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Abstract

Does corporate finance literature accurately identify firms facing homogeneous financing constraints when studying the impact of financing constraints on corporate investment? The short answer is no. The common practice of using pre-determined percentiles of a financing constraint metric compromises the validity of conclusions. Our empirical framework identifies four classes of firms facing homogenous financing constraints independently of the financing constraints metric used. Moreover, we show that while popular metrics of financing constraints may capture financing constraints reasonably well, differently from previous studies the sensitivity of investment to cash flow is inverse basin-shaped. We provide an understanding of this shape by studying investment and financial policies jointly, under different regimes of financing constraints.

Keywords: Homogeneous Financing constraints; Sorting scheme; Inverse basin shaped investment–cash flow sensitivity; Interdependence of financial policies.

JEL codes: C13, D25, G30, G31, G32.

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1. Introduction

Despite the critique by Kaplan and Zingales (1997), the sensitivity of investment to cash flow (ICFS hereinafter) continues to be taken as a measure of financing constraints (Bond and Söderbom 2013, Mclean and Zhao 2014, Erel, Jang and Weisbach 2015, Adelino, Lewellen and Sundaram 2015, Lewellen and Lewellen 2016, Mulier, Schoors and Merlevede 2016, Ağca and Mozumdar 2017, Larkin, Ng and Zhu 2018, Sprenger and Lazareva 2022, Liao, Nolte and Pawlina 2023). This because, among other reasons, corporate investment are key determinants of economic growth, and cash flows are a transmission channel for monetary and fiscal policies. Therefore, tracing the ICFS behavior helps predicting the effects of these policies on investment and economic growth (Erickson and Whited 2000, Gomes 2001, Alti 2003, Abel and Eberly 2011, Abel 2018, Wang and Zhang 2021).

When studying the role that financing constraints play on corporate investment, it is key to identify classes of firms that face homogenous financing constraints. Comparing the ICFS across these classes provides a deeper understanding than just focusing on the investment behavior of an average firm. This comparison is crucial for assessing whether, and to what extent, financing constraints influence investment decisions (Fazzari, Hubbard and Petersen, 1988). The first question we ask is therefore: do we adequately identify firms facing homogeneous financing constraints when studying whether financing constraints matter for investment? The short answer is no. The literature typically compares ICFS results based on conventional percentiles of popular *a-priori* financing constraint metrics, such as the top and bottom 30% of observations, or quintiles of the sample (Hovakimian 2009, Farre-Mensa and Ljungqvist 2016). Unfortunately, there is neither theoretical nor empirical reason for relying on these conventional percentiles to identify financing constraints. We find that all conclusions drawn in the literature about the shape of the ICFS depend on seemingly minor empirical choices. The resulting ICFS proves to be either increasing, or decreasing, or take different shapes under any of the popular metrics of financing constraints which we use to sort the sample. Our results show that, because the sample separation points define the number of classes, their size and homogeneity, they are key in determining the conclusion about whether and how financing constraints matter for investment. To the best of our knowledge this is a novel result.

This result raises several further considerations. First, it shows that misidentification of the sample separation point casts doubts on the conclusion about the role of financing constraints for investment, independently of the financing constraint metric used to sort the sample of observations. Since financing constraints are not directly observable, it is difficult to identify classes of homogeneous financing constraints. While the literature has debated only whether the sorting metric is appropriate (Fazzari, Hubbard and Petersen, 2000, Kaplan and Zingales, 2000, Hadlock and Pierce 2010), our results show that the sorting scheme entails more than simply choosing the appropriate metric of financing constraints. Homogeneous classes of financing constraints observations must be identified also by choosing the appropriate number and location of the sample separation points, which are both unknown. Since the sorting scheme is instead generally imposed at the outset of the empirical framework, researchers do not know whether it is sufficiently adequate to capture homogenous classes of financing constraints. Consequently, the resulting ICFS and the inference framework must be treated with caution. The conflicting ICFS results that we find might be due either to the metrics not correlating enough well with the financing constraints or to the sample separation points not adequately identifying classes of homogeneous financing constraints. Alternatively, the ICFS might be genuinely non-monotonic and varying randomly with financing constraints. Hence, the second question we ask is, when studying whether the ICFS increases as financing constraints increase, can we design an empirical framework which is robust to the details of the sorting scheme.

Another consideration is that samples differ across studies. Despite having the common objective of studying the impact of financing constraints on investment, sample sizes, sample periods, sample characteristics and definition of variables vary, and so it does the value of the sorting metrics at the conventional percentiles, that is, the "first 30% of the observations" may identify classes of firms with different size and characteristics. This, in turn, raises questions about the comparisons between results from various articles that support the use of ICFS as a measure of financing constraints and those that argue against it (Cleary 1999; Moyen 2004; Lyandres 2007; Almeida and Campello 2007; Cleary, Povel, and Raith 2007; Hadlock and Pierce 2010). Because the results from previous studies are not easily comparable, a closer examination reveals that the role of financing constraints on investment, if any, remains unclear. To put it simple, "there is no test of the fundamental assumption—implicit in all these tests—that investment-cash flow sensitivities increase monotonically with the degree of financing constraints" (Kaplan and Zingales 1997:170) whose conclusions are independent of the details of the sorting scheme. Once the uncertainty regarding the sorting scheme is properly considered, the null hypothesis should be regarded as a joint hypothesis: that the sorting scheme captures adequately homogenous classes of financing constraints, and/or that the ICFS increases along with financing constraints. Rejection of the joint hypothesis does not necessarily imply rejection of the latter. To reach a conclusion about whether the ICFS increases along with financing constraints, we therefore need evidence about the extent to which the sorting scheme is appropriate. Thus, the second question we ask is, when testing for this joint hypothesis, can the empirical framework also provide information about whether the sample is sorted according to financing constraints.

Third, our result suggests that the conflicting evidence about the various shapes of the ICFS, differently from what the relevant literature suggests, does not depend on the sorting metrics—at least to some extent. Theoretical investment models suggest that this result may arise because the true relation between investment and financing constraints is intrinsically non-monotonic (Moyen 2004, Cleary, Povel and Raith 2007). If this is the case, the ambiguity around the monotonicity of ICFS stems from the model selection. Accordingly, the conflicting theoretical and empirical findings of previous studies may be explained by the different models' assumptions. However, even theoretical models are silent on how to identify homogenous classes of financially constrained firms. Consequently, the empirical evidence is too weak to either support or disprove these models. The third question we ask is, once the empirical framework is made robust to the sorting scheme, can the analysis provide information about whether the ICFS has an underlying shape? If so, why does the ICFS varies with financing constraints?

Answering these questions is key for understanding the impact of financing constraints on investment. Additionally, it will cast doubts about the interpretation of most previous research. We begin by proposing an analysis of the ICFS that explicitly builds on the joint nature of the condition that the sorting metric sorts the sample monotonically with respect to financing constraints and that the ICFS is monotonic. We show that, under this joint condition, the inequality relationship between the sensitivities of two complementary classes of observations must be preserved at any sample separation point. We test this hypothesis using complementary classes of observations at different sample separation points, keeping constant all other empirical details, including the sample size, the estimation model, and the sample period under analysis. We perform this exercise using four popular metrics of financing constraints: the Kaplan and Zingales (1997) metric as developed by Lamont, Polk and Saa-Requejo (2001), as well as those proposed by Whited and Wu (2006), Almeida and Campello (2007) and Hadlock and Pierce (2010). Our approach bypasses all the major uncertainties surrounding the sorting scheme. Remarkably, our methodology obviates the need to choose the best metric of financing constraints, to state whether financing constraints increase or decrease with the sorting metric, to specify the correct number of classes of homogeneous financing constraints, and to identify the correct location of the sample separation points. Given the state of the debate, these are all key advantages. This is our second contribution to the literature.

We report robust and convincing evidence against the joint condition that the ICFS is monotonic and that the sorting scheme sorts the sample monotonically with respect to financing constraints. The evidence shows that this hypothesis is rejected under popular metrics of financing constraints. The analysis also suggests that there are three regimes of homogeneous financing constraints. The true splitting points differ according to the metric adopted to sort the sample. However, they are all different from terciles or quintiles or other fixed percentiles. Our conclusions are robust not only to the details of the sorting scheme, such as the sorting metric and the location and number of sample separation points, but also to alternative definitions of the variables under analysis, to measurement errors, to the sample period, to the model's specifications, to whether we balance the sample, and to whether we restrict the sample to positive cash flow observations. This is our third contribution to the literature.

Since our findings reject the joint null hypothesis, we check whether the monotonicity condition of the financing constraints metric is satisfied. To this aim we build on the literature, which suggests that all metrics convey some information about financing constraints. Since we are only interested in investigating whether the metrics are monotonic with respect to financing constraints, we test for pairs of complementary classes of observations whether each sorting metric monotonically sorts all other metrics and several firm characteristics that the literature associates with financing constraints. This approach builds on milder assumptions with respect to the practice of imposing at the outset of the empirical framework the best metric of financing constraints, the number of homogenous groups of observations, and the location of the sample separation points. Results suggest that each metric sorts monotonically all others: observations that are more (or less) constrained according to Hadlock and Pierce (2010) metric are also more (or less) constrained according to the metrics by Lamont, Polk and Saa-Requejo (2001), Whited and Wu (2006), and Almeida and Campello (2007). Moreover, the metrics sort several other variables that the literature uses to infer financing constraints. We take this result as sufficient evidence to support the hypothesis that the metrics are monotonic with respect to financing constraints. Consequently, we reject the monotonicity of the ICFS. This constitutes our fourth contribution to the literature.

The finding that the ICFS behaves consistently across all financing constraints metrics calls into question the interpretation of most of the literature's results. It challenges the conclusion that the different ICFS shapes result from the use of different sorting metrics. We ask whether the ICFS has an underlying shape that encompasses all those reported in the literature. To answer this question, we generalize the semi-parametric threshold model proposed by Hansen (1999, 2000) to the case of non-constant slope parameters in each regime of financing constraints. Our results indicate that ICFS behaves differently depending on the regime. In the first regime, it is monotonically increasing; in the second, it is constant; and it is monotonically decreasing in the third regime, where the ICFS is eventually negative for sufficiently high financing constraints. We therefore conclude that the true underlying ICFS is inverse basin shaped independently of the sorting metric. To the best of our knowledge, this conclusion is novel to the literature and represents our fifth contribution to the field.

Finally, we investigate why the ICFS exhibits an inverse basin shape. To do this, we consider corporate financial decisions as interdependent. Decisions regarding how to finance investment are part of the company's broader financial policy, which involves determining what portion of each additional dollar of cash flow should be allocated to investment, what portion should be retained as cash reserves, what portion should be distributed as cash dividends, and what portions should be used to repurchase debt and equity (Gatchev, Pulvino, and Tarhan, 2010; Chang, Dasgupta, Wong, and Yao, 2014). This approach helps in understanding how firms allocate cash flows across various financial policies. While it is somewhat acknowledged that financial policies are interdependent— as highlighted by the originators of this debate— financing constraints depend on dividends, as noted by Fazzari, Hubbard, and Petersen (1988), and on cash reserves and leverage, according to Kaplan and Zingales (1997). Despite this recognition, investment decisions have been studied somewhat surprisingly in isolation, as if they are mere residuals.

