the baseline model, the capital quality shock causes a typical recessionary response in investment and output, but the novelty of the model is the dynamics of leverage and balance sheet variables it can replicate. As already shown in the typical recession experiment in Section 4.1, shock to the value and productivity of capital generates higher default rates which induce interesting financial dynamics which mimic the dynamics of the banking sector in the Great Recession.

In the baseline model, the higher default rates amplify and propagate the initial shock through the *financial accelerator* mechanism, generating pro-cyclical leverage dynamics. The financial accelerator in the model operates through two channels: *i)* the balance sheet channel and *ii)* the bank lending channel. I will discuss each of these channels in turn.

Balance Sheet Channel — As default rates soar, banks have to adjust their balance sheet accordingly to abide by their solvency constraint. The banks, to keep their deposits risk-free, must ensure that these deposits are always backed by financial assets. As such, the banks' equity must be enough to cover the losses from asset defaults in each period. Consequently, as default rates increase the banks need to increase equity and by doing so must decrease their leverage ratio. As shown in Figure 5, the baseline model displays a sharp rise in the banks' equity, whilst in the model without endogenous default, such a rise does not occur. Indeed, in

Figure 5: Financial Crisis Experiment



Notes: *The figure shows the effects of the endogenous default mechanism and the leverage dynamics when the economy suffers a two percent capital quality shock, with a persistence of two-thirds. The red line shows the baseline model response with endogenous default. The blue dashed line displays the impulse response of the model without endogenous defaults. The unit of time is in quarters. All the plots are in percentage deviations from the steady-state, apart from the last row which is percentage point deviations from the steady-state.*

the model without endogenous default, the banks' equity features a small decline due to the lower asset prices.

