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Abstract

In *A Mathematical Formulation of the Ricardian System*, Pasinetti (1960) lays out the foundations of what has been dubbed the *canonical classical model*. He proves the model to be logically consistent and determinate in all its macro-economic features, and derives the solutions for all key variables independently of demand conditions. The model thus provides macroeconomic foundations to the classical theory of distribution. This paper examines the decentralised, competitive mechanism underlying the macroeconomic outcomes. First, we model a classical economy with capitalists, workers, and landlords and define the notion of a Classical Competitive Equilibrium (CCE). A unique CCE exists in a large class of concave classical economies and the resulting income distribution is proved to coincide with that of Pasinetti's canonical classical model. Second, we use an agent-based model in order to examine more explicitly the decentralised competitive mechanisms at play in the classical economy. We show that a realistic competitive interaction between boundedly rational agents with localised knowledge generates classical gravitational dynamics with the key distributive variables oscillating around their equilibrium values.

Keywords: Luigi Pasinetti; Income distribution; Classical competition; Agent-based model.

JEL Classification Numbers: B51, C63, D50.

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*We started working on classical competition models while writing our joint book with Peter Flaschel. For over a decade, Peter's unflinching support and sharp insights have been invaluable for our research. We dedicate this paper to his memory: we had planned to complete it with him but his sudden death in October 2021 made this impossible. Thanks are owed to participants in the 2023 European Association for Evolutionary Political Economy conference and the 2024 Eastern Economic Association conference. The usual disclaimer applies.

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1 Introduction

In *A Mathematical Formulation of the Ricardian System*, Pasinetti (1960) lays out Ricardo's "complete system in a rigorous and concise form . . . stating explicitly the assumptions needed in order to eliminate the ambiguities" (Pasinetti 1960, p.78) of Ricardo's theoretical construction. The result is a remarkably elegant, and simple, model which sheds light on some crucial characteristics of classical economics.

Pasinetti (1960) considers an economy with three classes: workers, capitalists, and landlords. Only one type of wage-good, say corn, is produced using land and labour in exactly one year. "[C]apital consists entirely of the wage bill, in other words, it is only circulating capital, which takes one year to be re-integrated" (Pasinetti 1960, p.82). There exists an invariable standard of value, say gold—a luxury good—, which always requires the same quantity of labour to be produced (ibid.). Technology is fixed and "the production of corn can be expressed by a technical production function, which we may assume to be continuously differentiable" (ibid.). Individual preferences, and demand more generally, play no role in the system which is closed by assuming Say's law to hold and considering two different wage determination mechanisms. In the (long run) "natural equilibrium of the Ricardian system" (Pasinetti 1960, p.84), the real wage rate is set at the level which keeps population constant.[1] In a (short-to-medium run) market equilibrium, the wage rate is set at the level that clears the labour market.

Pasinetti (1960, p.92) proves this parsimonious analytical framework to be "logically consistent and determinate in all its macro-economic features" and uses it to draw a number of relevant theoretical conclusions. In particular, "the system shows that wage-goods and luxury-goods play two different roles in the system. The production function for the wage commodity turns out to be of fundamental importance, while the conditions of production of the luxury-goods . . . have in the system a very limited influence. . . . [T]he solutions for all variables, except [the price of luxuries], depend on the [production] function [of corn] or on its first derivative" (Pasinetti 1960, p.85).

Further, the model provides clear macroeconomic foundations to the classical theory of distribution. "Whatever the demand equations for luxury-goods may be, i.e., independently of them, all the variables referring to the wage-goods part of the economy, all prices, the rate of profit, and all the macro-economic variables of the system—like total employment, national income, total profits, total rent, total wages, total capital—are already determined by the system" (Pasinetti 1960, p.91), independent of individual preferences and demand.[2]

However, the microeconomic foundations of the model and the decentralised, competitive mechanism underlying the macroeconomic outcomes are less clearly specified. As Pasinetti (1960, p.85) notes, Ricardo "mentions the behaviour of the capitalists, whose readiness to move their capital towards the most profitable sectors of the economy always cause the rates of profit to equalise in all sectors. . . . [Yet] Ricardo does not really say much more than

[1] Pasinetti (1960) distinguishes the natural equilibrium from the stationary state in which the capital stock is also constant. This distinction is not relevant for the main thread of our argument and we shall ignore it in what follows.

[2] Pasinetti (1960, pp.84-85) also proves that Ricardo's model "contains a theory of value which is completely and (owing to our explicit assumptions) rigorously independent of distribution". While this is an important insight, Ricardo's theory of value is not the main focus of our analysis and therefore we will not explore it.

[this]. He does not find it useful to enter into complicated details . . . Simply he allows for the process and carries on his analysis . . . on the assumption that the equalisation of the rates of profit has already been permanently achieved".

This analytical gap carries over to Pasinetti (1960), and much of the subsequent literature, and it is somewhat unsatisfactory as "ruthless competition" (Samuelson 1978) between agents plays an important role in classical theories, as the driving force of wage rate and profit rate equalisation, but also in the determination of rents over different qualities of land.[3] This paper aims to explicitly analyse the microeconomic process of ruthless competition over wages, profits, and rents underlying the canonical classical model in Pasinetti (1960).

To be specific, we analyse the functioning of decentralised, ruthless competition between agents in the short-run, where total labour supply and total capital are fixed. One of the core features of the model is that capital is not a physical good tied up in production. Rather, capital is conceptualised as a magnitude of purchasing power that can be freely allocated to different land/labour combinations. Consistent with this interpretation, the key behavioural assumption is that as long as the expected profit rate is positive, capitalists put all their capital to productive use and, in particular, they try to allocate capital to the lands that yield the highest rate of profit.

We define a *Classical Competitive Equilibrium* (CCE) as an allocation in which: a unique, market-clearing real wage rate prevails; land/labour combinations are chosen by capitalists so as to maximise the rate of profit; rents adjust so that a unique profit rate emerges on all lands in operation; and all capital is put to productive use. The CCE, in classical terminology, is thus defined as a type of *market* equilibrium, or a *moving* equilibrium (Pasinetti 1960), in that the economy has not reached its natural, or stationary state.

The existence, uniqueness and even optimality of the CCE can be proved in a large class of economies. In equilibrium the types of lands in operation, and the rents paid to their owners are uniquely determined in a model where both extensive and intensive diminishing marginal returns (and therefore rents) are accounted for. Perhaps more importantly, in the CCE, ruthless competition between agents leads to a well-defined distribution of income between the three classes which is determined independently of demand and provides microfoundations to (that is, coincides with) the income distribution in Pasinetti (1960).

While the concept of CCE allows us to show the consistency of Pasinetti's (1960) macroeconomic model with a decentralised classical equilibrium, it has two limitations. First, it implicitly relies on strong assumptions concerning individual rationality and knowledge diffusion. Second, it provides a static, one-shot picture of the *outcome* of "ruthless competition" without considering the actual competitive *process*. In order to tackle both issues, we build an agent-based model (ABM) of the classical economy.

An ABM is a computational model where sets of heterogeneous agents make decisions and interact according to given rules.[4] Agent decisions and interactions are decentralised and based on a given scope of information. One motivation for using ABMs is to *grow* or *generate* economic phenomena of interest from these specified agent sets and rules (Epstein 2006). The decentralised nature of decisions and interactions based on possibly limited in-

[3]An interesting exception is the recent contribution by Salvadori (2004), which analyses the Ricardian theory of rent in a game-theoretic perspective.