Results from estimating the five equations for investment, cash holdings, dividends, net debt issuance, and net equity issuance indicate that, in the first regime where financing constraints are low, firms primarily allocate additional cash flow to investment. In this scenario, the sensitivities of cash holdings, dividends, and debt and equity issuance to cash flow are negative, suggesting that firms prioritize investment over liquidity, dividends, or debt reduction. In the second regime, characterized by higher financing constraints, firms redirect cash flow toward other uses, such as increasing cash reserves or issuing debt and equity, reflecting the tightening of their constraints. In the third regime, marked by more severe financial constraints, the sensitivities of investment, cash holdings, and financing policies to cash flow all become negative. Firms are more likely to delay investment, reduce liquidity accumulation, and refrain from issuing debt or equity.

Our results are of interests to researchers and policymakers. The shape we document encompasses all the shapes the literature proposes, as all of them can be obtained by imposing further restrictions to the estimating model. Therefore, our findings help to resolve the seemingly conflicting results in the literature. The inverse U-shape ICFS is the result we get if a quadratic structure is parametrically imposed when designing the estimating model. However, this shape does not inform about observations having a constant ICFS and located at about the median values of the sorting metrics. On the other hand, the inverse U-shape, as proposed by Hansen (1999), does not inform about the non-constant ICFS for observations facing either low or high financing constraints in the first and third regime.

Our findings, finally, provide policymakers with insights about the investment's response to cash flow within the three regimes we identify. In the first regime, firms are relatively financially unconstrained and additional cash flow increases investment's response to cash flow. In the second regime, the ICFS is the highest. Finally, in the third regime where firms are the most financially constrained, additional cash flow reduces both investment's response to cash flow and the investment level.

2. Points of sample separation and classes of financing constraints

2.1. The empirical context

The study of corporate investment decisions occupies a prominent place in macroeconomics, microeconomics, and corporate finance literature. This literature is driven both by debates over which model offers the best explanation of firm investment and by policy questions about how monetary and fiscal policies affect investment. The finance literature has extended the conventional models of fixed investment to incorporate the role of financing constraints. In these models, financing constraints arise from asymmetric information and incentive problems in capital markets that drive a wedge between the cost of internal and external finance making investment dependent on internal funds.

To identify the impact of financing constraints on investment, hence, the literature investigates the extent to which cash flow matters for investment. The approach entails sorting the sample of firm-year observations according to a given metric of financing constraints, splitting the sample into *J* classes of homogenous financing constraints and, for each class, estimating the following equation separately:

$$(1)\quad \left(\frac{I}{K}\right)_{ijt} = \alpha_j Q_{ijt} + \beta_j \left(\frac{Cash\ flow}{K}\right)_{ijt} + \mu_{ij} + \tau_t + \varepsilon_{ijt}.$$

In Equation (1), $j=1,...,J$ denotes the class of financing constraints to which firm $i=1,...,n$ belongs at time $t=1,...,T$ (n and T are the sample's cross-section and time dimensions, respectively), I is investment, K is capital, and Q is Tobin's Q. The conclusion about the shape of the ICFS with respect to financing constraints is reached by comparing the parameters for each class, β_j. The working assumption is that ICFS increases monotonically across the J classes—the so-called monotonicity condition. If this condition holds, then the estimated parameters $\hat{\beta}_J$ should increase across the J classes of observations.

Fazzari, Hubbard and Petersen (1988), Hoshi, Kashiap and Scharfstein (1991), Oliner and Rudebusch (1992), Fazzari and Petersen (1993), Bond and Meghir (1994), Calomiris and Hubbard (1995), Gilchrist and Himmelberg (1995), Lamont (1997) among others provide substantial evidence in favor of this condition and take the ICFS as a measure of financing constraints. Kaplan and Zingales (1997) challenge it. They argue that the monotonicity condition is not theoretically grounded, and show that—if sorted by cash, leverage, and management statements—the most constrained subsample in Fazzari, Hubbard and Petersen (1998) exhibits a decreasing rather than an increasing ICFS.

Despite Kaplan and Zingales (1997) puts the framework under question, most of the subsequent studies find that the ICFS increases along financing constraints when they are measured by metrics such as firm size, age, dividends, free cash flow, bond rating and the Kaplan and Zingales (1997), Whited and Wu (2006) and Hadlock and Pierce (2010) indexes (see, among others, Lewellen and Lewellen 2016, Ağca and Mozumdar 2017, Larkin, Ng and Zhu 2018, Sprenger and Lazareva 2022, Liao, Nolte and Pawlina 2023). Few papers offer some mixed empirical evidence. The ICFS is decreasing if the sample is sorted by the probability of paying dividends (Cleary 1999); it is inverse U-shaped when firm size, age, and tangibility are used as sorting metrics (Almeida and Campello 2007, Hadlock and Pierce 2010), and it is U-shaped when using firm age and dispersion of earnings forecasts as metrics of financing constraints (Lyandres 2007).

2.2. Sample separation points and the impact of financing constraints on investment

Conclusions about ICFS's monotonicity are robust only if the sorting metric is monotonic with respect to financing constraints and the sample separation points identify classes of homogenous financing constraints, that is, if the sorting scheme is adequate. Unfortunately, it is difficult to identify whether the sorting scheme is successfully determined because financing constraints are not directly observable. Several metrics have been proposed for sorting the sample according to the degree of financing constraints, and an intense debate is ongoing about their ability to capture financing constraints (Fazzari, Hubbard and Petersen 2000, Kaplan and Zingales 2000, Hennessy and Whited 2007, Hadlock and Pierce 2010, Hoberg and Maksimovic 2015, Farre-Mensa and Ljungqvist 2016, Buehlmaier and Whited 2018). Sorting schemes, including the sample separation points, are therefore imposed at the outset without knowing whether the schemes sort the sample monotonically with respect to financing constraints or whether they identify groups of firms facing homogenous financing constraints. As a result, the validity of the conclusion about monotonicity of ICFS is questionable.

We illustrate our argument with an empirical exercise. We use a sample whose composition, size, variables, and period closely follow most of the literature, as detailed in Appendix 1. We adopt as metrics of financing constraints the Kaplan and Zingales (1997), Withed and Wu (2007), and Hadlock and Pierce (2010) indexes—KZ, WW, and HP, respectively. We generate four different sorting schemes (i.e., Schemes 1–4) for each metric, each featuring three classes of observations. These schemes differ in that, given the metric we adopt to sort the sample, the classes are defined by different *ad hoc* sample separation points. We then estimate Equation (1) for each of the three classes and compare ICFS. Table 1 reports the results. The results show that when HP is used as a sorting metric, under Scheme 1, where $HP \leq -3.61$, $-3.61 < HP \leq -3.33$, and $HP > -3.33$, the ICFS increases from Class 1 to Class 2 and from Class 2 to Class 3. In contrast, in Scheme 2, where $HP \leq -2.65$, $-2.65 < HP \leq -2.09$, and $HP > -2.09$, ICFS decreases from Class 1 to Class 2 and from Class 2 to Class 3. In Scheme 3, where $HP \leq -3.28$, $-3.28 < HP \leq -2.52$, and $HP > -2.52$, ICFS increases from Class 1 to Class 2 and decreases from Class 2 to Class 3. Finally, in Scheme 4, where $HP \leq -2.65$, $-2.65 < HP \leq -2.55$, and $HP > -2.55$, ICFS decreases from Class 1 to Class 2 and increases from Class 2 to Class 3. The differences in the parameter values between the adjacent classes are always statistically significant. Strikingly, the evidence is similar if the sample is sorted using the schemes that build upon KZ and WW.

Insert Table 1

The above results lead to four conclusions. First, the literature has shifted from the study of ICFS given the metric of financing constraints to its analysis under alternative metrics. However, the sorting schemes that this literature adopts build on different sorting metrics and on other empirical details, such as the sample separation points and the value of the sorting metric at terciles, for example. Our results show that the conclusions crucially depend on such seemingly unimportant details. Therefore, it is risky to compare the results from Scheme 1 under the HP metric with those from Scheme 2 under the KZ metric because both the metrics and the sample separation points differ. In addition, the values of the sorting metrics at terciles differ because studies adopt different sample sizes and sample periods, thus casting further doubts on the comparability of the results the various studies offer.

The second conclusion is that, different from the conventional wisdom, the different ICFS shapes are not necessarily due to the adoption of different sorting metrics. Indeed, we obtain any shape of the ICFS proposed by the literature independently of the metric. The sample separation points define class composition and class size, which are imposed at the outset of the empirical framework. This approach again casts doubt on the robustness of the conclusion about monotonicity of the ICFS. The findings leave doubt as to whether the parameter's variation across classes is the result of the average financing constraints being different across classes, of a misidentified number of classes, of the presence of misclassified observations in some classes, of the sorting metric not being correlated to financing constraints, or of the ICFS being genuinely non-monotonic.

The third conclusion is that the evidence does not necessarily show that ICFS is non-monotonic. Across all financing constraints metrics, under Scheme 1, because the ICFS increases along with the financing constraints, the evidence supports the conclusion by Fazzari, Hubbard and Petersen (1988) and much of the subsequent literature. In this case, the ICFS can be taken as a measure of the financing constraints. Under Scheme 2, because the ICFS decreases as financing constraints increase, the evidence supports the conclusion reached by Kaplan and Zingales (1997) and Cleary (1999). In this case, ICFS is not a measure of the financing constraints. However, because it is monotonic, the discussion should revolve around whether financing constraints increase or decrease across classes. In the latter case, the ICFS could still be taken as a measure of financing constraints. Under Schemes 3 and 4 the ICFS is non-monotonic and therefore it should not be taken as a measure of the financing constraints. However, because we have no information on which, if any, of the sorting schemes is appropriate the results do not necessarily reject ICFS's monotonicity.

The last conclusion is that, once the uncertainty surrounding the sorting scheme is considered, the results should provide information about whether the following joint condition holds: the sorting scheme sorts the sample adequately with respect to financing constraints, and the ICFS is monotonic. Rejection of monotonicity would require evidence also about whether the schemes sort the sample according to the financing constraints.

3. The impact of financing constraints on investment: A robust framework

3.1. From unobservable marginal ICFS to observable average ICFS

The sorting scheme consists of the metric of financing constraints, $\lambda_m(k)$, the number of classes J, and the positions of the $J-1$ sample separation points. The literature is silent about the appropriate number and location of the sample separation points. These empirical details are unimportant if the metric monotonically sorts the sample with respect to the financing constraints and if ICFS is monotonic. However, they are crucial if this joint condition does not hold. This is the starting point for our analysis. We identify a systematic relationship between the metric's monotonicity with respect to financing constraints, ICFS's monotonicity, and the direction of the inequality between the ICFS of two complementary classes of observations. We show that if the metric sorts the sample monotonically with respect to the financing constraints and ICFS is monotonic, then the position of the sample separation point *s* does not affect the direction of the inequality between the ICFS for the first *s* observations and that for the remaining *n-s* observations. Conversely, if the direction of the inequality depends on the position of *s*, the metric is not monotonic with respect to the financing constraints and/or ICFS is non-monotonic.

To illustrate our argument, let the observations be sorted by the metric $\lambda_m(k)$, where *m=1,…,M*. Assume that the metric monotonically increases as financing constraints k increase

(2) $\lambda_m(k)_1 < \cdots < \lambda_m(k)_i < \cdots < \lambda_m(k)_n$,

and ICFS parameter β_i increases monotonically across the sorted observations *i =1,…,n*

(3) $\beta_1 \leq \cdots \leq \beta_i \leq \cdots \leq \beta_n$.