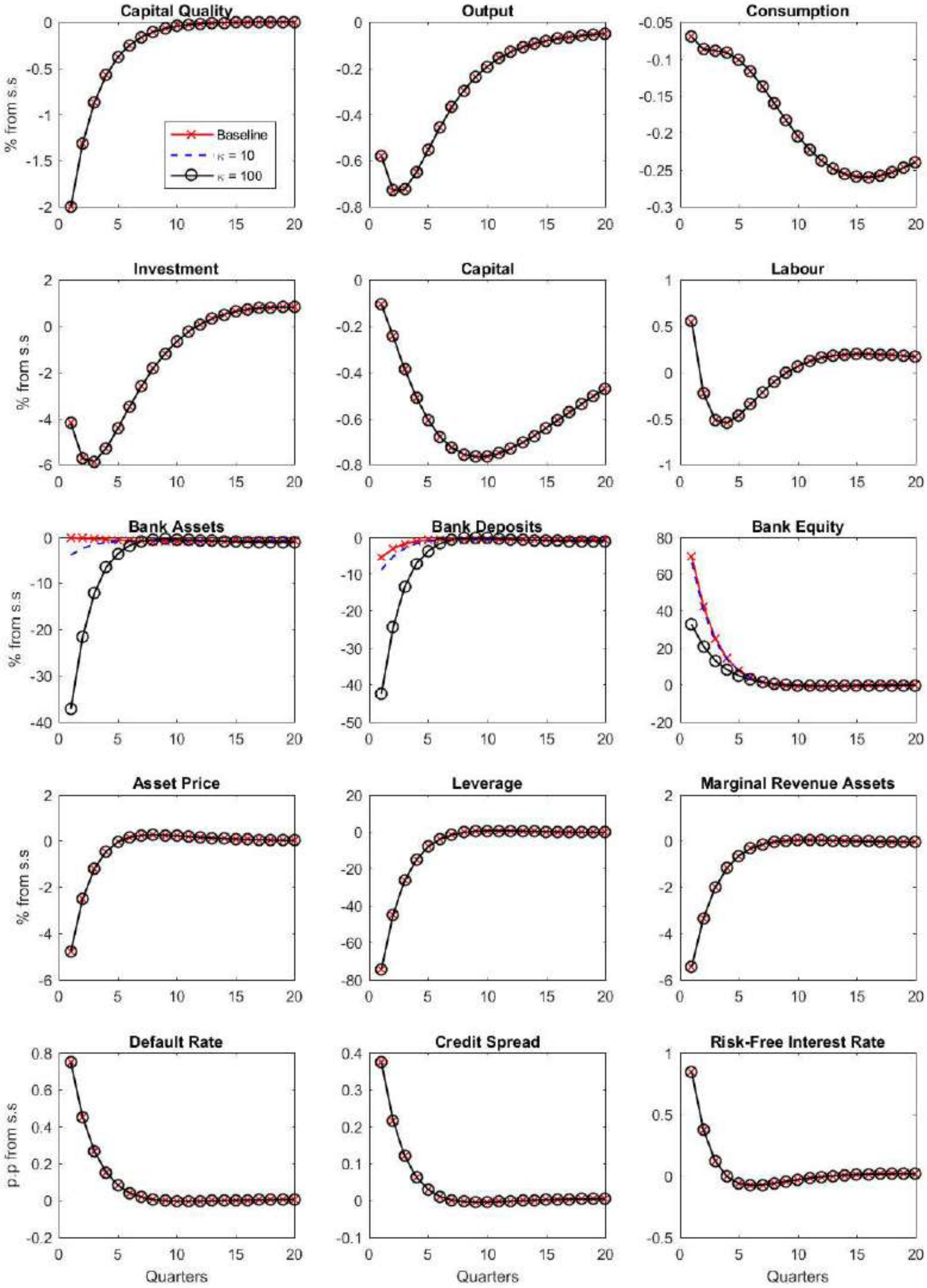
Bank Lending Channel — A second effect of soaring default rates comes through the bank lending channel. Recall that the marginal revenue of assets is defined as the expected return of a financial claim net of defaults, minus the cost of raising the funds through deposits, $\mathbb{E}R^{k'}(1-\gamma')-R'$. Despite there being an increase in the rate of return of capital due to the lower capital demand by final good firms, the higher default rates provoke a collapse in the marginal revenue of assets. Indeed, Figure 5 displays the different reaction of the marginal revenue of assets between the model with endogenous defaults and the model without. Whilst the latter features a rise in the marginal revenue of assets as a consequence of the rise in the rate of return of capital, the former model generates a fall in the marginal revenue of assets thanks to the rise in defaults. As the profitability of assets fall, the banks are less willing to purchase them, resulting in a fall in asset acquisitions and a further fall in the leverage ratio.

Overall, the financial sector's leverage ratio due to the *balance sheet channel* and the *bank lending channel* decreases by 75%, as bank equity rises by almost as much and bank assets fall by nearly 1%. Output falls by 0.75% and investment by 6%. Notably, the model with endogenous defaults amplifies and propagates the initial shock. This results in a heavier fall in economic aggregates of around a third, in contrast to a model without the financial accelerator mechanism driven by endogenous defaults.

The leverage dynamics differ substantially from the financial accelerator literature such as Gertler and Karadi (2011). In the related literature, the financial sector is assumed to have its equity equal to a constant fraction of assets. Such financial friction, although not strictly microfounded, is meant to represent the agency problem between depositors and banks. The banks' constraint in these models are strictly exogenous as a fraction of assets never changes, unlike the solvency constraint in this paper.

Furthermore, in such models, a financial shock decreases banks' equity through lower asset prices. The leverage dynamics in this literature relies on the inverse relationship between bank equity and credit spreads. More precisely, as credit spreads rise in a financial crisis, banks cut assets proportionally less than equity by taking on more debt. As the marginal rate of return of assets is equal to the credit spread absent defaults, banks try to exhaust all gains from asset intermediation by levering up in a crisis. However, evidence of such counter-cyclical leverage dynamics is lacking, as demonstrated by Figure 2 and empirical studies by Adrian and Shin (2010), and Kalemli-Ozcan et al. (2012). By contrast, in this model the positive relationship between credit spreads and leverage present in financial sector models in the literature is broken. This is achieved though the novel financial friction imposed using endogenous default of assets.