[4]See Axtell and Farmer (2023), Epstein (2006), Farmer and Foley (2009), and LeBaron and Tesfatsion (2008) for surveys and discussions of the methodology of ABMs.

formation implies that agents may make decisions outside of equilibrium. A natural question is then: can an equilibrium like the CCE be attained—perhaps as a centre of gravity, or even tendentially—from the evolution of agent interactions and out-of-equilibrium decisions?

As Cogliano, Veneziani, and Yoshihara (2022a) discuss in a recent survey, computational methods—and especially ABMs—are not yet widely used in classical-Marxian economics. Computational simulations have recently been developed to study the dynamics of exploitation and class, the relation between inequalities in exploitation intensity and conventional measures of income and wealth inequality, and imperialism and exploitative unequal exchange between countries in classical-Marxian economies (Cogliano, Veneziani, and Yoshihara 2016, 2019, 2022b, 2024). The specific distributive focus of these contributions is rather different than ours, however, and they examine almost exclusively equilibrium behaviour. Closer to our current approach are recent ABMs developed by Cogliano and Jiang (2016), Jiang (2015), and Wright (2008), but these contributions primarily focus on issues in value theory and the Marxian circuit of capital, which we ignore here.

2 The economy[5]

The formal framework closely follows the model in Pasinetti (1960) with two main exceptions. First, we focus exclusively on wage goods and ignore luxuries. As Pasinetti (1960) shows, this is without any loss of generality for our purposes. Second, we explicitly introduce a multitude of heterogeneous agents and lands.

Consider an economy with a set $\mathcal{L} = \{1, \dots, n\}$ of lands with distinct fertility. Farming requires only labour, L, as an input, and the period of production is uniform and normalised to one on each piece of land. Each land $i \in \mathcal{L}$ is characterised by a production function $Y_i = f_i(L_i)$, which is twice differentiable, with positive but decreasing marginal productivity, and such that no output can be obtained without labour.[6] Formally, let $\mathcal{F} = \{f_1(.), ..., f_n(.)\}$: for all $f_i \in \mathcal{F}$, $f_i(0) = 0$, $f_i'(.) \equiv \frac{\partial f}{\partial L_i} > 0$ and $f_i''(.) \equiv \frac{\partial^2 f}{\partial L_i^2} < 0$, for all $L_i \in \mathbb{R}_+$.

Without loss of generality, we assume that if land $i \in \mathcal{L}$ has a higher harvest than land $j \in \mathcal{L}$ for some labour input $L > 0$, it also has a higher harvest for every other positive labour input, so that lands can be unambiguously ordered in terms of their fertility.[7] Formally:

Assumption 1. For all $f_i \in \mathcal{F}$, $f_1(L) > f_2(L) > \dots > f_n(L)$ for all $L > 0$.

There are three classes: workers, capitalist farmers, and landowners. At the beginning of each period, landowners own land, possibly of different types, and maximise rent. Workers are endowed with a given, equal amount of (homogeneous) labour and they supply labour inelastically. Aggregate labour supply is therefore given and is denoted by $\bar{L}$. Workers compete in order to obtain the highest possible (real) wage. Each capitalist farmer, denoted as c, is endowed with a certain amount of capital Ω^c, which is used only to hire workers ex

[5]The analysis in this section draws heavily from Cogliano, Flaschel, Franke, Fröhlich, and Veneziani (2018, Chapter 2).

[6]The use of differentiable production functions on every piece of land allows us to analyse both extensive and intensive rent (Kurz and Salvadori 1992).

[7]We note in passing that an ordering of lands in terms of fertility is more problematic in models where intermediate goods are also used in production (see Kurz and Salvadori (1992, p.232)).

ante. Capital is thus a wage fund and is not used as an intermediate good in production. Capitalists can decide to pool their wage funds to form production coalitions. Since wages are paid in advance, gross profits include the return on the wage fund. Rents are paid out of the net product.[8]

Each period is divided into two stages. In the first stage, capitalists compete in order to hire workers: insofar as the expected rate of return on productive investments in agriculture is positive, they use all of their wage-funds in order to hire as many workers as possible to be used in agricultural production. In the second stage, they compete in order to get those lands which yield the highest profit rate, taking as given the rent paid to landowners and the real wage rate.

Formally, let $\mathcal{C}$ be the set of capitalists in the economy. Let ω be the real wage rate. For all $i \in \mathcal{L}$, let ρ_i be the rent paid on land of type i and let $r_i = \frac{f_i(L_i) - \rho_i - \omega L_i}{\omega L_i}$ be the profit rate earned on i, if L_i workers are employed on it. Let L^c denote the labour demand of capitalist c. Then, our behavioural assumption implies that for all $c \in \mathcal{C}$, $L^c = \Omega^c/\omega$, whenever there is some i such that $r_i \geq 0$, for some $L_i > 0$, supposing that each capitalist owns a sufficiently small proportion of the total capital stock.

Let $\mathcal{C}_i \subseteq \mathcal{C}$ be a subset of capitalists (possibly, a singleton) forming a productive coalition, and let $\Omega^{\mathcal{C}_i} = \sum_{c \in \mathcal{C}_i} \Omega^c$ be the total wage funds of coalition $\mathcal{C}_i$. We assume that there are no costs and no barriers in the formation of coalitions, and that each capitalist may belong to more than one coalition. The maximum profit rate that $\mathcal{C}_i$ can obtain on $i \in \mathcal{L}$ is given by

$$r_i\left(\rho_i, \omega\right) \equiv \max_{L_i} r_i = \frac{f_i(L_i) - \rho_i - \omega L_i}{\omega L_i} = \frac{f_i(L_i) - \rho_i}{\omega L_i} - 1, \qquad (MP_1)$$

$$\text{subject to } 0 \leq L_i \leq \frac{\Omega^{\mathcal{C}_i}}{\omega}.$$

The classical notion of competition adopted here is different from the neoclassical concept of perfect competition. First, we do not assume that all agents have exactly zero market power. In the Ricardian theory of rent, "some concentration of landed property is [compatible] with free competition. And free competition is ... perfectly compatible with the existence of rent" (Gehrke and Kurz 2001, 474). The notion of ruthless competition requires only that competitive forces be strong enough to enforce the law of one price, thus establishing a unique price for labour, a unique rent for each type of land, and a unique rate of profit across the economy.[9]

Second, we depart here from the standard neoclassical emphasis on maximising *profits* by focusing on the profit *rate* as the key variable driving competition between capitalists. To be sure, the assumption that capitalist farmers choose land/labour combinations that maximise the profit rate implies that they obtain the highest total profits. Yet our behavioural

[8]Pasinetti (1960) assumes extreme classical saving habits: there is no saving out of wages or rents, whereas all profits are saved. Thus the wage-fund for the next production period is equal to current aggregate gross profits. Because we are looking at the short-to-medium run market equilibrium of the economy with fixed capital and labour endowments, and given that no theory of expenditure is necessary, we need not make any assumptions concerning consumption and savings here. More on this in section 3.1 below.