Let $1 \leq s \leq n$ be the sample separation point. If Equations (2) and (3) hold, then the condition

(4) $$\frac{\beta_1 + \cdots + \beta_s}{s} \leq \frac{\beta_{s+1} + \cdots + \beta_n}{n-s},$$

holds independently of the sample separation point *s*, because β_s is the upper bound for the average value of parameters β_i located in the first *s* positions. If Equation (4) holds true for any *s*, then β_s cannot be greater than β_{s+1} because the latter is the lower bound for the average value of parameters β_i in the remaining *n-s* positions:

(5) $$\frac{\beta_1 + \cdots + \beta_s}{s} \leq \frac{s\beta_s}{s} = \beta_s \leq \beta_{s+1} = \frac{(n-s)\beta_{s+1}}{(n-s)} \leq \frac{\beta_{s+1} + \cdots + \beta_n}{n-s}.$$

For Equation (4) to hold with equality, regardless of s, all the β_i parameters must be equal. However, the evidence in Table 1 suggests that equality does not hold. Hence, strict inequality holds:

$$(6) \quad \frac{\beta_1 + \cdots + \beta_s}{s} < \frac{\beta_{s+1} + \cdots + \beta_n}{n-s}.$$

The analysis builds on a relationship between the conditions in Equations (2) and (3), the two average ICFS parameters in Equation (6), and the position of the sample separation point s. This eliminates the need to decide both how many classes to generate and the position of the sample separation points, that according to the results reported earlier, are risky decisions. If the evidence contradicts Equation (6), because the direction of the inequality depends on the location of s, either Equation (2) is false, or Equation (3) is false, or both are false. This result makes clear that a rejection of Equation (6) does not necessarily imply a rejection of ICFS's monotonicity. Indeed, if the condition in Equation (6) does not hold, it is difficult to distinguish whether ICFS's monotonicity is rejected because of a genuine violation of the monotonicity condition in Equation (3) or because of a violation of Equation (2).

Equation (2) is violated if the sorting metric is not monotonic with respect to the financing constraint. Obviously, the metrics are correlated to the unobservable financing constraints: all of them build on firm-specific characteristics that correlate with the cost of or need for external finance (Hoberg and Maksimovic 2015, Bodnaruk, Loughran, and McDonald 2015, Farre-Mensa and Ljungqvist 2016). It is debated whether the sorting metric measures the financing constraints directly or inversely. For example, Whited (1992) and Kaplan and Zingales (1997) argue that debt is a direct measure of financing constraints as firms with low debt have large debt capacities that make it easier for them to obtain external funds when needed. In contrast, Calomiris and Hubbard (1995) and Fazzari, Hubbard, and Petersen (2000) argue that debt is an inverse measure of financing constraints as firms with low debt are those that cannot convince lenders to provide them with credit, perhaps due to the lack of collateral. Similarly, the literature does not agree on how cash should measure financing constraints, with some arguing that cash is a direct measure of financing constraints (Calomiris and Hubbard 1995, Fazzari, Hubbard and Petersen 2000) and others taking cash as an inverse measure of financing constraints (Kashyap, Lamont, and Stein 1994, Kaplan and Zingales 1997, Cleary 1999). A similar argument holds for size, where Devereux and Schiantarelli (1990) suggest that large (vs. smaller) firms are likely to be more financially constrained due to agency problems, whereas Whited and Wu (2006), Hennessy and Whited (2007), and Hadlock and Pierce (2010) suggest the opposite. Therefore, whether the different metrics correlate to financing constraints is not at debate. The debate centers on *how well* the metrics correlate to financing constraints. However, because financing constraints are unobservable, this evidence is difficult, if not impossible, to provide. Our approach does not need to state the strength of this correlation. Instead, we require evidence about whether the metric sorts the observations monotonically. Therefore, we build on the view that all metrics convey information about financing constraints, and we check whether they are monotonically correlated with each other. We do so by testing whether the most popular metrics of financing constraints sort monotonically one another, and various firm characteristics commonly associated with the presence of financing constraints. Such a monotonic relationship between each metric and financing constraints will lead to a genuine rejection of the monotonicity reported in Equation (3).

Specifically, we assume that if Equation (2) holds for metric *m*, then the following condition holds for all metrics other than *m*:

$$(7) \quad \lambda_{\neq m}(k)_1 \leq \cdots \leq \lambda_{\neq m}(k)_i \leq \cdots \leq \lambda_{\neq m}(k)_{n.}$$

Similarly to the analysis that we have proposed for the ICFS, we test whether the following inequality is preserved at the sample separation points used when deriving evidence for Equation (6):

$$(8) \quad \frac{\lambda_{\neq m}(k)_1 + \cdots + \lambda_{\neq m}(k)_s}{s} \leq \frac{\lambda_{\neq m}(k)_{s+1} + \cdots + \lambda_{\neq m}(k)_n}{n-s}.$$

Our framework has the advantage of not requiring knowledge about which sorting metric is the best metric for financing constraints, which is an unresolved question (Farre-Mensa and Ljungquist 2016). What we borrow from the literature is the assumption that all metrics convey information about financing constraints (Fazzari, Hubbard and Petersen 1988, Oliner and Rudebusch 1992, Whited 1992, Bond and Meghir 1994, Kaplan and Zingales 1997, Cleary 1999, Whited and Wu 2006, Almeida and Campello 2007, Lyandres 2007, Hadlock and Pierce 2010, Hoberg and Maksimovic 2015, Buehlmaier and Whited2018). This is a milder approach than imposing all the discussed empirical details at the outset of the empirical framework.

The proposed methodology also has the advantage of not needing to state whether financing constraints increase or decrease as the sorting metric increases. This is key in the context of the ongoing debate about whether the metrics increase or decrease with financing constraints (Fazzari, Hubbard and Petersen 2000, Kaplan and Zingales 2000). Our conclusion holds even if the true ICFS parameter, β_i, monotonically decreases across *i* observations. Specifically, let:

(9) $\lambda_m(k)_1 < \cdots < \lambda_m(k)_i < \cdots < \lambda_m(k)_n$ and $\beta_1 \geq \cdots \geq \beta_i \geq \cdots \geq \beta_n$,

hold for any sample separation point *s*. In this case, the average of parameters β_i for the first *s* observations is greater than the average of parameters β_i for the remaining *n-s* observations:

(10) $$\frac{\beta_1 + \cdots + \beta_s}{s} > \frac{\beta_{s+1} + \cdots + \beta_n}{n - s}.$$

If the direction of the inequality in Equation (10) changes with the position of *s*, then Equation (9) is false. However, the conditions stated in Equations (2) and (3) would also be rejected. Similarly, if the direction of the inequality in Equation (6) changes with the position of s, then Equations (2) and (3) are false and so is Equation (9). Therefore, if the direction of the inequality between the two ICFS values changes with *s*, the joint condition in Equations (2) and (3) and the condition reported in Equation (9) are false, independent of whether the analysis builds on the condition in Equation (6) or (10). This result implies that we do not need to state whether the sorting metric increases or decreases with the financing constraints. Therefore, our approach bypasses much of the debate concerning whether the financing constraints increase or decrease with the sorting metric.

The third key advantage is that the conclusions based on Equation (6) also hold if, instead of having *n* ICFS parameters, we have *J* classes for which the ICFS parameter β_i is homogeneous within each class but different across the *J* classes:

(11) $$\underbrace{\beta_1 = \cdots = \beta_1}_{n_1} < \cdots < \underbrace{\beta_j = \cdots = \beta_j}_{n_j} < \cdots < \underbrace{\beta_J = \cdots = \beta_J}_{n_J},$$

where n_j is the number of observations with the same ICFS parameter. If this is the case, Equations (6) and (10) both hold with strict inequality. This result shows that the conclusion about ICFS's monotonicity, whether built on Equation (6) or (10), does not require information about the true number of classes of homogenous financing constraints and the correct positions of sample separation points. Both the number and the correct position of s are unknown. In turn, they define the number of classes and the class's numerosity, both key details.

Finally, we can observe how the behavior of the two average ICFSs change as only s varies. We identify each class and its complementary class of observations by using two dummy variables and estimate the model using the entire sample of observations. This approach makes the resulting ICFS independent of the class size, so the results are less sensitive to fluctuations due to the possible presence of outliers or misclassified observations. Therefore, our conclusion is robust to all details of the sorting scheme: the empirical framework builds on the entire sample without needing to specify whether the financing constraints increase or decrease with the metric, the best metric, the correlation direction between the metric and the true unobservable financing constraints, or the true number and positions of the sample separation points.

3.2. Testing strategy

Our empirical strategy is as follows:

1. Adopt one metric of financing constraints, $\lambda_m(k)_1$, to sort the sample of firm–year observations.

2. Assume that $\beta_1 \leq \cdots \leq \beta_i \leq \cdots \leq \beta_n$ hold.

3. Select sample separation point s corresponding to the bottom 5% of the sample of firm–year observations, and estimate the average ICFS parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$ using the following model:

$$(12) \quad \left(\frac{I}{K}\right)_{it} = \alpha Q_{i,t} + \gamma_s D_s + \bar{\beta}_s D_s \left(\frac{Cash\,flow}{K}\right)_{it} + \bar{\beta}_{n-s} D_{n-s} \left(\frac{Cash\,flow}{K}\right)_{i,t} + \mu_i + \tau_t + \varepsilon_{it} ,$$

   where D_s and D_{n-s} are the dummy variables for the two classes s and n-s, respectively.

4. Test hypothesis H_0: $\bar{\beta}_s = \bar{\beta}_{n-s}$ *vs* H_A: $\bar{\beta}_s \neq \bar{\beta}_{n-s}$. Repeat the test with s corresponding to 10, 15, 20, …, 95% of firm–year observations. Figure 1 illustrates the 19 sample separation points. If the direction of inequality between $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$ is preserved for all sample separation points s, then the

conditions under Step 2 hold; if not, the null hypothesis is rejected.

Insert Figure 1

3.3. Empirical results

Table 2 reports the estimation results for the model in Equation (12) using the full unbalanced sample of 93,107 firm–year observations. The observations are sorted by the HP index of financing constraints. As in previous studies, we use a fixed effects estimation to account for unobserved relationships between investment and the independent variables as well as to capture business-cycle effects (Cleary 1999). We use the end-of-year market-to-book ratio to proxy for Tobin's Q. Fazzari, Hubbard, and Petersen (1988) and Kaplan and Zingales (1997) argue that to proxy for investment opportunities, year-end market values are preferable to beginning-of-year market values because the former can capture information about investment opportunities arriving in the current period that is not captured in beginning-of-year market values. Column (a) reports the point of sample separation s as a percentage of firm–year observations. Column (b) shows coefficient γ_s associated with class of observations $i \leq s$. Column (c) shows coefficient α associated with the market-to-book ratio. Columns (d) and (e) report ICFS coefficients $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$ for classes of observations $i \leq s$ and $i > s$, respectively. Heteroskedasticity-consistent standard errors are reported in parentheses. Column (f) shows the adjusted R^2. Column (g) displays the F-statistic associated with the null hypothesis of equality of parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$.

Under the HP index, coefficient α for the market-to-book ratio is positive and statistically significant for all classes. Our results show that for sample separation points between 5% and 30% of the sample, the test rejects the equality of parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$. For sample separation points between 35% and 80% of the sample, the test does not reject the equality of parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$. This evidence rejects the conditions in Equations (6) and (10). This result is reinforced by the finding that for sample separation points between 85% and 95% of the sample, the average ICFS of the lower class, $\bar{\beta}_s$, is statistically significantly greater than that of upper class, $\bar{\beta}_{n-s}$. One potential concern associated with our procedure is that, if the observations across different sample separation points are the same, then the statistics for the different thresholds are not independent. This problem arises when adopting multiple testing procedures. If the joint null hypothesis is rejected at the first sample separation point, the null should not be rejected at another sample separation point. For this reason, Column (i) reports the Bonferroni-adjusted *p*-values associated with the tests. The results confirm our conclusions.