Figure 6: Credit Easing Policy During a Financial Recession



Notes: *The figure shows the effects of credit easing policy when the economy suffers a two percent capital quality shock, with a persistence of two-thirds. The red line shows the baseline scenario without central bank intervention. The blue dashed line displays the impulse response with a small-sized credit easing policy ($\kappa = 10$), whilst the black line with o symbols shows the responses following a large credit easing policy ($\kappa = 100$). The unit of time is in quarters. All the plots are in percentage deviations from the steady-state, apart from the last row which is percentage point deviations from the steady-state.*

5 Credit Easing Policy during a Financial Recession

In this section, the model is used to investigate the effectiveness of credit easing policy in alleviating a financial crisis and its ability to minimise the fall in aggregate output and investment during a financial crisis. As discussed in the previous section, a financial crisis is simulated by a negative capital quality shock of two percent, with an auto-regressive coefficient of two thirds. In contrast to the previous exercise, monetary authorities are now allowed to intervene using the credit easing policy rule described in Section 2.6. Figure 6 displays the impulse responses with no credit easing policy using the red line with $\times$ symbols, a small-sized credit easing policy intervention using a blue dashed line and a large credit easing policy intervention using the black line with *o* symbols. The size of the credit easing policy is controlled by varying the κ coefficient of the policy rule, which takes the value of ten for the small-sized credit easing policy intervention and one hundred for the larger intervention. The values are chosen in accordance with previous studies in the field, such as Gertler and Karadi (2011).

Following the exogenous drop in capital quality, a financial crisis erupts but this time the central bank expands its balance sheet by purchasing financial claims from financial good firms. The purpose of the credit easing policy is to increase asset demand, and therefore credit supply. This would stop asset prices falling causing a decrease in borrowing costs, therefore muting the decline in output and investment. Indeed, by intervening in the private asset intermediation market, the central bank aims to use its balance sheet to directly borrow financial funds to final good firms when the banks are unable to do so.

As shown by Figure 6, the policy fails to mitigate the recession because there is a crowding-out effect in the asset market. As the central bank increases the purchases of private assets, it tends to reduce the marginal rate of return of assets, and as a result, banks cut down their demand for assets exactly one-for-one. Consequently, the overall credit supply is unchanged and neither credit spreads fall, nor do asset prices rise. The failure to reduce borrowing costs and stop asset prices from falling means that output and investment remain unchanged, even with the central bank's intervention.

With respect to the banking sector, credit easing policy cannot slow down the banks' deleveraging process because it cannot counter the rise in defaults. The policy does not alter the final good firm decision to default as the borrowing costs remain high due to the crowding out effect. Since defaults continue to rise, the balance sheet channel and the bank lending channel continue to reduce bank' leverage and dampen credit supply. Indeed, as central banks increase the credit supply, the banks reduce their credit supply, causing bank assets and deposits to decrease. Finally, the banks' equity now only increases marginally as the number of privately intermediated assets falls, so the losses faced by banks also falls.

6 Credit Subsidy Policy during a Financial Recession

So far the paper has established the ineffectiveness of credit easing policies by monetary authorities in mitigating the macroeconomic effects of a financial crisis. Recently, a novel non-monetary policy tool for governments has been discussed: *credit subsidies*. These subsidies, instead of providing funds directly to final good firms, are intended to reduce the costs of the recession faced by firms. In previous literature, namely Correia et al. (2018), the credit subsidy policy aims to directly reduce the firms' borrowing costs. However, given that the model explored in this paper revolves around the endogenous default mechanism, I shall modify the policy to reduce the fixed costs of production for firms to directly reduce the default rates in the economy.