[9]Yet a significant concentration of land ownership may require an explicit analysis of strategic interdependence between landowners. Salvadori (2004) develops an interesting analysis of Ricardian extensive rent theory in a game-theoretic framework.

assumption is different from assuming that they maximise total profits $f_i(L_i) - \rho_i - \omega L_i$. Formally, the solution to the latter problem would imply $f_i'(L_i^*) = \omega^*$ and would therefore not give rise to the classical theory of rent.

We can now provide a formal definition of the concept of *Classical Competitive Equilibrium* (CCE), which is conceptually related to Pasinetti's (1960) "market solutions", whereby equalisation of profit rates is achieved and capital and labour endowments (rather than the real wage) are given. Let a *classical economy* be defined by the sets $\mathcal{L}, \mathcal{F}, \mathcal{C}$, a vector of wage funds $(\Omega^c)_{c\in\mathcal{C}}$, and an aggregate labour endowment $\bar{L} > 0$. Let $\bar{\Omega} = \sum_{c\in\mathcal{C}} \Omega^c$.

The CCE can be defined as follows:

Definition 1. *The classical competitive equilibrium (CCE) of a classical economy is a non-negative tuple* $\left(\omega_o, \{r_i^*\}_{i=1,\dots,n}, \{\rho_i^*\}_{i=1,\dots,n}\right)$ *and the associated actions* $\{L_i^*\}_{i=1,\dots,n}$ *such that*

(i) L_i^* *solves* MP_1 *for all* $\mathcal{C}_i \subseteq \mathcal{C}$ *(profit rate maximisation);*

(ii) $r_i(\rho_i^*, \omega^*) = r^* \geq 0$, *for all* $i \in \mathcal{L}$ *such that* $L_i^* > 0$ *(profit rate equalisation);*

(iii) $\sum_{i=1}^n L_i^* = \bar{\Omega}/\omega^*$ *(capital market equilibrium).*

(iv) $\omega^* = \bar{\Omega}/\bar{L}$ *(labour market equilibrium);*

In other words, in equilibrium (i) capitalist farmers choose land/labour combinations that maximise the rate of profit on each type of land; (ii) profit rates are equalised across lands; and (iii) all capital advanced for the purchase of labour is actually used. Together with condition (ii), this implies that in equilibrium employment decisions taken in the first stage are also rational *ex post*.[10] Finally, condition (iv) states that the real wage rate clears the labour market. To see this, observe that labour supply is completely inelastic and equal to $\bar{L}$. As for labour demand, at the beginning of the period, provided the expected rate of profit is nonnegative, capitalist farmers invest all of their wage-fund $\bar{\Omega}$ to hire workers at the given real wage rate, ω. For each such ω the aggregate labour demand of capitalist farmers is $L^C = \bar{\Omega}/\omega$. Setting $\bar{L} = L^C$ yields Definition 1(iv).

Cogliano et al. (2018, Theorem 2.1) prove the existence and uniqueness of a CCE for a large class of economies with a concave technology and any initial labour endowment, provided capital is not abundant. Further, if only $k < n$ types of land are operated, then these should be the k most productive ones, as defined in Assumption 1. Further, if a CCE exists, then it is efficient: decentralised ruthless competition leads to the maximum corn production that can be obtained from a given amount of labour (Cogliano et al. 2018, Theorem 2.2).

What is the distribution of income between the three classes at a CCE? Definition 1(iv) determines the equilibrium real wage rate: $\omega^* = \bar{\Omega}/\bar{L}$. Given ω^*, Definition 1(i)-(iii) imply that there exists a number $k \leq n$ such that for all lands $i = 1, ..., k \leq n$:

$$r_i(\rho_i^*, \omega^*) = r^* = \frac{f_i'(L_i^*) - \omega^*}{\omega^*} \geq 0, \quad \rho_i^* = f_i(L_i^*) - f'(L_i^*)L_i^* > 0, \quad L_i^* > 0,$$

[10]Condition (iii) implies that at any CCE at least one type of land must be operated, and therefore it must be $\bar{\Omega}/\bar{L} < f_i'(0)$ for at least one $i \in \mathcal{L}$. Note also that coalition formation is not a problem in equilibrium because farmers are indifferent between various coalitions at a CCE. For by condition (ii) every type of land yields the same rate of profit and by conditions (iii)-(iv) all capital is used.

while for all $k+1 \leq i \leq n$:[11]

$$f_i'(0) \leq \omega^*, \quad \rho_i^* = L_i^* = 0.$$

The number k of utilised lands is a cut off in the fertility hierarchy. If more productive land of type $j \leq k$ were idle, and less fertile soils $l > k$ were used, there would be an offer for j that would increase rent payments and profits as compared to l. There must therefore be a unique $k \in \mathcal{L}$—which depends on $\bar{L}$—that separates utilised from non-utilised land.

The fully decentralised CCE can be aggregated up to map precisely into the (market) equilibrium income distribution in Pasinetti (1960). One way to show this is to note that a well defined relation can be identified between aggregate labour and income distribution. To see this, observe that if $\bar{L}$ changes, so does the equilibrium vector $\mathbf{L}^* = (L_1^*, \ldots, L_n^*)$. Therefore, we can write the optimal labour inputs as functions of $\bar{L}$: $L_i^* = \phi_i(\bar{L})$, $i \in \mathcal{L}$. Similarly, we denote total output by $F(\bar{L})$,

$$Y^p = F(\bar{L}) \equiv \textstyle\sum_{i=1}^n f_i[\phi_i(\bar{L})].$$

Cogliano et al. (2018, Propositions 2.1 and 2.2) prove that $F(\bar{L})$ is well-defined and twice differentiable with $F'(\bar{L}) > 0$ and $F''(\bar{L}) < 0$, for all $\bar{L} > 0$. Then, it can be used to provide an alternative representation of the income distribution between workers, capitalist farmers and landlords—determined by decentralised ruthless classical competition. Given $\bar{L}$, and noting that at the CCE, the marginal products in each type of land operated must be equalised, the aggregate incomes of workers, capitalists and landlords are, respectively,

$$\begin{aligned}
\omega^* \bar{L} &= \bar{\Omega}, \\
(1+r^*)\omega^* \bar{L} &= F'(\bar{L})\bar{L}, \\
\rho^* &= \sum_{i=1}^n \rho_i^* = F(\bar{L}) - F'(\bar{L})\bar{L}.
\end{aligned}$$

This is exactly the income distribution in Pasinetti (1960, p.84). Unlike in Pasinetti (1960), however, they are explicitly derived from the decentralised competitive behaviour of economic agents.[12]

3 An agent-based approach to competitive equilibrium

While the analysis in the previous section shows the consistency of Pasinetti's (1960) macroeconomic model with a decentralised classical equilibrium, it has two limitations. First, it implicitly relies on strong assumptions concerning individual rationality and knowledge diffusion. Second, it provides a static, one-shot picture of the *outcome* of "ruthless competition"

[11] Observe that the set of idle lands may be empty, depending on technology. It is empty, for example, if $f_i'(0) = \infty$, for all $i \in \mathcal{L}$.

[12] Samuelson (1978) derives the equation for total rents from the assumption that labour (together with capital) gets its marginal product and total rent is a residual after the payment of labour (and capital). For, given full employment of labour and capital, marginal productivity determines the wage rate (and similarly for the profit rate). In our model the causality is the other way round, consistent with the classical approach.

without considering the actual competitive *process*. In order to tackle both issues, we build an agent-based model (ABM) of the classical economy.