Insert Table 2

The conclusion we reach is independent of the sorting metric. Hereafter, for brevity and without loss of generality, we report results only for some sample separation points. Full results are available upon request. The evidence reported in Panel A of Table 3 confirms the rejection of the claims in both Equations (6) and (10) when the observations are sorted using the metrics by KZ, WW, and Almeida and Campello (2007) (AC hereafter).

Following prior studies, we perform several tests of the robustness of the non-monotonicity finding. Researchers studying the ICFS behavior split the sample into classes of financing constraints and estimate the ICFS of each class. For each class, the model in Equation (12) simplifies to the model in Equation (1). Similarly, we estimate the model in Equation (1) for different classes and test for differences in parameters. Estimation results are reported in Table 3, Panel B. The χ^2 test for equality of the ICFS parameters rejects the monotonicity condition under analysis.

Insert Table 3

Second, we run the exercise using the beginning-of-year market-to-book ratio to proxy for Tobin's Q. Third, we check whether our finding of non-monotonicity is robust to measurement errors in Tobin's Q by using a GMM estimator. Fourth, we adopt gross capital stock as a scaling variable to scale investment and cash flow, as suggested by Erickson and Whited (2012). Fifth, we estimate the model both for a restricted sample of observations with positive cash flow and for the balanced sample to check whether our results are affected by firms' financial health. Sixth, because our main sample period includes the 2008 global financial crisis, we check whether our conclusion holds for the observations before and after this event. Finally, we check whether the results depend on the model's specifications. None of these robustness checks affect our conclusion about the joint hypothesis under examination, which remains robust and compelling. Details of these tests are in Appendix 2.1.

4. The analysis of the ICFS's as a joint condition

4.1. Taking the uncertainty about the sorting metric into consideration

At a close look, whether the different metrics correlate to financing constraints is not debated. The debate centers on how well the metrics correlate to financing constraints. However, because financing constraints are unobservable, this evidence is difficult, if not impossible, to provide. Our approach does not need to state the strength of this correlation. Instead, we require evidence about whether the metric sorts the observations monotonically. Therefore, we build on the view that all metrics convey information about financing constraints, and we check whether they are monotonically correlated with each other. We do so by testing whether the most popular metrics of financing constraints sort monotonically one another, and various firm characteristics commonly associated with the presence of financing constraints. Such a monotonic relationship between each metric and financing constraints will lead to a genuine rejection of the monotonicity reported in Equation (3).

4.2. Monotonicity of the metrics with respect to the financing constraints

To investigate whether the metrics adopted to sort the sample are monotonic with respect to financing constraints, we begin by sorting the sample of observations using the HP metric. Then, we test the hypothesis that, for all other metrics of financing constraints, the estimated mean of the first 5% of ordered observations is statistically significantly different from the estimated mean of the remaining 95%. We then repeat this exercise at every 5% increments up to 95% of the observations. As reported in Table 4, Panel A, the results show that the WW, KZ, and AC metrics monotonically increase along with HP. We repeat these exercises to investigate whether the KZ, WW, and AC metrics also sort one another monotonically. We find that the three metrics of financing constraints are all mutually and monotonically correlated to one another. The estimation results are reported in Panels B, C, and D. This evidence supports the hypothesis that these common metrics of financing constraints sort each other monotonically, which increases our confidence that they are monotonically correlated with unobservable, underlying financing constraints.

Insert Table 4

To provide further supporting evidence that the metrics are monotonically correlated with the financing constraints, we check whether each metric monotonically sorts several firm characteristics that are used in the literature to infer the presence of financing constraints. These include firm size, age, cash, coverage ratio, debt, equity, and dividends. Our results for the HP metric, reported in Table 5, suggest that all firm characteristics move monotonically with the HP metric, and similar results hold for the other metrics, see Appendix 2.2. This evidence suggests that the rejection of Equation (6) is not caused by a non-monotonic relationship between the sorting metric and the financing constraints, and that ICFS's monotonicity is genuinely violated under all sorting metrics.

Insert Table 5

5. The shape of the ICFS

5.1. Has the ICFS a common underlying shape?

The evidence points to the existence of three regimes of homogenous financing constraints. ICFS's monotonicity is rejected if we consider the entire sample. However, it is not rejected if we consider the three regimes separately. Regime 1 is defined by $\lambda_m(k) \leq s_{1m}$, with s_{1m} being the sample separation point at which the ICFS parameters for the first s_{1m} observations are statistically lower than the ICFS parameters for the remaining $n - s_{1m}$ observations. Moreover, as the sorting metric increases, the difference between the ICFS parameters decreases, i.e., ICFS monotonically increases. Regime 2 is defined by $s_{1m} < \lambda_m(k) \leq s_{2m}$, with s_{2m} being the sample separation point at which the ICFS parameters for the first s_{2m} observations are consistently higher than the ICFS parameters for the complementary $n - s_{2m}$ observations. In this regime, the ICFS parameters are equal, i.e., ICFS is constant. Regime 3, finally, is defined by $\lambda_m(k) > s_{2m}$. As the metric increases, the difference between the ICFS parameters of the two subsamples increases, i.e., ICFS is monotonically decreasing. Therefore, these results suggest that within each of the three regimes, ICFS is monotonic.

Building on this evidence, we assume that ICFS is a function $f(.)$ of the metric

$$(13) \quad \frac{\mathrm{d}(I/K)}{\mathrm{d}(Cash\,flow/K)} = f(\lambda_m(k)),$$

whose shape must be estimated. Integrating Equation (13) over $Cash\,flow/K$ yields

$$(14)\quad \frac{I}{K} = \int f(\lambda_m(k))\,\mathrm{d}\left(\frac{Cash\,flow}{K}\right) = const + f(\lambda_m(k))\left(\frac{Cash\,flow}{K}\right),$$

where $const$ is the constant of integration. To estimate the ICFS's shape, we adopt a parsimonious semi-parametric approach that extends the threshold model proposed by Hansen (1999, 2000) to the case of non-constant parameters in regimes. This approach allows us to model the potential heterogeneity of the slope parameters. We assume that:

$$(15)\quad f(\lambda_m(k)) = \beta_1 \times h(\lambda_{mit}, \theta)\,,$$

where β_1 and θ are vectors of the parameters, λ_{mit} is the empirical counterpart of threshold variable $\lambda_m(k)$, and $h(\lambda_{mit}, \theta)$ is the transition function, defined as follows:

$$(16)\quad h(\lambda_{mit}, \theta) = \begin{cases} 1\ if\ \lambda_{mit} \leq s_{1m} \\ 0\ if\ \lambda_{mit} > s_{1m} \end{cases},$$

where s_{1m} is the first threshold or location parameter, i.e., the position of the first sample separation point. If the transition function is binary, we have different *J* regimes for the slope parameters that depend on the threshold variable and the *J−1* location parameters, $s_{1m}, \dots, s_{J-1m}$.

Because the sample separation points are predetermined, this approach is helpful because it allows us to estimate the threshold model using the within-group estimator. Therefore, we define:

$$(17)\quad f(\lambda_{mit}) = (\beta_1 + \delta_1 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \leq s_{1m})} + (\beta_2 + \delta_2 T\lambda_{mit})\mathbb{I}_{(s_{1m} < \lambda_{mit} \leq s_{2m})} + (\beta_3 + \delta_3 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} > s_{2m})},$$

where $\mathbb{I}_{(.)}$ is the indicator function and s_{1m} and s_{2m} are the threshold parameters as defined above. To ensure that the signs and magnitudes of the interaction parameter estimates are comparable, we adopt the following monotonic transformation of the financing constraints metric:

$$(18)\quad T\lambda_{mit} = [\lambda_{mit} - Min(\lambda_{mit})]/[Max(\lambda_{mit}) - Min(\lambda_{mit})]\,,$$

where $0 \leq T\lambda_{mit} \leq 1$. This approach has the advantage of retaining informative content from the financing constraints metrics in the estimating model. By plugging Equation (17) into Equation (14), we obtain:

$$(19)\quad \left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + [(\beta_1 + \delta_1 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \leq s_{1m})} + (\beta_2 + \delta_2 T\lambda_{mit})\mathbb{I}_{(s_{1m} < \lambda_{mit} \leq s_{2m})}$$

$$+ (\beta_3 + \delta_3 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} > s_{2m})}] \left(\frac{Cash\,flow}{K}\right)_{it} + \mu_i + \tau_t + \varepsilon_{it,},$$

where, following the reasoning above, we expect $\delta_1 > 0, \delta_2 = 0,$ and $\delta_3 < 0$.

5.2. Empirical results

Table 6 reports the results from estimating the threshold regression model in Equation (19), using the HP metric and the full unbalanced sample of firm–year observations. The results reported in Panel A suggest that ICFS is inverse basin-shaped, i.e., increasing in Regime 1, constant in Regime 2, and decreasing in Regime 3. To support this finding, we check for the presence of ICFS's potential non-linearity within Regime 2. The results for this augmented model (i.e., the second model reported in Panel A) confirm that ICFS is constant in Regime 2. Therefore, we report the third and best performing model, which supports the conclusion that ICFS is inverse basin-shaped. We also estimate this model using the KZ, WW, and AC metrics, and find that ICFS is inverse basin-shaped for all sorting metrics of financing constraints. This shape is robust to the sample separation points (see Appendix 2.3).

Finally, we estimate the models without imposing the number and locations of threshold parameters at the outset, as in Hansen (1999, 2000) and Medeiros (2019). This approach involves first removing the fixed effects with an auxiliary regression. Second, we determine the number and locations of the sample separation points jointly with the ICFS parameters by minimizing the concentrated sum of squared errors as a function of the sample separation points. The results of this analysis, using the HP metric, are displayed in Table 6, Panel B. The findings favor the presence of two thresholds, located at $s_{1HP} = -3.0439$ and $s_{2HP} = -2.0010$. These thresholds are similar to those identified in our previous analysis. The evidence again supports an inverse basin-shaped ICFS.

Insert Table 6

6. Why is the ICFS inverse basin-shaped?

6.1. Financing constraints and the interdependence of financial policies

When trying to model the impact of financing constraints on corporate investment, the literature has often considered this issue in isolation from other financial decisions. This is surprising, as managers typically do not make financial decisions in isolation (Gatchev, Pulvino, and Tarhan, 2010; Chang, Dasgupta, Wong, and Yao, 2014). Decisions regarding how to finance investments should be viewed as part of the broader financial strategy of the company, which encompasses all sources and uses of funds. These include, in addition to investment, cash reserves, dividends, cash flow, net debt, and equity issuance. The interdependence of these factors is clearly illustrated by examining the following cash-flow identity:

(19) $I_t + \Delta Cash_t + Div_t = Cash\ flow_t + \Delta D_t + \Delta E_t$,

According to the identity in Equation (19), the use of funds include the change in cash holdings ($\Delta Cash$), the payment of cash dividends (Div) and investment. While, cash flow, debt issuance (ΔD), equity issuance (ΔE) are the sources of internal and external finance respectively. By rearranging the equation above:

(20) $Cash\ flow_t = I_t + \Delta Cash_t + Div_t - \Delta D_t - \Delta E_t$.

Equation (20) nicely shows the interdependence between the uses and sources of funds that are of interest. Specifically, it shows how financial policies involve deciding how to allocate each additional dollar of cash flow. This includes determining what portion should fund investments, what portion should be held as cash reserves, what portion should be paid out as dividends, and how much should be used for repurchasing debt and equity. This ex-ante budget constraint helps explain how firms allocate their cash flow across different financial policies. In turn, these decisions are influenced by financing constraints. For instance, as highlighted in the seminal contributions to the debate on the ICFS, financing constraints are linked to dividends, as discussed by Fazzari, Hubbard, and Petersen (1988), and to cash holdings and leverage, as shown by Kaplan and Zingales (1997).