6.1 Credit Subsidy Policy

The credit subsidy policy works as follows. As in the baseline model, the final good firms face a fixed cost of production, which induces firms to decide whether to default on their debt obligations or not. However, instead of facing a fixed cost of c^f, the firms now face a fixed cost of $c^{f^*} = (1-\iota)c^f$, where ι represents the credit subsidy received by the firm.

The credit subsidy is supplied by the government and is funded using non-distortionary lump-sum taxes on the household. Similarly to the credit easing rule, the government uses a credit subsidy policy rule which states that the size of the subsidies depends on the increase in default rates during the crisis

$$\iota = \theta\left[\gamma - \gamma^*\right], \tag{40}$$

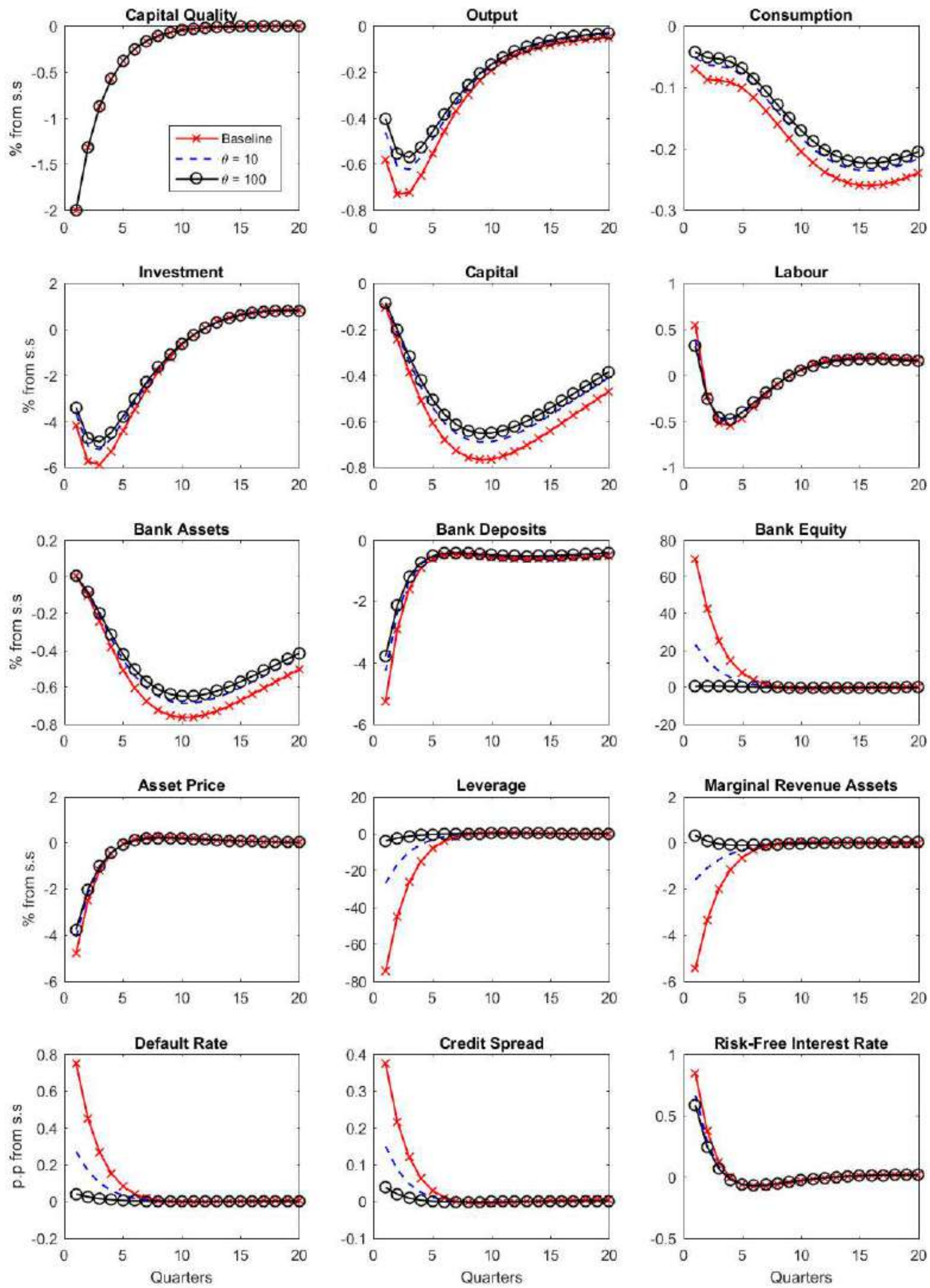
where γ^* is the steady-state level of default rates and θ is the credit subsidy intensity parameter, which will be manipulated to simulate different size interventions.