The ABM uses the same general framework presented above, with some more concrete specifications of certain components and the behaviour of agents. For the set of lands, $\mathcal{L}$, we assume $n = 10$ and that each land $i \in \mathcal{L}$ is characterised by a production function $Y_i = \alpha_i L_i^{\beta}$, with $\alpha_i < \alpha_j$ for all $1 \leq i < j \leq 10$, and $\beta < 1$. The simulation runs for T periods and each time period is denoted by $t = 1, \ldots, T$. In the rest of this section time subscripts are omitted to simplify notation where appropriate, but all variables and computational procedures described below can be read as occurring during any time period t.

3.1 Agents

Each agent is either a landlord, or a capitalist, or a worker. There is one landlord for each of the n pieces of land and landlords want to obtain as a high a rent, ρ_i, as possible on their land. All rents are consumed during t.

Each capitalist farmer $c \in \mathcal{C} = \{1, \ldots m\}$ is endowed with a wage-fund Ω^c, which can be used to hire workers in advance. Capitalists aim to obtain the highest possible profit rate on the land on which they operate. We set $m = 100$.

Workers supply the required amount of labour inelastically at the given real wage ω. They consume all earnings. All wages are paid prior to production taking place.

Because we are interested in the gravitational processes around the CCE, rather than the dynamics of the system when labour and capital endowments are allowed to change, we assume that the aggregate labour supply, $\bar{L}$, is fixed across time periods and capitalists consume all *net* profits during each t (while the wage-fund is replenished for $t + 1$).

3.2 Simulation Procedure

For the simulation, the aggregate wage-fund is constant at $\bar{\Omega}$ and, because we are not interested in exploring labour market dynamics, the real wage is $\omega^* = \bar{\Omega}/\bar{L}$ (Definition 1(iv) holds). Because $\bar{\Omega}$ and $\bar{L}$ are assumed to be constant, ω^* is also fixed across time periods.

At the start of the simulation, each capitalist's Ω^c is randomly drawn from a Pareto distribution such that (i) $\bar{\Omega} = \sum_{c \in \mathcal{C}} \Omega^c$, and (ii) no capitalist can operate a piece of land optimally on their own. Coalitions are necessary to operate lands at optimal levels.

At the beginning of each t, the n landlords announce rents, ρ_i, at which they are willing to supply their land. The announced rents $(\rho_1, \ldots, \rho_n)$ are take-it-or-leave it offers and each ρ_i is communicated to a random subset of capitalists $c^{\rho_i} \subset \mathcal{C}$ with cardinality η. Each capitalist may receive multiple rent announcements, and some may receive none. Capitalists who do not receive any ρ_i are placed in a subset of inactive capitalists $\mathcal{U} \subset \mathcal{C}$ who may receive a rent announcement later during t.[13]

Capitalists select which land proposal to accept based on the maximum profit rate, r_i^{max}, they can expect to attain on the different lands. For each ρ_i received by a capitalist, they

[13]See Rule 1 in Appendix A.1 for a full description of the rent communication algorithm.

compute r_i^{max} by solving the following problem:

$$r_i^{max} \equiv \max_{L_i} r_i = \frac{\alpha_i L_i^\beta - \rho_i - \omega L_i}{\omega L_i} = \frac{\alpha_i L_i^\beta - \rho_i}{\omega L_i} - 1. \qquad (MP)$$

If they receive only one rent announcement, then the capitalist chooses i. If they receive multiple rent announcements, they select i with the highest r_i^{max}. Capitalists will only operate on lands with $r_i^{max} > 0$.[14]

Suppose $\mathcal{U} \neq \varnothing$. Then, after the initial proposals $(\rho_1, \dots, \rho_n)$ are accepted by capitalists in $\mathcal{C} \backslash \mathcal{U}$, the announcements $(\rho_1, \dots, \rho_n)$ are sent again to initially unmatched capitalists in $\mathcal{U}$. This can be thought of as an initiative of the initially allocated capitalists communicating with the unallocated ones in order to form coalitions. Each ρ_i is communicated to $\eta/2$ unmatched capitalists who again select the land with the highest r_i^{max}. Any capitalists who do not receive a rent announcement in this second round remain inactive for the rest of t.[15]

Because wages are paid ex ante, each capitalist uses their entire Ω^c to hire as much labour as possible. Thus, barring the unlikely case that some capitalists receive no rent announcements,[16] both labour and capital are fully utilised during each t and Definition 1(iii) also holds.

However, nothing guarantees that the allocation is a CCE. Capitalists effectively "bring" their workers to the selected piece of land i forming a coalition $\mathcal{C}_i \subseteq \mathcal{C}$ with capital $\Omega^{\mathcal{C}_i} = \sum_{c \in \mathcal{C}_i} \Omega^c$ in order to operate it at the highest possible activity level: $Y_i = \alpha_i L_i^\beta$, where $L_i = \Omega^{\mathcal{C}_i}/\omega$ and $L_i \lesseqqgtr L_i^*$ depending on the size of $\mathcal{C}_i$. Any lands that are not selected by *any* capitalists will be inactive during t.[17]

After production concludes, the surplus is distributed between capitalists and landlords.[18] Because $L_i \lesseqqgtr L_i^*$, realised profit rates, r_i and rents, $\tilde{\rho}_i$, may differ from those initially expected. When the amount of labour allocated to a piece of land is greater than, or equal to, the optimum, $\gamma_i \equiv L_i/L_i^* \geq 1$, landlords will realise their advertised rent: $\tilde{\rho}_i = \rho_i$. However, when $\gamma_i < 1$, the landlords will be flexible in the rent they accept, within some boundaries. If the amount of labour is close to the ideal amount (within 10%), then landlords still take their advertised rent as long as it does not cause a negative profit rate: $\tilde{\rho}_i = \rho_i$. If L_i is less than $\kappa_1 = 90\%$ of the ideal amount, then landlords will accept some portion of their advertised rent down to a floor of half the surplus generated on their land, but they do so only grudgingly and approach this floor slowly—i.e. the misallocation of labour must be fairly extreme for landlords to accept the rent floor. In other words, landlords can be taken as being somewhat benevolent and understanding the need to ensure they can get something from what is produced on their land without driving capitalists away in future time periods.

[14]For a complete description of land selection and rent proposal acceptance, see Rule 2 in Appendix A.1. In section 2 capitalists are assumed to produce even if the expected profit rate on a piece of lands is zero. We exclude this possibility here merely for computational reasons because computers do not handle zero well. This is without any loss of generality, however, because given our parameter selection, zero expected profit rates are a zero measure event.

[15]See Rule 3 in Appendix A.1 for a complete description of the procedure.

[16]The parameter η is chosen to minimise the likelihood of this happening.

[17]This does not violate Definition 1(iii) since capital (and labour) will simply be allocated to other lands.

[18]See Rule 5 in Appendix A.1 for a full description.

Realised profit rates are then determined based on $\tilde{\rho}_i$ according to

$$r_i = \frac{\alpha_i L_i^{\beta} - \tilde{\rho}_i - \omega L_i}{\omega L_i}.$$

For any land i, r_i, may differ from both r_i^{max} and r^*.