Therefore, we estimate the equivalent of Equation (19) for all the remaining uses and sources by through the following set of equations:

$$(21)\quad Y_{ait} = \alpha_a Q_{it} + [(\beta_{a1} + \delta_{a1} T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \leq s_{1m})} + (\beta_{a2} + \delta_{a2} T\lambda_{mit})\mathbb{I}_{(s_{1m} < \lambda_{mit} \leq s_{2m})} + ((\beta_{a3} + \delta_{a3} T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} > s_{2m})}] \left(\frac{Cash\ flow}{K}\right)_{it} + \beta_{a4} T\lambda_{mit} + \mu_{ai} + \tau_{at} + \varepsilon_{ait,},$$

with $a = 1, \ldots 4$ for $\Delta Cash, Div, \Delta D, \Delta E,$ respectively. This analysis sheds light on how these policies vary across the different classes of financing constraints we have identified. Finally, comparing the results from estimating Equations (21) with those from estimating Equation (19) offers insights into why the investment-cash flow sensitivity (ICFS) exhibits an inverse basin shape—specifically, why investment responds differently to cash flow depending on the financial regime.

Two points require clarification. First, according to Chang, Dasgupta, Wong, and Yao (2014), if the variables are consistently defined, the sum of the cash flow sensitivities must equal unity. However, they do not expect this to be the case if variables, as in Gatchev, Pulvino and Tarhan (2010), are defined using both the cash flow statement and the income statement. In our case, since we have estimated a model based on the ICFS literature, the variables have been constructed in accordance with this literature's empirical framework. Therefore, the sum of the estimated cash flow sensitivity coefficients from the various equations is not expected to equal unity. However, we present the analysis using only cash flow statement variables in the appendix, which demonstrates that the constraint holds once the variables are adequately defined.

Second, the analysis we propose is not intended to examine the shape of each use and source within the identified classes. Upon closer inspection, the sample separation points have been chosen based on the analysis of the ICFS as the metrics of financing constraints increase. Our primary interest lies in understanding how other uses and sources vary as the ICFS changes within each class. Therefore, the appropriate sample separations for studying the shapes of these other uses and sources depending upon financing constraints are likely different from those established at the outset of Equations (21). Whether these sample separation points coincide with those that would be obtained by optimally choosing them for each policy is a separate research question, which we address in the appendix.

6.2. Empirical results

Our results from the five estimated equations—investment, changes in cash holdings, dividends, net issuance of debt, and net issuance of equity—are reported in Table 7. Analyzing these results suggests that, in the first regime of low financing constraints, firms prefer to allocate one additional dollar of cash flow to investment, resulting in an ICFS of 0.1202. In this regime, the sensitivities of cash holdings, dividends, net equity issuance, and net debt issuance to cash flow are negative. This aligns with the perspective that less financially constrained firms are more inclined to invest additional cash flow in opportunities rather than diverting it to cash reserves, dividends, or debt reduction (Chang, Dasgupta, Wong, and Yao, 2014). In the second regime, where financing constraints are substantial, the level of investment remains relatively high, but the ICFS parameter is not significant. Conversely, the cash flow sensitivities for cash holdings, dividends, net debt issuance, and net equity issuance turn positive. This suggests that as financing constraints tighten, firms may reduce their allocation of cash flow to investment and instead redirect it to alternative uses based on their priorities and constraints. Finally, in the third regime, characterized by significant financial constraints, the cash flow sensitivities for investment, cash holdings, dividends, net debt issuance, and net equity issuance become negative. This may indicate that the severity of financing constraints leads firms to respond to changes in cash flow by postponing investment, dividends, and liquidity buildup, as well as delaying the issuance of debt and equity.

The exercise above shows that, since financial decisions are interdependent, the allocation of each dollar of cash flow across various uses can fundamentally shape a firm's investment response to changes in cash flow under the different regimes of financing constraints (Chang, Dasgupta, Wong, and Yao, 2014).

Insert Table 7

In addition to analyze cash flow sensitivities within each regime, we compare firm-specific characteristics across the three regimes. In order to do so, we sort firm–year observations by the HP metric and for the three regimes we calculate the mean values of the other financing constraints metrics and variables commonly used to infer financing constraints. We then test whether the means for the first regime are significantly different from those for the second regime, and whether the latter are significantly different from those for the third regime. Our results, reported in Table 8, suggest that firms in the first regime are larger, older, have less cash flow, cash stock and investment. They pay less cash dividends, and issue less debt and equity than the firms in the second regime. This is consistent with the behavior of the cash flow sensitivities of the various decisions: these firms having not high level of investment prefer to allocate extra dollar of cash flow to investment rather than to other financial policies.

At high levels of financing constraints, i.e., in the third regime with respect to the second regime, firms are the most financially constrained according to HP, KZ and WW indexes. Statistics indicate that they are smaller, younger, invest less, have lower debt, equity and dividends. However, they have more cash flow and cash stock relatively to the firms in the second regime. This is in line with the view that these firms being highly constrained they tend to cancel or delay investment projects and stockpile cash out of cash flow stocks (Bates, Kahle and Stulz 2009). The reduced investment and ICFS in the third regime may also be due to the higher adjustment costs that the most financially constrained firms are likely to face (Liao, Nolte, and Pawlina 2023).

Insert Table 8

7. Discussion

Our study's findings have several key implications. First, the inverse basin-shaped ICFS that we have documented is a generalization of all shapes reported in previous studies. These shapes can be obtained from the inverse basin shape by imposing further restrictions on the estimating model. Specifically, if the sample separation points are chosen in Regime 1, ICFS will be increasing. This is because most of the observations are concentrated in the first two regimes, in which ICFS is increasing and then constant. In this case, the average ICFS of the observations belonging to Regime 1 will be lower than the average ICFS of the observations belonging to Regime 2 because the parameter is increasing. If, instead, the first sample separation point is chosen to include most of the observations in Regime 2, in which ICFS is constant, and the second sample separation point is chosen to include some observations located in Regime 3, in which ICFS is decreasing, ICFS will be decreasing. Because Kaplan and Zingales (1997) focus on the subsample of the most constrained firms from Fazzari, Hubbard, and Petersen (1988), their result of a decreasing ICFS is unsurprising. Finally, if the two sample separation points are chosen in Regime 2, ICFS may be U-shaped or inverse U-shaped if these points are misidentified.

A second implication of our findings is that if a parametric polynomial function, rather than the number of regimes, is imposed at the outset, the ICFS's resulting shape would be an inverse U. To see this, we adopt a parametric approach to study the ICFS's shape over the entire range of the financing constraints metric. The approach approximates the function $f(\lambda_m(k))$ with the polynomial function $f(FC) = \sum_{r=0}^{R} \beta_r \lambda_{m,it}^r$ and generalizes the baseline investment model in Equation (1) as follows:

$$(20) \quad \left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + \sum_{r=0}^{R} \beta_r \lambda_{mit}^r \times \left(\frac{Cash\ flow}{K}\right)_{it} + \mu_i + \tau_t + \varepsilon_{it} \, ,$$

for which we empirically determine the optimal degree of polynomial *R*. The estimation model in Equation (20) is especially useful because it nests all the hypotheses proposed in prior studies. If *R=0*, the model reduces to the baseline investment model in Equation (1). If *R=1*, we augment this baseline model with the interaction between cash flow and the sorting metric. If *R=2*, we investigate ICFS's non-monotonicity. If this were the prevailing model, then the sign of the estimated parameters would help to determine ICFS's non-monotonic shape. Finally, if *R=3*, ICFS would take shapes not yet explored in the literature.

The results for the HP metric are reported in the upper part of Table 9. When *R=1*, the *F*-statistic for the test of the significance of β_1 on the HP metric interacted with cash flow confirms that the ICFS parameter depends on the HP metric. When *R=2*, the *F*-statistic for the test of statistical significance of β_1 and β_2 doubles, and the adjusted R^2 increases. However, when HP^3 interacted with cash flow is added to the set of regressors, the *F*-statistic for the test of significance of β_1, β_2, and β_3 decreases significantly. Therefore, Equation (20) with *R=2* yields the best specification of the model for ICFS. To investigate the shape of ICFS, we use the fitted quadratic model:

$$(21) \quad \widehat{I/K} = 0.0231 \times Q + 0.0048 \times Cash\ flow/K - 0.0536 \times Cash\ flow/K \times HP - 0.0103 \times Cash\ flow/K \times HP^2.$$

This model implies that ICFS is the following derivative:

$$(22) \quad ICFS = \frac{d\ (\widehat{I/K})}{d(Cash\ flow/K)} = -0.0103 \times HP^2 - 0.0536 \times HP + 0.0048\ ,$$

which is the second-order polynomial in HP. ICFS equals zero for HP = -5.31 and 0.09. The polynomial yields an inverse U-shaped ICFS with its peak at HP = -2.62 (see the bold dotted line in Figure A.1 in Appendix 3). In addition, the inverse U-shape is robust to the choice of the degree of the polynomial. Indeed, if we take the model in Equation (20) with *R=3* as the best specification

$$(23) \quad \widehat{I/K} = 0.0230 \times Q + 0.0107 \times Cash\ flow/K - 0.0371 \times Cash\ flow/K \times HP - 0.0007 \times Cash\ flow/K \times HP^2 + 0.00152 \times Cash\ flow/K \times HP^3\ ,$$

we would have the following derivative as ICFS:

$$(24) \quad ICFS = \frac{d\ (\widehat{I/K})}{d(Cash\ flow/K)} = 0.0107 - 0.0371 \times HP - 0.0007 \times HP^2 + 0.00152 \times HP^3\ ,$$

and we would again observe an inverse U-shaped ICFS because we do not have a high enough HP index for this part of the ICFS function. We also report the estimation results for the model in Equation (20) with *R=2* using KZ, WW, and AC as interaction variables. The estimation results confirm an inverse U-shaped ICFS. The true underlying inverse basin-shaped ICFS that we document makes the inverse U-shape the prevailing empirical shape if parametrically imposed at the outset of the estimating model. However, this conclusion would omit information about a large proportion of observations that have a constant ICFS. Finally, if we follow Hansen (1999, 2000) and impose the assumption that ICFS is constant in each regime, the resulting shape will again be an inverse U-shape, with three constant parameters. The lower panel of Table 9 shows the results for the threshold regression model with constant parameters in each regime. The results are reported for all four metrics of financing constraints.

Insert Table 9

The third implication of our findings is that ICFS takes an inverse basin shape regardless of whether the metric captures an inelastic supply of funds (Whited and Wu 2006, Farre-Mensa and Ljungquist 2016), a large wedge between the cost of internal and external finance (Fazzari, Hubbard and Petersen 1988), or the need for external finance (Kaplan and Zingales 1997, Cleary, Povel and Raith 2007). Nevertheless, given the ICFS's regime-level monotonicity, we can conclude that the magnitude of the ICFS parameter can still be used as a measure of the severity of financing constraints if we know in which regime of financing constraints the firm is located. In the first regime, firms are relatively financially unconstrained, and the ICFS increases with the financing constraints. In the second regime, the ICFS is the highest, and further financing constraints do not change the response of investment to cash flow. Finally, in the third regime, firms are the most financially constrained, and an increase in the degree of financing constraints reduces the ICFS. The evidence implies that, if the objective is to increase investment using cash flow, information is needed about which regime of financing constraints the firm is facing. The implication of this result is that when financing constraints are severe, policies designed to increase cash flow may lead to a decrease in investment.