For simplicity, I assume that all firms receive the subsidy, even though it may be more efficient for the government to provide credit subsidies to the firms at the margin of default. Such simplification, however, manages to avoid the problems of modelling the monitoring costs incurred by the government in identifying the firms at the margin.

6.2 Credit Subsidy Policy Experiment

As modelled before, a financial crisis is simulated by a negative capital quality shock of two percent, with an auto-regressive coefficient of two thirds. In contrast to the previous exercise, the government is now allowed to intervene using the credit subsidy policy rule described in Section 40. Figure 7 displays the impulse responses with no credit subsidy policy using the red line with $\times$ symbols, a small-sized credit subsidy intervention using a blue dashed line and a large credit subsidy policy intervention using the black line with o symbols. The size of the credit easing policy is controlled by varying the θ coefficient of the policy rule, which takes a value of ten for the small-sized policy intervention and one hundred for the larger intervention.

Figure 7: Credit Subsidy Policy During a Financial Recession



Notes: *The figure shows the effects of credit subsidy policy when the economy suffers a two percent capital quality shock, with a persistence of two-thirds. The red line shows the baseline scenario without credit subsidy intervention. The blue dashed line displays the impulse response with a small-sized credit subsidy policy ($\theta = 10$), whilst the black line with o symbols shows the responses following a large credit subsidy policy ($\theta = 100$). The unit of time is in quarters. All the plots are in percentage deviations from the steady-state, apart from the last row which is percentage point deviations from the steady-state.*

Highlighted by Figure 7, in contrast to the experiment with the credit easing policy in Section 5, the credit subsidy manages to mitigate the effects of the financial recession. By providing subsidies to final good firms aiming to reduce the fixed costs of production, the policy counters the rise in defaults. As the financial shock increases the threshold idiosyncratic productivity, the credit subsidies are able to dampen defaults directly. As defaults fall, so does the effect on the banks' leverage and balance sheet. As a result of the policy, banks need not deleverage as much because the losses from defaulting loans are reduced. Overall, the responses of macroeconomic variables are muted by the intervention of the credit subsidies, which reduce the fall in output and investment by 40% and 30% respectively in the case of the large intervention.

Correia et al. (2018) have provided an initial inspection of the credit subsidy policy. They assume that the government uses subsidies to reduce the interest rate paid by firms to banks. Nonetheless, their exposition of the credit subsidy policy is based on a model, like previous literature on the subject, with a financial sector a lá Gertler and Karadi (2011). As a result, Correia et al. (2018) utilise a model that does not feature the endogenous default of assets and is unable to capture the pro-cyclical dynamics of leverage.

7 Conclusion

During the U.S. financial recession of 2007, default rates on bank assets soared and the financial sector began to heavily deleverage. To tackle the fallout from the financial recession, unconventional monetary policy, specifically credit easing policies, have been used by the Federal Reserve. The objective of this paper was to build a dynamic stochastic general equilibrium model with a financial sector and endogenous asset default that can account for the observed default and leverage dynamics. I then examined whether credit easing policies have been effective in mitigating the effects of a financial recession.