At the start of the next time period, $t+1$, rent announcements are updated based on γ_i. If landlords realised expectations, $\gamma_i = 1$, then they make no change in their advertised rents: $\rho_{i;t+1} = \rho_{i,t}$. If labour was over-allocated to their land and they could have potentially extracted more rent, $\gamma_i > 1$, they tentatively adjust the rent advertisement upward, $\rho_{i;t+1} = (1+\nu_{i;t})\rho_{i;t}$ where $\nu_{i;t}$ is randomly determined in each t. If landlords obtained less than expected, $\gamma_i < 1$, they reluctantly adjust the next period advertisement downward, but the adjustment is slower than rent increases, $\rho_{i;t+1} = (1-\nu_{i;t}/3)\rho_{i;t}$. All rent adjustments are based on *advertised*—not *realised*—rents.[19] The simulation then repeats as described above.

3.3 Parameters and results

The parameters for the simulation reported here are shown in table 1.[20] They are constant throughout the simulation. The $\rho_{i;1}$'s described in table 1 are the initial rents announced at the beginning of the simulation, $t = 1$. The CCE for the parameters in table 1 is shown in table 2. All simulation results are shown *relative to these equilibrium values*.[21]

[Table 1 here]

[Table 2 here]

Figure 1 reports the results of a typical simulation run with parameters and initial conditions from table 1 and $T = 300$. Strikingly, the profit rates on different lands gravitate around r^* within roughly ten percentage points above or below equilibrium, and the average profit rate, r^{avg}, moves rather tightly around the equilibrium. Figure 1(a) shows the profit rates on each piece of land for the full run of the simulation. At the beginning of the simulation, profit rates move slightly above equilibrium but then after roughly twenty-five time periods they start oscillating relatively close to the CCE (see figure 1(b)). Sometimes the profit rate on land i falls to zero: this happens when land i remains inactive because landlord i has announced a rent that makes it unprofitable to operate on it.

Figures 1(c) and 1(d) further highlight the patterns of gravitation of profit rates and the tight movement of the average profit rate around the equilibrium by zooming in on select t and, in the latter case, by excluding the zero profit rates on inactive lands.

[Figure 1 here]

[19] For a full description of the rent update procedure see Rule 6 in Appendix A.1.

[20] All simulations are done in Mathematica version 14 and the simulation codebook is available at: https://www.dropbox.com/scl/fi/xh31z9uxpr7tf20exu1h9/ClassicalCompetitionEquilibriumABM_SimulationCode_Redacted.pdf?rlkey=gn9egfy4thav7ya9xcvcwoxcf&dl=0.

[21] The parameters κ_2, and μ determine the speed of the downward adjustment of realised rents caused by the misallocation of labour on individual tracts of land. See Rule 5 in Appendix A.1.

Figure 1 shows the most important result from the simulation: a classical competitive process ensuring tendential profit rate equalisation is at play. Notably, the competitive mechanism is driven by rent adjustments in response to localised (land-specific) market conditions. Each landlord's realised rents depend on their advertised rent and the utilisation of their land. Realised rents then factor into how advertised rents are updated in the next period. Rent adjustments drive the profit rate dynamics and how capitalists select lands on which to produce, driving the allocation of wage-funds and labour to each piece of land in each time period. As competition between landlords and capitalists over the surplus unfolds, the equilibrium emerges as a centre of gravity for profit rate fluctuations. Perhaps most importantly, agents do not have *any* knowledge of the equilibrium, yet it provides a relevant reference point for the dynamics of the economy.

The gravitational pattern around the CCE can also be observed in the behaviour of realised rents, $\tilde{\rho}_i$, whose dynamics are shown in figure 2—and in figures 2(c) and 2(d) for select t—relative to their equilibrium values, ρ_i^*. After a relatively short initial phase during which the boundary for fluctuations in rents is below the equilibrium (see figure 2(b)), all rents move around their equilibrium values, either above or below them, but in a pattern where the realised rent on each land spends time both above and below its equilibrium value. (In figure 2, rents drop to zero when a land is inactive.)

[Figure 2 here]

Further simulation results for the allocation of labour, capital (wage-funds), and head-counts of the number of capitalists in any $\mathcal{C}_i$ during each t are shown in Appendix A.2. These results are all consistent with the expected behaviour of the simulation, with labour and capital allocations gravitating around their respective equilibria.

4 Robustness

We have analysed many variations of the ABM in order to assess the robustness of our results. In this section, we briefly summarise the main points.[22]

Different rules to allocate the aggregate wage-funds—e.g. using a uniform distribution—make virtually no difference. As long as wage-funds are distributed in a way that allows multiple possible combinations of capitalists on a piece of land to achieve or surpass the optimal amount of labour L_i^*, the dynamics of the simulation unfold in similar ways.

Alternative parameter sets—to include, for example, $\beta < 0.9$, much smaller or much larger α_i's, or wider variation in the fertility of land—affect the equilibrium values of key variables, but do not fundamentally alter the dynamics of the simulation. However, wider variation in the α_i's can make attaining combinations of capitalists on pieces of land that put the simulation close to equilibrium difficult. More extreme differences in land fertility entail a need for large differences in the number of capitalists (and quantity of wage-funds) needed to operate lands at their optimal levels.

Similarly, we have considered both larger and smaller values of η in order to examine how information flows affect the allocation of capitalists and workers to different pieces of

[22]Results are available from the authors upon request.

land and the subsequent profit rate and rent dynamics. More restricted information leads to weaker adjustment processes in profit rates and rents, and in extreme cases can keep the simulation out of equilibrium by preventing the appropriate matching of capitalists to lands.

Alternative initial conditions also have a minor impact on the results. Randomly drawing the initial rents $\rho_{i;1}$ from a 20% interval either above or below equilibrium rents, or randomly setting them far below equilibrium values simply extends the initial phase of the simulation where the key variables turbulently move toward the neighborhood of the equilibrium.

Different distribution mechanisms have also been tested. Notably, a mechanism that allows landlords to realise rents greater than ρ_i. In this case, $\tilde{\rho}_i = \gamma_i \rho_i$, but when $\gamma_i < 1$, landlords have a floor on the lowest rent they will accept, subject to the profit rate remaining positive. This alternative specification presents strong competitive pressures inducing gravitation of profit rates very close to equilibrium, with other variables exhibiting similar behaviour to the results reported above. However, in the competitive framework of the model, it seems difficult to imagine that capitalists will passively accept an upward revision of the rents.

Alternative rent updating mechanisms have also been tested. Some are based on how realised rents compare to advertised rents $\tilde{\rho}_i/\rho_i$, and others include stronger upward and downward pressures on rents. These present similar dynamics in rents over time, but the amplitude of changes in rents across time periods varies widely.

5 Conclusions

We have examined the microeconomic foundations of the *canonical classical model* laid out by Pasinetti (1960). We have introduced the concept of classical competitive equilibrium (CCE) in order to analyse income distribution in a general equilibrium model that captures the process of competition envisaged by Classical economists. In a large class of economies, the CCE exists and is unique, and it identifies a well-defined distribution of income between profits, rents, and wages that corresponds to that in Pasinetti (1960).