These findings return us to the unsolved problem of measuring financing constraints. The literature on financing constraints acknowledges that no financing constraints metric is free of criticism. This acknowledgment is not surprising since each metric relies on certain empirical and/or theoretical assumptions that may or may not be valid. In addition, many of these metrics rely on endogenous financial choices that may not be related directly to constraints (Hadlock and Pierce 2010). Farre-Mensa and Ljungqvist (2016) argue that the HP metric is unlikely to capture financing constraints but may reflect differences in growth and financing policies at different stages of the firm's life cycle. They suggest that high values of the HP index capture firms in their fast-growth stage. However, our results support this idea only partially because firms in the third regime of the HP index, in contrast to the evidence provided by Farre-Mensa and Ljungqvist (2016), report the lowest levels of debt, equity, dividends, return on asset, sale growth, coverage ratio and asset tangibility. These are all characteristics commonly attributed to the presence of more severe financing constraints. These firms are similar to the most-constrained firms in Hoberg and Maksimovic (2015), Hadlock and Pierce (2010), and Brown, Fazzari, and Petersen (2009) in that they are smaller, younger, have higher R&D, higher need for external equity and debt financing, and the highest cash stocks, likely due to high adjustment costs and the inability to quickly exercise investment options.

8. Conclusion

This paper proposes a novel approach to identify classes of firms facing homogenous financing constraints and to study the impact of financing constraints on investment via the ICFS's behavior. We identify four classes of homogenous financing constraints and we show that the popular metrics of financing constraints do capture financing constraints reasonably well. Moreover, we provide robust and convincing evidence against the ICFS monotonicity.

Since our framework bypasses all concerns about the method used to sort firms according to financing constraints, our results hold regardless of the true sample separation points, class sizes and composition, regardless of whether financing constraints increase or decrease with the sorting metric, and regardless of whether the metric is the best measure of financing constraints.

Being the monotonicity of the CFS rejected, we search for the ICFS's true underlying shape. We adopt a generalized threshold regression approach that relaxes the hypothesis that the ICFS parameter is constant within each regime. We find that the true underlying ICFS is inverse basin-shaped. The above evidence suggests that the prior conflicting findings about the shape of the ICFS are due to the additional restrictions imposed at the outset of the estimating model. These restrictions arise from assumptions about the location of the sample separation points, the parametric shape of the ICFS function, or the homogenous ICFS in each regime.

In addition, our findings suggest that the shape of the ICFS cannot be explained independently of the firm's financial policies, nor regardless of the financing constraints the firm faces. Our study opens new avenues for future research, particularly in modeling the interaction between ICFS and financial policies. By advancing these models, researchers can enhance the understanding of how firms navigate financial challenges. Future studies may also focus on developing more robust measures of financing constraints, potentially leading to more effective financial strategies and policies.

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Table 1
Investment–Cash Flow Sensitivity Under Alternative Sorting Schemes

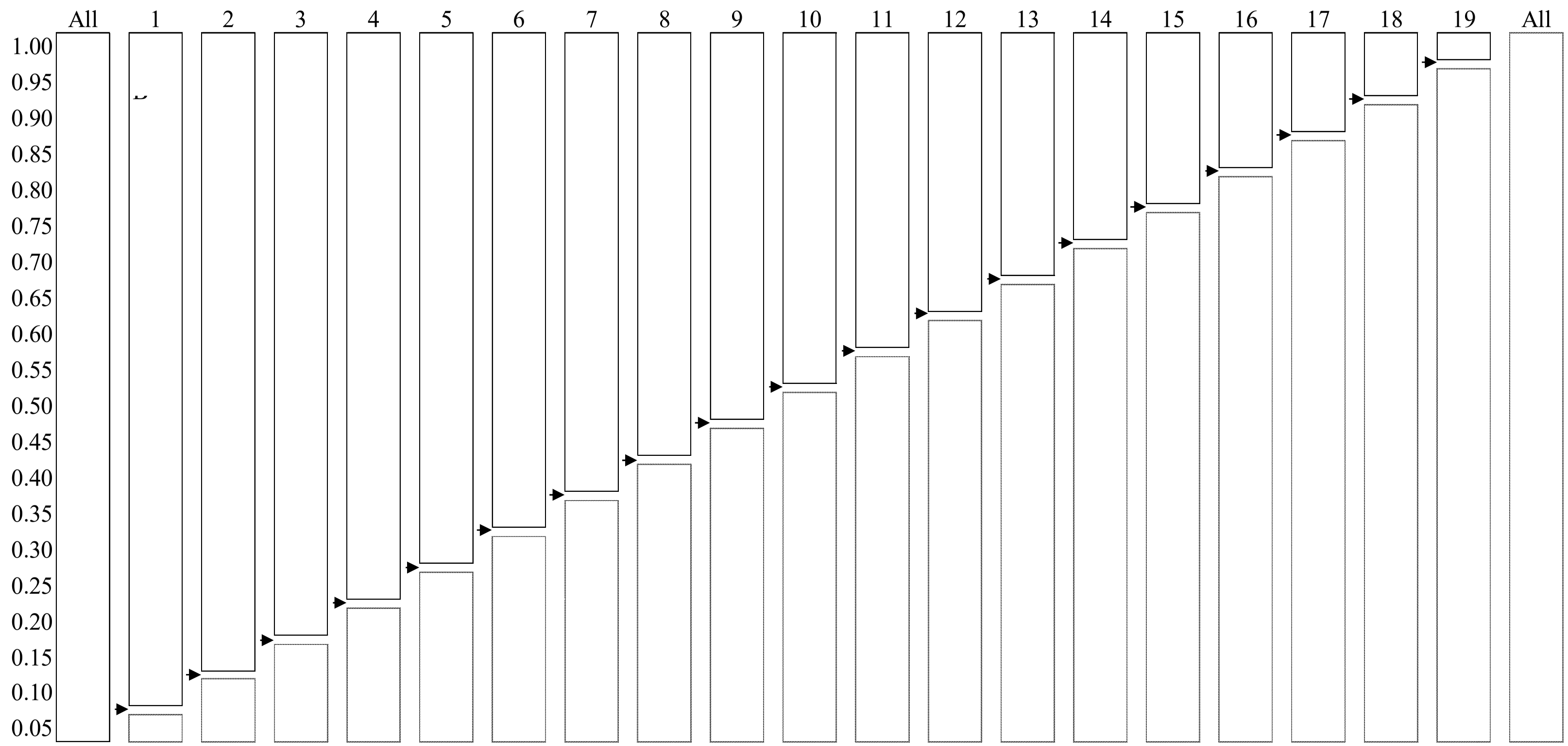
Sample sorted according to					
Hadlock and Pierce (2010)		Kaplan and Zingales (1997)		Whited and Wu (2006)	
Points of sample separation	β $(s.e.)$	Points of sample separation	β $(s.e.)$	Points of sample separation	β $(s.e.)$
Sorting scheme 1		Sorting scheme 1		Sorting scheme 1	
$HP \leq -3.61$	0.0438	$KZ \leq -0.44$	0.0525	$WW \leq -0.30$	0.0563
	(0.0077)		(0.0060)		(0.0043)
$-3.61 < HP \leq -3.33$	0.0605	$-0.44 < KZ \leq -0.01$	0.0623	$-0.30 < WW \leq -0.27$	0.0590
	(0.0072)		(0.0054)		(0.0129)
$HP > -3.33$	0.0651	$KZ > -0.01$	0.0729	$WW > -0.27$	0.0623
	(0.0026)		(0.0030)		(0.0026)
Sorting scheme 2		Sorting scheme 2		Sorting scheme 2	
$HP \leq -2.65$	0.0639	$KZ \leq 1.08$	0.0639	$WW \leq -0.20$	0.0648
	(0.0031)		(0.0024)		(0.0034)
$-2.65 < HP \leq -2.09$	0.0619	$1.08 < KZ \leq 1.60$	0.0614	$-0.20 < WW \leq -0.13$	0.0597
	(0.0056)		(0.0115)		(0.0047)
$HP > -2.09$	0.0610	$KZ > 1.60$	0.0332	$WW > -0.13$	0.0554
	(0.0053)		(0.0129)		(0.0050)
Sorting scheme 3		Sorting scheme 3		Sorting scheme 3	
$HP \leq -3.28$	0.0505	$KZ \leq 0.63$	0.0615	$WW \leq -0.32$	0.0517
	(0.0043)		(0.0026)		(0.0049)
$-3.28 < HP \leq -2.52$	0.0720	$0.63 < KZ \leq 0.72$	0.1032	$-0.32 < WW \leq -0.18$	0.0707
	(0.0040)		(0.0240)		(0.0038)
$HP > -2.52$	0.0614	$KZ > 0.72$	0.0619	$WW > -0.18$	0.0562
	(0.0037)		(0.0053)		(0.0036)
Sorting scheme 4		Sorting scheme 4		Sorting scheme 4	
$HP \leq -2.65$	0.0639	$KZ \leq 0.53$	0.0609	$WW \leq -0.41$	0.0637
	(0.0031)		(0.0026)		(0.0067)
$-2.65 < HP \leq -2.55$	0.0336	$0.53 < KZ \leq 0.59$	0.0515	$-0.41 < WW \leq -0.34$	0.0507
	(0.0167)		(0.0311)		(0.0058)
$HP > -2.55$	0.0602	$KZ > 0.59$	0.0694	$WW > -0.34$	0.0638
	(0.0037)		(0.0050)		(0.0024)

This table presents results from estimating the investment model in eq. (1) under four different sorting schemes, characterized by a given metric of financing constraints and some sample separations points. All coefficient estimates are for the full unbalanced sample of 93,107 firm-year observations with *cash flow* ≥ -1, with firm and year fixed effects. The sample period is from 1990 to 2013. The dependent variable is investment, normalized by the beginning-of-period net capital stock. Market-to-book is measured at the beginning of the observation year; cash flow is measured contemporaneously with the investment decision. Observations are sorted by the Hadlock and Pierce (2010), Kaplan and Zingales (1997), and Whited and Wu (2006) metrics of financing constraints. We report estimated coefficients on cash flow. Heteroskedasticity-consistent standard errors are in parentheses.