The model developed in this paper expands on an otherwise standard real business cycle model to include a financial sector, final good firms that sell non-enforceable debt obligations, asset price dynamics and, finally, a monetary authority which can intervene in the private intermediated asset market using credit easing policies. The novelty of the model lies in the financial accelerator mechanism that arises when asset defaults are modelled. Since assets held by banks are subject to default, and deposits are assumed to be risk-free, the bank is constrained in its lending. Specifically, as deposits have to be backed by non-defaultable assets at all times, the banks' equity must be such that the asset losses from defaults can be covered.

The endogenous default mechanism, which is the main driver of the financial accelerator in the model, is able to reproduce the default and leverage dynamics experienced by the U.S. economy during the Great Recession. An adverse financial shock, which reduces capital productivity and asset prices, exogenously generates higher asset defaults as firms cannot repay their debt obligations. The higher defaults cause banks to start deleveraging because they have to raise equity to cover the new losses (*balance sheet channel*), and because they are unwilling to purchase new assets as they are less profitable (*bank lending channel*). The endogenous default mechanism, through the deleveraging dynamics, amplifies and propagates the initial shock causing a bigger and deeper recession.

When the central bank intervenes using credit easing policies, that is, by directly lending to final good firms when the private financial sector cannot, the economic fallout from a financial recession is not alleviated. Since credit easing policy does not alter the decision to default, instead of lowering borrowing costs and raising prices, it simply substitutes privately intermediated assets with publicly intermediated assets.

The paper proposes an alternative policy to tackle a financial recession: credit subsidies. Instead of lending to final good firms, the government can subsidise them in order to reduce the fixed costs of production. Credit subsidies would lower default rates directly, removing the need for banks to deleverage, and, as such, mitigate the effects of the financial recession by muting the response of the financial accelerator mechanism.

In conclusion, the paper provides evidence in favour of the view that credit easing policies used in the U.S. have not been effective in reducing the burden of the financial crisis of 2007 on the wider economy. Specifically, when accounting for pro-cyclical financial leverage and counter-cyclical default dynamics, credit policy is unable to stop asset prices from falling, nor borrowing costs from rising. Other policies, such as credit subsidies, which tackle the source of the problem - that is, the spike in default rates - are more effective in mitigating the recession. These non-monetary policy prescriptions may become relevant again as, at the time of writing, most of the developed economies may not be able to use conventional monetary tools to deal with the next recession, as they operating near to, or even at, the Zero Lower Bound.

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Appendix A Baseline Model Equations

The model's equations are laid out in this section. The model is solved using Dynare 4.3.3. The programme solves the model using stochastic simulation computing a Taylor approximation of the decision and transition functions for the model and impulse response function. Due to the simplicity of the model, only a first order approximation is considered, but robustness checks with higher order approximation are carried out.

Household

Euler Equation:

$$\mathbb{E}\beta\lambda' r' = 1, \tag{41}$$

Stochastic Discount Factor:

$$\lambda' = \frac{\varrho'}{\varrho}, \tag{42}$$

Marginal Utility of Consumption:

$$\varrho = (c - hc_{-1})^{-1} - \mathbb{E}\beta h(c' - hc)^{-1}, \tag{43}$$

Labour First Order Condition:

$$\varrho w = \chi l^{\varphi}. \tag{44}$$

Final Good Firms

Output:

$$y = \int_{\underline{a}}^{\infty} za(\xi k^{\alpha} l^{1-\alpha})^{\zeta} f(a)\, da \tag{45}$$

Labour:

$$l = \int_{\underline{a}}^{\infty} \frac{(1-\alpha)y(k, a, z, \xi)}{w} f(a)\, da, \tag{46}$$

Investment Decision:

$$k = \mathbb{E}_{-1}\left[\frac{\alpha y(k, a, z, \xi)}{r^k q_{-1} + \delta\xi - q\xi}\right], \tag{47}$$

Default Decision:

$$0 = \underline{y} - w\underline{l} - r^k q_{-1} k - \delta k + qk - c^f, \tag{48}$$