Perhaps more strikingly, we have shown that the CCE represents the centre of attraction of a classical gravitational process emerging from the competitive interaction of boundedly rational agents with only localised knowledge. While agents do not know the equilibrium, it acts as an attractor for the dynamics of the economy as competitive pressures guide the fluctuations in profit rates and rents toward the neighbourhood of the equilibrium. The macroeconomic equilibrium in Pasinetti (1960) can thus be seen as the emergent property of many uncoordinated actions.

In closing this paper, it is worth remarking on two limitations of our analysis, which point to some avenues for further research. First, both in Pasinetti (1960) and in our microeconomic analysis, workers are exceedingly passive. It would be interesting to extend the analysis to incorporate more explicit labour market dynamics.

Second, our analysis is limited to *market* equilibria, taking aggregate capital and labour as given: a natural extension is to look at the long-run *natural* equilibria, and the Ricardian stationary state. Pasinetti (1960) and Cogliano et al. (2018) consider the long-run macrodynamics of capital and labour and derive the conditions for the convergence of the market equilibria to a (unique) stationary state. A natural extension of the ABM would be to relax the assumption that the aggregate endowments of capital and labour are constant and examine the emergent properties of the multitude of individual decisions on the long-run dynamics of the economy.

Disclosure Statement

No potential conflict of interest was reported by the author(s)

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A Online Appendix (NOT FOR PUBLICATION)

A.1 Simulation procedures

This appendix describes the simulation procedure in section 3 in more detail. Each step of the simulation is formalised as a computational rule, or algorithm, describing how agent decisions are made and how relevant variables in the model are updated over the course of the simulation. The simulation rules are run in the order presented here, as discussed in section 3. Rule 1 describes how advertised rents are communicated to capitalists.

Rule 1 (Rent communication)**.** *Let η be the number of rent announcements each landlord sends out. For any announced rent $\rho_i \in \boldsymbol{\rho} \equiv \{\rho_1, \ldots, \rho_{10}\}$, the capitalists receiving notification of ρ_i, $c^{\rho_i} \subset \mathcal{C}$ are randomly selected from the set of capitalists such that each ρ_i is communicated to η capitalists.*

Each $c \in \mathcal{C}$ may receive multiple rent announcements. Some $c \in \mathcal{C}$ may receive no rent announcements. Any $c \in \mathcal{C}$ who receive no rent announcement are placed in a subset of inactive capitalists $\mathcal{U} \subset \mathcal{C}$ who may receive rent announcements later within t.

Rule 2 describes how rent proposals are accepted by capitalists. Specifically, how they evaluate the different advertised rents and decide which will yield the highest expected profit rate. This determines the land on which capitalists operate and how much labour they bring to their respective coalitions.

Rule 2 (Proposal acceptance)**.** *Every capitalist $c \in \mathcal{C} \setminus \mathcal{U}$ who receives notice of rents $\boldsymbol{\rho}^c \subseteq \boldsymbol{\rho}$ solves the following problem for all $\rho_i \in \boldsymbol{\rho}^c$ in order to compute the expected profit rates on all lands:*

$$r_i^{max} = \max_{L_i} r_i = \frac{\alpha_i L_i^{\beta} - \rho_i - \omega L_i}{\omega L_i} = \frac{\alpha_i L_i^{\beta} - \rho_i}{\omega L_i} - 1. \qquad (MP)$$

If $\boldsymbol{\rho}^c$ is a singleton, then there is only one r_i^{max} and c accepts ρ_i and operates on land i. If $\boldsymbol{\rho}^c$ contains more than one element, then c accepts ρ_i such that $r_i^{max} > r_j^{max}$ for all $j \neq i$ and operates on land i. Capitalists only operate on lands with $r_i^{max} > 0$.

Rule 3 describes how the round of secondary rent announcements takes place and rent signal are sent to initially unallocated capitalists. It also explains how initially unallocated capitalists receiving secondary rent announcements evaluate their options based on Rule 2.

Rule 3 (Secondary rent communication)**.** *Suppose $\mathcal{U} \neq \varnothing$. Then, after the initial proposals $(\rho_1, \ldots, \rho_n)$ are accepted by capitalists in $\mathcal{C} \setminus \mathcal{U}$, the announcements $(\rho_1, \ldots, \rho_n)$ are sent again to initially unmatched capitalists in $\mathcal{U}$. Each ρ_i is communicated to $\eta/2$ (rounded down to the nearest integer) $c \in \mathcal{U}$. Each $c \in \mathcal{U}$ may receive multiple rent announcements and selects the best proposal following Rule 2. Any $c \in \mathcal{U}$ who do not receive rent announcements are placed in a subset of inactive capitalists $\mathcal{U}' \subset \mathcal{C}$ for the remainder of t.*

Production then occurs according to Rule 4.

Rule 4 (Production)**.** *For each land i, $Y_i = \alpha_i L_i^{\beta}$, where $L_i = \Omega^{\mathcal{C}_i}/\omega$.*

Rule 5 explains how realised rents are determined after production and how landlords may or may not realise their advertised rents. Cases where they will realise less than the advertised rent are explained below. They are willing to decrease their rents slowly to a floor of half the surplus generated on their land.

Rule 5 (Distribution)**.** *Let $\gamma_i = \frac{\Omega^c/\omega}{L_i^*}$ and let κ_1 and κ_2 be numbers between 0 and 1 with $\kappa_1 > \kappa_2$. There are four cases:*

(i) If $\gamma_i \geq 1$, landlords realise their advertised rent and $\tilde{\rho}_i = \rho_i$.

(ii) If $\kappa_1 \leq \gamma_i < 1$, $\tilde{\rho}_i = \rho_i$ provided $r_i > 0$ at $\tilde{\rho}_i$.

(iii) If $\kappa_2 \leq \gamma_i < \kappa_1$, $\tilde{\rho}_i = \gamma_i \rho_i$, provided $r_i > 0$ at $\tilde{\rho}_i$.

(iv) When $\gamma_i < \kappa_2$ landlords accept $\mu\gamma_i\rho_i$, $0.5\rho_i$, or $0.5(Y_i - \omega L_i)$, whichever is highest without causing $r_i < 0$.

The realised profit rate on land i is then:

$$r_i = \frac{\alpha_i L_i^{\beta} - \tilde{\rho}_i - \omega L_i}{\omega L_i}.$$

Rule 6 explains how rents for the next time period $t+1$ are determined at the end of each simulation round. The simulation then repeats as described in section 3.

Rule 6 (Rent Updates)**.** *At the beginning of $t+1$ landlords announce updated rents based on the outcomes at the end of t. Let $\nu_{i;t} \in (0,1)$. There are three cases:*

(i) If $\gamma_{i;t} = 1$ then $\rho_{i;t+1} = \rho_{i,t}$.

(ii) If $\gamma_{i;t} > 1$ then $\rho_{i;t+1} = (1 + \nu_{i;t})\rho_{i;t}$.

(iii) If $\gamma_{i;t} < 1$ then $\rho_{i;t+1} = (1 - \nu_{i;t}/3)\rho_{i;t}$.