Figure 1

Alternative Sample Separation Points

The vertical axis shows the 19 alternative sample separation points s used in testing for the monotonicity of ICFS. The full sample of firm-year observations (Column "All") is sorted by the HP index of financing constraints, so that the mean level of HP increases progressively across both classes of observations $i > s$ and $i \leq s$. The test for difference in means, reported in the figure, indicates that the mean HP of the first class ($i \leq s$) is always significantly different from that of the complementary class ($i > s$) for any sample separation point.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Class $i > s$																			
Mean of HP	-2.92	-2.88	-2.83	-2.79	-2.75	-2.70	-2.66	-2.61	-2.56	-2.50	-2.44	-2.38	-2.31	-2.22	-2.13	-2.02	-1.87	-1.68	-1.36
Class $i \leq s$																			
Mean of HP	-3.93	-3.84	-3.77	-3.70	-3.65	-3.60	-3.56	-3.52	-3.48	-3.44	-3.41	-3.37	-3.33	-3.29	-3.25	-3.21	-3.17	-3.12	-3.06
t-test	403.1	379.1	364.9	354.2	345.5	338.5	332.9	328.7	325.9	323.6	320.9	317.6	313.3	308.1	301.5	292.7	281.5	265.6	232.5

Table 2
Analysis of the Joint Condition

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Position of s (%)	γ_s	α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic	Bonferroni corrected p-values
Sample sorted according to the Hadlock and Pierce (2010) index of financing constraints							
5	0.0418	0.0222	0.0455	0.0688	38.64%	19.60***	0.0000
	(0.0049)	(0.0019)	(0.0051)	(0.0021)			
10	0.0341	0.0223	0.0502	0.0694	38.66%	21.07***	0.0000
	(0.0044)	(0.0019)	(0.0041)	(0.0021)			
15	0.0295	0.0222	0.0511	0.0701	38.67%	19.78***	0.0000
	(0.0042)	(0.0019)	(0.0040)	(0.0021)			
20	0.0182	0.0223	0.0580	0.0698	38.63%	8.06***	0.0023
	(0.0040)	(0.0019)	(0.0038)	(0.0022)			
25	0.0067	0.0223	0.0615	0.0694	38.60%	3.73*	0.0268
	(0.0039)	(0.0019)	(0.0038)	(0.0022)			
30	0.0013	0.0223	0.0610	0.0698	38.61%	5.15**	0.0117
	(0.0037)	(0.0019)	(0.0035)	(0.0022)			
35	0.0007	0.0223	0.0638	0.0694	38.59%	2.13	0.0721
	(0.0036)	(0.0019)	(0.0034)	(0.0023)			
40	-0.0040	0.0223	0.0655	0.0690	38.59%	0.97	0.1626
	(0.0037)	(0.0019)	(0.0032)	(0.0023)			
45	-0.0067	0.0223	0.0666	0.0688	38.59%	0.42	0.2589
	(0.0038)	(0.0019)	(0.0030)	(0.0024)			
50	-0.0053	0.0223	0.0655	0.0694	38.59%	1.27	0.1298
	(0.0039)	(0.0019)	(0.0028)	(0.0024)			
55	-0.0010	0.0224	0.0672	0.0688	38.58%	0.23	0.3156
	(0.0041)	(0.0019)	(0.0027)	(0.0025)			
60	0.0054	0.0225	0.0691	0.0676	38.59%	0.20	0.3290
	(0.0044)	(0.0019)	(0.0027)	(0.0026)			
65	0.0094	0.0226	0.0694	0.0673	38.59%	0.40	0.2640
	(0.0045)	(0.0019)	(0.0026)	(0.0026)			
70	0.0162	0.0227	0.0707	0.0656	38.62%	2.11	0.0732
	(0.0048)	(0.0019)	(0.0026)	(0.0028)			
75	0.0283	0.0229	0.0695	0.0661	38.65%	0.95	0.1654
	(0.0051)	(0.0019)	(0.0025)	(0.0029)			
80	0.0368	0.0230	0.0701	0.0644	38.69%	2.33	0.0637
	(0.0056)	(0.0019)	(0.0025)	(0.0031)			
85	0.0488	0.0233	0.0708	0.0603	38.78%	7.39***	0.0033
	(0.0061)	(0.0018)	(0.0023)	(0.0034)			
90	0.0703	0.0237	0.0704	0.0564	38.86%	10.38***	0.0007
	(0.0070)	(0.0018)	(0.0022)	(0.0040)			
95	0.0795	0.0239	0.0696	0.0538	38.80%	7.19***	0.0037
	(0.0094)	(0.0018)	(0.0021)	(0.0057)			

This table presents results from estimating the investment model in eq. (1) when observations are sorted by the Hadlock and Pierce (2010) index of financing constraints. Coefficient estimates are the within fixed firm and year estimates for the full unbalanced sample of 93,107 firm-year observations. The sample period is 1990–2013. The dependent variable is investment, normalized by beginning-of-period net capital stock. Market-to-book ratio is measured at the end of the observation year, and cash flow is measured contemporaneously with the investment decision. Column (a) reports the point of sample separation. Column (b) reports the coefficient on the class of observations i ≤ s. Column (c) reports the coefficient on the market-to-book ratio. Columns (d) and (e) report the coefficient on ICFS for the classes of observations i ≤ s and i > s, respectively. Column (f) reports the adjusted R^2. Column (g) reports the F-statistic testing the null hypothesis of equality of parameters in Columns (d) and (e). Column (i) report the Bonferroni corrected p-values. Heteroskedasticity-consistent standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1, 5, and 10% levels, respectively.

Table 3

The Joint Condition Under Alternative Sorting Metrics, and the Sample Splitting Approach

Panel A: results based on different sorting metrics

(a)	(b)	(c)	(d)	(e)	(f)	(g)
Position of s (%)	γ_s	α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic
Sample sorted according to the Kaplan and Zingales (1997) index						
10	-0.0183	0.0220	0.0532	0.0723	38.78%	21.15***
	(0.0043)	(0.0019)	(0.0039)	(0.0021)		
65	-0.0086	0.0211	0.0674	0.0716	38.61%	1.28
	(0.0028)	(0.0019)	(0.0022)	(0.0034)		
95	0.0158	0.0243	0.0687	0.0570	38.62%	3.10*
	(0.0057)	(0.0019)	(0.0021)	(0.0065)		
Sample sorted according to the Whited and Wu (2006) index						
10	0.0223	0.0225	0.0532	0.0685	38.60%	5.63**
	(0.0046)	(0.0019)	(0.0063)	(0.0020)		
55	0.0292	0.0223	0.0709	0.0660	38.68%	1.80
	(0.0045)	(0.0019)	(0.0032)	(0.0024)		
95	0.0812	0.0236	0.0701	0.0420	38.92%	19.84***
	(0.0078)	(0.0018)	(0.0021)	(0.0061)		
Sample sorted according to Almeida and Campello (2007) index						
15	0.0043	0.0222	0.0631	0.0695	38.60%	2.87*
	(0.0037)	(0.0019)	(0.0035)	(0.0022)		
55	-0.0054	0.0221	0.0671	0.0689	38.59%	0.33
	(0.0028)	(0.0019)	(0.0025)	(0.0026)		
95	-0.0164	0.0224	0.0699	0.0604	38.61%	4.46**
	(0.0091)	(0.0019)	(0.0021)	(0.0043)		

Panel B: Results based on the sample splitting approach

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
	Class $i \leq s$			Class $i > s$			
Position of s (%)	α	β	Adjusted R^2	α	β	Adjusted R^2	χ^2-Statistic
10	0.0142	0.0498	61.29%	0.0212	0.0696	37.90%	5.25**
	(0.0065)	(0.0085)		(0.0015)	(0.0017)		
60	0.0357	0.0658	47.25%	0.0120	0.0689	34.59%	0.72
	(0.0023)	(0.0028)		(0.0021)	(0.0024)		
90	0.0348	0.0699	42.31%	-0.0005	0.0584	23.72%	5.23**
	(0.0017)	(0.0019)		(0.0033)	(0.0047)		

Panel A presents results from estimating the investment model in eq. (1) when observations are sorted by alternative metrics of financing constraints. Coefficient estimates are the within fixed firm and year estimates for the full unbalanced sample of 93,107 firm-year observations. The sample period is 1990–2013. Panel B reports results when observations are sorted by the Hadlock and Pierce (2010) index of financing constraints and we use the sample splitting approach. In this case, we report the $\chi 2$ statistic testing the null hypothesis of equality of the two cash flow parameters. Heteroskedasticity-consistent standard errors are reported in parentheses. ***, **, and * denote statistical significance at the 1, 5, and 10% levels, respectively.

Table 4
Are Sorting Metrics Monotonic?

		Percentiles						
Subsample		10	20	40	50	60	80	90
Panel A: Hadlock and Pierce (2010)								
Whited and Wu (2006)	f	-0.395	-0.381	-0.362	-0.348	-0.332	-0.302	-0.285
	c	-0.251	-0.237	-0.202	-0.184	-0.165	-0.122	-0.087
	diff	0.14***	0.14***	0.16***	0.16***	0.17***	0.18***	0.20***
Kaplan and Zingales (1997)	f	0.241	0.254	0.268	0.276	0.271	0.258	0.255
	c	0.274	0.275	0.272	0.265	0.269	0.319	0.407
	diff	0.03***	0.02***	0.00	-0.01*	-0.001	0.06***	0.15***
Almeida and Campello (2007)	f	0.456	0.467	0.482	0.489	0.498	0.514	0.520
	c	0.532	0.539	0.553	0.560	0.565	0.568	0.568
	diff	0.08***	0.07***	0.07***	0.07***	0.07***	0.05***	0.05***
Panel B: Kaplan and Zingales (1997)								
Hadlock and Pierce (2010)	f	-3.014	-2.974	-2.902	-2.886	-2.878	-2.883	-2.893
	c	-2.875	-2.867	-2.880	-2.891	-2.905	-2.912	-2.853
	diff	0.14***	0.11***	0.02***	-0.00	-0.03***	-0.03***	0.04***
Whited and Wu (2006)	f	-0.321	-0.305	-0.281	-0.276	-0.272	-0.269	-0.269
	c	-0.259	-0.256	-0.255	-0.256	-0.256	-0.251	-0.238
	diff	0.06***	0.05***	0.03***	0.02***	0.02***	0.02***	0.03***
Almeida and Campello (2007)	f	0.563	0.579	0.566	0.557	0.550	0.537	0.530
	c	0.520	0.511	0.496	0.492	0.487	0.476	0.478
	diff	-0.04***	-0.07***	-0.07***	-0.07***	-0.06***	-0.06***	-0.05***
Panel C: Whited and Wu (2006)								
Hadlock and Pierce (2010)	f	-3.510	-3.512	-3.413	-3.351	-3.284	-3.132	-3.037
	c	-2.820	-2.733	-2.539	-2.427	-2.295	-1.917	-1.558
	diff	0.69***	0.78***	0.87***	0.92***	0.99***	1.22***	1.48***
Kaplan and Zingales (1997)	f	-0.024	0.069	0.155	0.187	0.206	0.226	0.240
	c	0.303	0.321	0.348	0.354	0.368	0.448	0.548
	diff	0.33***	0.25***	0.19***	0.17***	0.16***	0.22***	0.31***
Almeida and Campello (2007)	f	0.450	0.462	0.483	0.491	0.500	0.514	0.520
	c	0.533	0.540	0.552	0.558	0.562	0.565	0.568
	diff	0.08***	0.08***	0.07***	0.07***	0.06***	0.05***	0.05***
Panel D: Almeida and Campello (2007)								
Hadlock and Pierce (2010)	f	-3.123	-3.115	-3.072	-3.051	-3.020	-2.952	-2.920
	c	-2.863	-2.832	-2.766	-2.727	-2.693	-2.637	-2.609
	diff	0.26***	0.28***	0.31***	0.32***	0.33***	0.31***	0.31***
Kaplan and Zingales (1997)	f	0.524	0.480	0.422	0.411	0.401	0.352	0.311
	c	0.242	0.218	0.169	0.130	0.075	-0.057	-0.097
	diff	-0.28***	-0.26***	-0.25***	-0.28***	-0.33***	-0.41***	-0.41***
Whited and Wu (2006)	f	-0.307	-0.306	-0.300	-0.295	-0.290	-0.277	-0.271
	c	-0.261	-0.256	-0.243	-0.236	-0.230	-0.219	-0.214
	diff	0.05***	0.05***	0.06***	0.06***	0.06***	0.06***	0.06***

This table reports estimated subsample mean values for the four metrics of financing constraints used in our analysis. Means are estimated by sorting observations according to the HP (Panel 1), KZ (Panel 2), WW (Panel 3), or AC index (Panel 4), then regressing the metric of financing constraints on two dummy variables corresponding to the first (f) 5% (10, 15, 20, etc.) and the remaining (c) 95% (90, 85, 80, etc.) of the sorting metric. ***, **, and * denote statistical significance of the difference between the two parameters at the 1, 5, and 10% levels, respectively.