Output at Threshold:

$$\underline{y} = z\underline{a}k^{\alpha}\underline{l^{1-\alpha}}, \tag{49}$$

Labour at Threshold:

$$(1-\alpha)\frac{\underline{y}}{\underline{l}} = w, \tag{50}$$

Default Rate:

$$\gamma = \int_0^{\underline{a}} f(a_i)\, da. \tag{51}$$

Banks

Balance Sheet:

$$qs^p = n + b, \tag{52}$$

Equity Constraint:

$$n = \gamma\tau q_{-1}s^p_{-1}, \tag{53}$$

Balance Sheet:

$$\omega + \nu = E\beta\lambda'[r^{k'}(1-\gamma') + \nu\tau\gamma' + (1-\tau)\gamma'] \tag{54}$$

Deposits Optimality Condition:

$$\nu = \mathbb{E}\beta\lambda'[r'] \tag{55}$$

Assets Optimality Condition:

$$C'(s^p) + \nu = \beta\mathbb{E}\Lambda'\left[r^{k'}(1-\gamma') + (1-\tau+\nu'\tau)\gamma'\right] \tag{56}$$

Leverage:

$$\phi = \frac{qs^p}{n} \tag{57}$$

Capital Producer Firm

Q Relation for Net Investment:

$$q = 1 + f(\cdot) + \left(\frac{i^n + i^{ss}}{i^n_{-1} + i^{ss}}\right) f'(\cdot) - \mathbb{E}\beta\Lambda\left(\frac{i'^n + i^{ss}}{i^n + i^{ss}}\right)^2 f'(\cdot), \tag{58}$$

Net Investment:

$$i^n = i - \delta k. \tag{59}$$

Government

Credit Policy Rule:

$$\psi = \kappa_\psi\mathbb{E}\{[log(r^{k'}) - log(r')] - [log(r^{k^*}) - log(r^*)]\}, \tag{60}$$

Government Assets:

$$qs^g = \psi qs \tag{61}$$

Economy's Leverage:

$$\phi^c = \frac{\phi}{1-\psi}, \tag{62}$$

Government Budget Constraint:

$$g + vqs^g = T + [r^k(1-\gamma) - r]\psi_{-1}q_{-1}s_{-1}. \tag{63}$$

Market Clearing

Total Assets:

$$qs = qs^p + qs^g, \tag{64}$$

$$qs = qk', \tag{65}$$

Capital Law of Motion:

$$k' = i + (1-\delta)k, \tag{66}$$

Aggregate Constraint:

$$y = c + i + g + C(s^p) + f\left(\frac{i^n + i^{ss}}{i^n_{-1} + i^{ss}}\right)(i^n + i^{ss}) + (1-\gamma)c^f \tag{67}$$

Exogenous Shocks

TFP Law of Motion:

$$log(z) = \rho^z log(z_{-1}) + \epsilon^z, \tag{68}$$

Capital Quality Law of Motion:

$$log(\xi) = \rho^\xi log(\xi_{-1}) + \epsilon^\xi. \tag{69}$$

Appendix B Baseline Model Steady State Equations

The model's steady state equations are laid out in this section. Denote the steady state variables with the *ss* superscript.

Household

Euler Equation:

$$\beta \lambda^{ss} r^{ss} = 1, \tag{70}$$

Stochastic Discount Factor:

$$\lambda^{ss} = 1, \tag{71}$$

Marginal Utility of Consumption:

$$\varrho^{ss} = \frac{[1 - \beta h]}{[(1-h)c^{ss}]}, \tag{72}$$

Labour First Order Condition:

$$\varrho^{ss} w^{ss} = \chi l^{ss\varphi}. \tag{73}$$

Final Good Firms

Output:

$$y^{ss} = \int_{\underline{a}^{ss}}^{\infty} z^{ss} a (\xi^{s} s k^{ss\alpha} l^{ss 1-\alpha})^{\zeta} f(a)\, da \tag{74}$$