A.2 Supplementary simulation results

[Figure 3 here]

[Figure 4 here]

[Figure 5 here]

Tables

Table 1: Parameters

Parameter	Value
n	10
m	100
α_i	$[50.5, 55]$ in increments of 0.5
β	0.9
η	20
$\bar{\Omega}$	10,000
Ω^c	randomly drawn from a Pareto distribution such that $\sum_{c=1}^{m} \Omega^c = \bar{\Omega}$
$\bar{L}$	500
$\rho_{i;1}$	randomly determined $\pm 10\%$ from equilibrium rents at $t = 0$
κ_1	0.9
κ_2	0.75
μ	2.5
$\nu_{i;t}$	randomly determined between 0.08 and 0.12

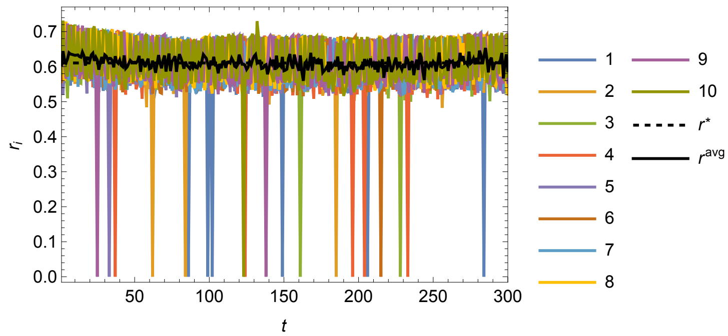
Table 2: Equilibrium

Variable	Lands				
	1	2	3	4	5
L_i^*	31.2841	34.5232	38.0611	41.9221	46.132
Ω_i^*	625.682	690.464	761.223	838.442	922.641
ρ_i^*	111.965	123.558	136.22	150.038	165.106
	Lands				
	6	7	8	9	10
L_i^*	50.7187	55.7118	61.1431	67.0463	73.4576
Ω_i^*	1014.37	1114.24	1222.86	1340.93	1469.15
ρ_i^*	181.521	199.391	218.83	239.957	262.903
r^*	0.61054				

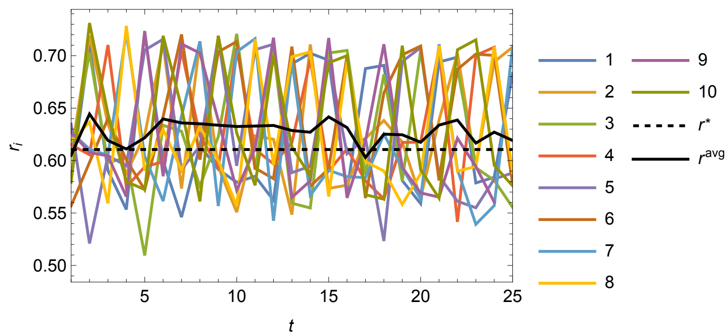
Figures

Figure 1: Sample simulation results - profit rates

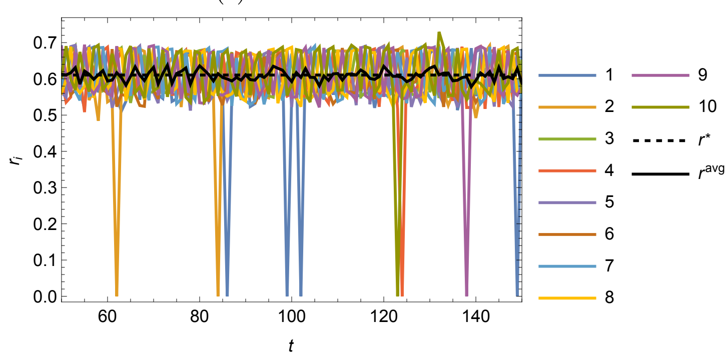
(a) Profit rates for $T = 300$



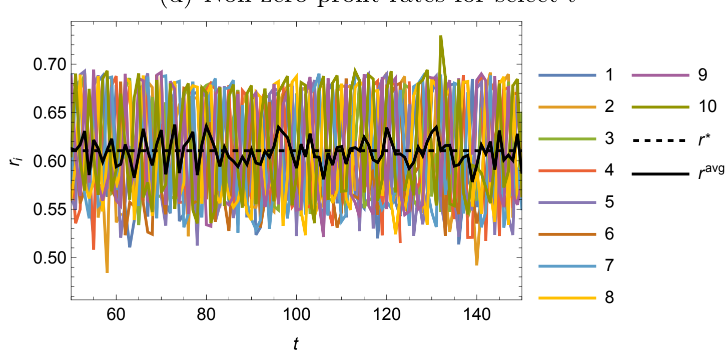
(b) Profit rates for early t



(c) Profit rates for select t



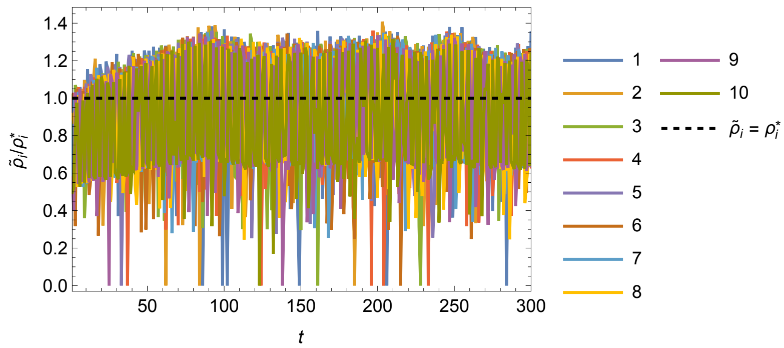
(d) Non-zero profit rates for select t



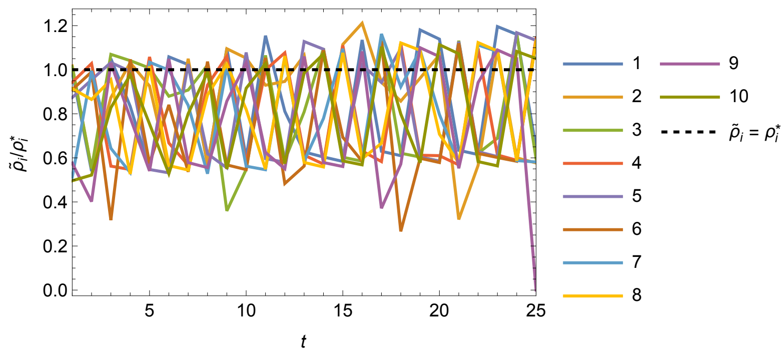
Note: The numbers in each legend correspond to a tract of land. Lands are numbered in order of increasing fertility. Calculation of the average profit rate, r^{avg}, excludes cases where $r_i = 0$.