Labour:

$$l^{ss} = \int_{\underline{a}^{ss}}^{\infty} \frac{(1-\alpha) y(k^{ss}, a^{ss}, z^{ss}, \xi^{ss})}{w^{ss}} f(a)\, da, \tag{75}$$

Investment Decision:

$$k^{ss} = \left[\frac{\alpha y(k^{ss}, a^{ss}, z^{ss}, \xi^{ss})}{r^{k} q^{ss} + \delta \xi^{ss} - q^{ss} \xi^{ss}} \right], \tag{76}$$

Default Decision:

$$0 = \underline{y}^{ss} - w^{ss} \underline{l}^{ss} - r^{k^{ss}} q^{ss} k^{ss} - \delta k^{ss} + q^{ss} k^{ss} - c^{f}, \tag{77}$$

Output at Threshold:

$$\underline{y}^{ss} = z^{ss} \underline{a}^{ss} k^{ss\alpha} \underline{l}^{ss 1-\alpha}, \tag{78}$$

Labour at Threshold:

$$(1-\alpha) \frac{\underline{y}^{ss}}{\underline{l}^{ss}} = w^{ss}, \tag{79}$$

Default Rate:

$$\gamma^{ss} = \int_{0}^{\underline{a}^{ss}} f(a_i)\, da. \tag{80}$$

Banks

Balance Sheet:

$$q^{ss}s^{pss} = n^{ss} + b^{ss}, \tag{81}$$

Equity Constraint:

$$n^{ss} = \gamma^{ss}\tau q^{ss}s^{pss}, \tag{82}$$

Balance Sheet:

$$\omega + \nu^{ss} = \beta\lambda^{ss}[r^{k^{ss}}(1-\gamma^{ss}) + \nu^{ss}\tau\gamma^{ss} + (1-\tau)\gamma^{ss}] \tag{83}$$

Deposits Optimality Condition:

$$\nu^{ss} = \beta\lambda^{ss}[r^{ss}] \tag{84}$$

Assets Optimality Condition:

$$C'(s^{pss}) + \nu^{ss} = \beta\Lambda^{ss}\left[r^{k^{ss}}(1-\gamma^{ss}) + (1-\tau+\nu^{ss}\tau)\gamma^{ss}\right] \tag{85}$$

Leverage:

$$\phi^{ss} = \frac{q^{ss}s^{pss}}{n^{ss}} \tag{86}$$

Capital Producer Firm

Q Relation for Net Investment:

$$q^{ss} = 1, \tag{87}$$

Net Investment:

$$i^{nss} = i^{ss} - \delta k^{ss}. \tag{88}$$

Government

Credit Policy Rule:

$$\psi^{ss} = 0, \tag{89}$$

Government Assets:

$$q^{ss}s^{gss} = \psi^{ss}q^{ss}s^{ss} \tag{90}$$

Economy's Leverage:

$$\phi^{css} = \frac{\phi^{ss}}{1-\psi^{ss}}, \tag{91}$$

Government Budget Constraint:

$$g^{ss} + vq^{ss}s^{gss} = T^{ss} + [r^{k^{ss}}(1-\gamma^{ss}) - r^{ss}]\psi^{ss}q^{ss}s^{ss}. \tag{92}$$

Market Clearing

Total Assets:

$$q^{ss}s^{ss} = q^{ss}s^{pss} + q^{ss}s^{gss}, \tag{93}$$

$$q^{ss}s^{ss} = q^{ss}k^{ss}, \tag{94}$$

Capital Law of Motion:

$$k^{ss} = i^{ss} + (1-\delta)k^{ss}, \tag{95}$$

Aggregate Constraint:

$$y^{ss} = c^{ss} + i^{ss} + g^{ss} + C(s^{pss}) + (1-\gamma^{ss})c^{f} \tag{96}$$

Exogenous Shocks

TFP Law of Motion:

$$log(z^{ss}) = 0, \tag{97}$$

Capital Quality Law of Motion:

$$log(\xi^{ss}) = 0. \tag{98}$$

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