Figure 2: Sample simulation results - realised rents

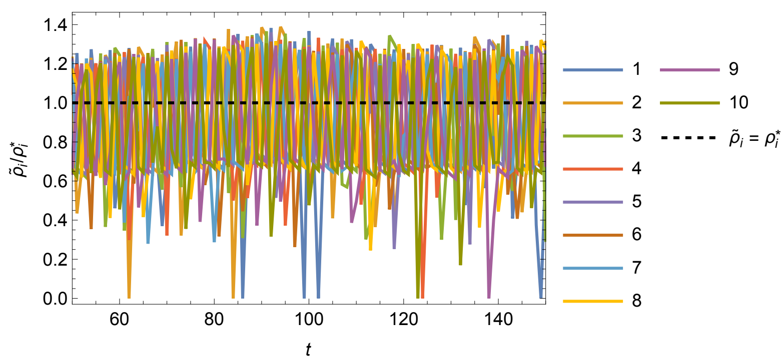
(a) Rents for $T = 300$



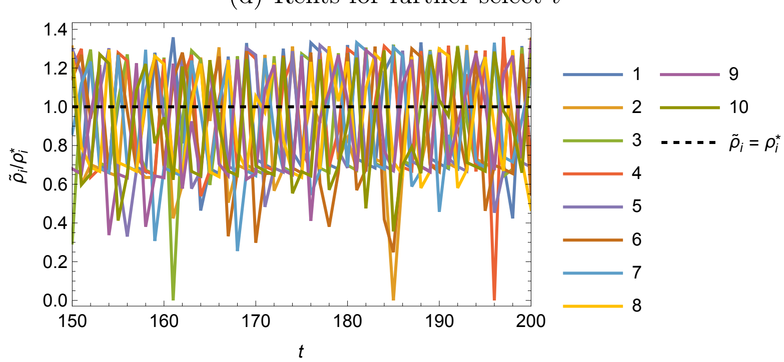
(b) Rents for early t



(c) Rents for select t



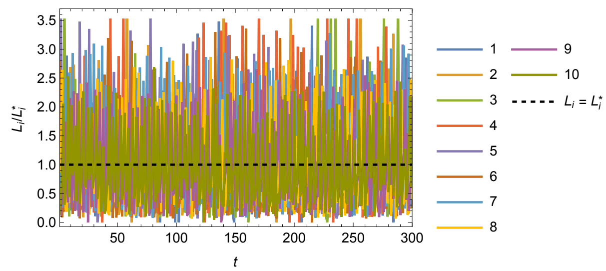
(d) Rents for further select t



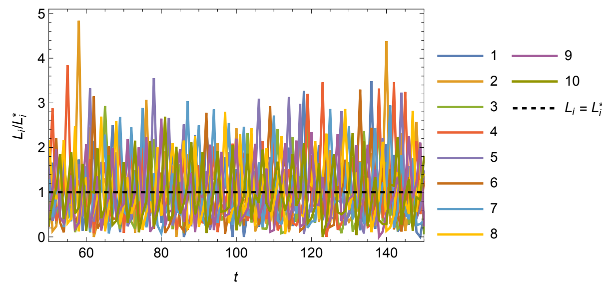
Note: The numbers in each legend correspond to a tract of land. Lands are numbered in order of increasing fertility.

Figure 3: Sample simulation results - labour allocation

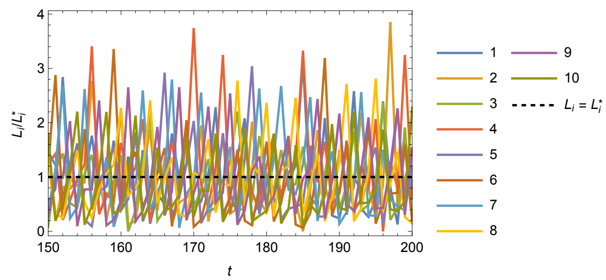
(a) Labour for $T = 300$



(b) Labour for select t



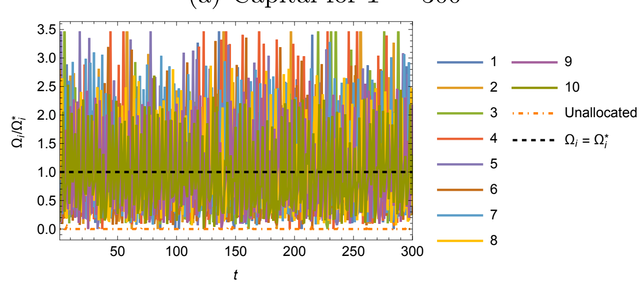
(c) Labour for further select t



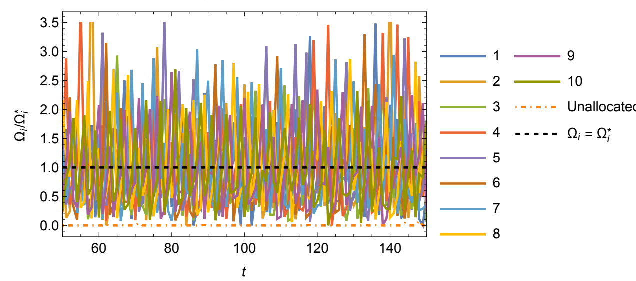
Note: The numbers in each legend correspond to a tract of land. Lands are numbered in order of increasing fertility.

Figure 4: Sample simulation results - capital allocation

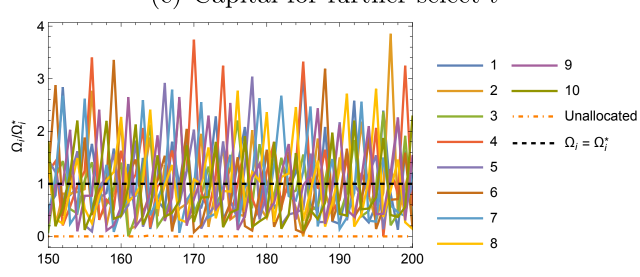
(a) Capital for $T = 300$



(b) Capital for select t



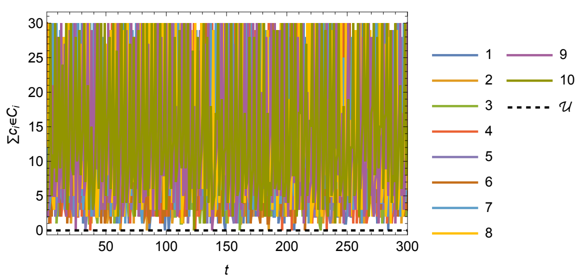
(c) Capital for further select t



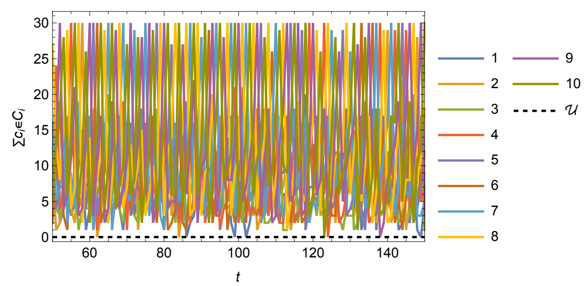
Note: The numbers in each legend correspond to a tract of land. Lands are numbered in order of increasing fertility.

Figure 5: Sample simulation results - coalition headcounts

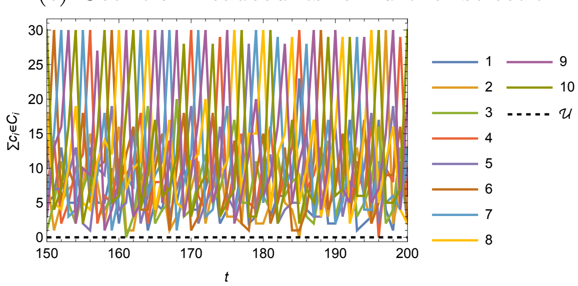
(a) Coalition headcounts for $T = 300$



(b) Coalition headcounts for select t



(c) Coalition headcounts for further select t



Note: The numbers in each legend correspond to a tract of land. Lands are numbered in order of increasing fertility.

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