Table 5

Monotonicity of Firm Characteristics with respect to the Hadlock and Pierce (2010) Index: Selected Percentiles

	Subsample	Percentiles						
		10	20	40	50	60	80	90
Hadlock and Pierce (2010)	f	-3.82	-3.68	-3.48	-3.40	-3.32	-3.15	-3.05
	c	-2.79	-2.69	-2.49	-2.37	-2.24	-1.84	-1.47
	diff	1.03***	0.98***	0.99***	1.02***	1.08***	1.31***	1.58***
Size	f	8.08	7.84	7.47	7.20	6.91	6.30	5.98
	c	5.31	5.02	4.33	3.97	3.61	2.72	2.03
	diff	-2.77***	-2.81***	-3.14***	-3.23***	-3.30***	-3.58***	-3.96***
Age	f	18.36	15.44	11.58	10.50	9.74	8.70	8.35
	c	6.88	6.17	5.65	5.55	5.45	5.32	5.13
	diff	-11.49***	-9.27***	-5.92***	-4.95***	-4.29***	-3.38***	-3.21***
Cash	f	0.11	0.11	0.10	0.10	0.11	0.11	0.12
	c	0.12	0.12	0.13	0.13	0.13	0.13	0.13
	diff	0.00***	0.01***	0.02***	0.02***	0.02***	0.01***	0.01***
Coverage ratio	f	90.99	133.89	99.57	95.08	92.14	82.65	75.43
	c	63.92	46.15	39.24	31.57	19.98	-8.55	-23.72
	diff	-27.07	-87.74**	-60.33***	-63.51***	-72.15***	-91.20***	-99.14***
Debt	f	0.19	0.18	0.17	0.16	0.16	0.14	0.13
	c	0.12	0.12	0.10	0.09	0.09	0.08	0.09
	diff	-0.07***	-0.06***	-0.07***	-0.07***	-0.07***	-0.05***	-0.04***
Equity	f	0.41	0.39	0.37	0.37	0.37	0.38	0.38
	c	0.31	0.30	0.29	0.28	0.25	0.10	-0.18
	diff	-0.09***	-0.08***	-0.08***	-0.09***	-0.12***	-0.28***	-0.55***
Dividends	f	0.014	0.013	0.012	0.011	0.010	0.009	0.009
	c	0.008	0.007	0.006	0.006	0.005	0.005	0.005
	diff	-0.007***	-0.006***	-0.006***	-0.005***	-0.005***	-0.004***	-0.004***

This table reports subsample estimated mean values of the most commonly used characteristics of financing constraints. Means are estimated by sorting observations according to the HP index and then regressing the metric of financing constraints on two dummy variables corresponding to the first (f) 5% (10, 15, 20, etc.) and the remaining (c) 95% (90, 85, 80, etc.) of the Hadlock and Pierce (2010) index. ***, **, and * denote statistical significance of the difference (diff) between the two parameters at the 1, 5, and 10% levels, respectively.

Table 6

The Shape of ICFS: Generalized Threshold Models

(a) α	(b) β_1	(c) β_2	(d) β_3	(e) β_4	(f) β_7	(g) β_5	(h) β_6	(i) Adj. R^2	(j) F-Stat. (p-value)
Panel A. Exogenous sample separation points									
Hadlock and Pierce (2010)									
Sample separation points: $s_1 = -3.2416, s_2 = -2.3484$									
$\left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + [(\beta_1 + \beta_4 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \le s_{1m})} + (\beta_2 + \beta_5 T\lambda_{mit})\mathbb{I}_{(s_{1m} < T\lambda_{mit} \le s_{2m})} + (\beta_3 + \beta_6 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit} > s_{2m})}](Cash\ flow/k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$									
0.0229	0.0509	0.1202	0.0755	-0.000038		0.1204	-0.1243	38.85%	193.89
(0.0018)	(0.0063)	(0.0506)	(0.0104)	(0.0403)		(0.0122)	(0.0247)		(0.000)
$\left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + [(\beta_1 + \beta_4 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \le s_{1m})} + (\beta_2 + \beta_5 T\lambda_{mit} + \beta_7 T\lambda_{mit}^2)\mathbb{I}_{(s_{1m} < T\lambda_{mit} \le s_{2m})} + (\beta_3 + \beta_6 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit} > s_{2m})}](Cash\ flow/k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$									
0.0228	0.0509	0.1194	0.0341	0.3345	-0.6479	0.1203	-0.1241	38.85%	166.67
(0.0018)	(0.0063)	(0.0506)	(0.0511)	(0.4101)	(0.7935)	(0.0122)	(0.0247)		(0.000)
$\left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + [(\beta_1 + \beta_4 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \le s_{1m})} + \beta_2 \mathbb{I}_{(s_{1m} < T\lambda_{mit} \le s_{2m})} + (\beta_3 + \beta_6 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit} > s_{2m})}](Cash\ flow/k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$									
0.0229	0.0509	0.1202	0.0755			0.1204	-0.1243	38.85%	228.21
(0.0018)	(0.0063)	(0.0507)	(0.0028)			(0.0122)	(0.0246)		(0.000)
Kaplan and Zingales (1997)									
Sample separation points: $s_1 = 0.5007, s_2 = 1.0178$									
0.0233	0.0363	0.0557	0.0827			0.1527	-0.1021	38.87%	246.44
(0.0018)	(0.0055)	(0.0093)	(0.0038)			(0.0446)	(0.0519)		(0.000)
Whited and Wu (2006)									
Sample separation points: $s_1 = -0.2420, s_2 = -0.2220$									
0.0223	0.0428	0.0780	0.0704			0.1560	-0.1435	38.82%	226.59
(0.0018)	(0.0110)	(0.0269)	(0.0049)			(0.0198)	(0.0306)		(0.000)
Almeida and Campello (2007)									
Sample separation points: $s_1 = 0.6001, s_2 = 0.6466$									
0.0217	0.0577	0.0308	0.0758			0.1265	-0.0694	38.94%	234.04
(0.0019)	(0.0043)	(0.0103)	(0.0047)			(0.0221)	(0.0256)		(0.000)
Panel B. Endogenous sample separation points									
Hadlock and Pierce (2010)									
Sample separation points: $s_1 = -3.0439, s_2 = -2.0010$									
0.0332	0.0388	0.1964	0.0859	-0.0491		0.1248	-0.1481		SSR_1: 6005.53
(0.0012)	(0.0040)	(0.0304)	(0.0095)	(0.0319)		(0.0125)	(0.0216)		SSR_2: 5983.27

This table presents estimated results from the threshold regression analysis. The upper panels report the ICFS estimates when the number and locations of the thresholds are exogenously determined. Coefficients are the within-group estimates for the full unbalanced sample of firm-year observations with *cash flow* ≥ -1 . The sample period is 1990–2013. The dependent variable is investment, normalized by the beginning-of-period net capital stock. Market-to-book ratio is measured at the end of the observation year, and cash flow is measured contemporaneously with the investment decision. In all models, Column (a) reports the coefficient on market-to-book ratio; Columns (b), (d), and (g) report coefficients on ICFS for the three regimes. Columns (c), (e), and (h) report coefficients on the interactions between ICFS and TM, a monotonic transformation of the financing constraints metric (M). Column (f) reports the coefficient on the interaction between ICFS and TM square. Heteroskedasticity-consistent standard errors are in parentheses. In the lower panel, we report ICFS estimates when the number and locations of the thresholds are endogenously determined.

Table 7

Investment and financing decisions within the regimes: Sample sorted by Hadlock and Pierce (2010) metric

(a) α	(b) β_1	(c) δ_1	(d) β_2	(e) δ_2	(f) β_3	(g) δ_3	(h) Adj. R^2	(i) F-Stat. (p-value)
Sample separation points: $s_1 = -3.2416, s_2 = -2.3484$								
$\left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + [(\beta_1 + \delta_1 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \le s_{1m})} + (\beta_2 + \delta_2 T\lambda_{mit})\mathbb{I}_{(s_{1m} < T\lambda_{mit} \le s_{2m})} + (\beta_3 + \delta_3 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit} > s_{2m})}](Cash\,flow/k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$								
0.0229	0.0509	0.1202	0.0755	-0.000038	0.1204	-0.1243	38.85%	193.89
(0.0018)	(0.0063)	(0.0506)	(0.0104)	(0.0403)	(0.0122)	(0.0247)		(0.000)
0.0229	0.0509	0.1202	0.0755		0.1204	-0.1243	38.85%	228.21
(0.0018)	(0.0063)	(0.0507)	(0.0028)		(0.0122)	(0.0246)		(0.000)
$(\Delta Cash)_{it} = \alpha Q_{it} + [(\beta_1 + \delta_1 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \le s_{1m})} + (\beta_2 + \delta_2 T\lambda_{mit})\mathbb{I}_{(s_{1m} < T\lambda_{mit} \le s_{2m})} + (\beta_3 + \delta_3 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit} > s_{2m})}](Cash\,flow/k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$								
0.0134	0.0226	-0.1537	-0.0141	0.0929	0.0292	-0.0395	36.09%	59.92
(0.0007)	(0.0023)	(0.0186)	(0.0031)	(0.0124)	(0.0038)	(0.0078)		(0.000)
$(Dividend)_{it} = \alpha Q_{it} + [(\beta_1 + \delta_1 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \le s_{1m})} + (\beta_2 + \delta_2 T\lambda_{mit})\mathbb{I}_{(s_{1m} < T\lambda_{mit} \le s_{2m})} + (\beta_3 + \delta_3 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit} > s_{2m})}](Cash\,flow/k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$								
0.0027	0.0066	-0.0540	-0.0067	0.0296	0.0042	-0.0071	62.82%	33.74
(0.0002)	(0.0008)	(0.0058)	(0.0008)	(0.0031)	(0.0009)	0.0018		(0.000)
$(\Delta Debt)_{it} = \alpha Q_{it} + [(\beta_1 + \delta_1 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \le s_{1m})} + (\beta_2 + \delta_2 T\lambda_{mit})\mathbb{I}_{(s_{1m} < T\lambda_{mit} \le s_{2m})} + (\beta_3 + \delta_3 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit} > s_{2m})}](Cash\,flow/k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$								
0.0018	0.0158	-0.1330	-0.0136	0.0503	0.0071	-0.0213	40.93%	7.42
(0.0009)	(0.0039)	(0.0298)	(0.0037)	(0.0140)	(0.0041)	0.0084		(0.000)
$(\Delta Equity)_{it} = \alpha Q_{it} + [(\beta_1 + \delta_1 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit} \le s_{1m})} + (\beta_2 + \delta_2 T\lambda_{mit})\mathbb{I}_{(s_{1m} < T\lambda_{mit} \le s_{2m})} + (\beta_3 + \delta_3 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit} > s_{2m})}](Cash\,flow/k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$								
0.0162	0.0080	-0.1149	-0.0190	0.0937	0.0214	-0.0409	56.55%	19.90
(0.0010)	(0.0034)	(0.0273)	(0.0036)	(0.0141)	(0.0041)	(0.0084)		(0.000)

Table 8
Hadlock and Pierce (2010) Index and Financing Constraints

(a)		(b)		(c)	(d)		(e)		(f)
Hadlock and Pierce (2010)	Bottom	-3.541	Middle	-2.861	Almeida and Campello (2007)	Bottom	0.478	Middle	0.540
	Middle	-2.861	Top	-1.798		Middle	0.540	Top	0.568
	t-stat	396.59***		262.61***		*t*-stat	60.39***		22.64***
Whited and Wu (2006)	Bottom	-0.368	Middle	-0.253	Coverage ratio	Bottom	106.108	Middle	60.762
	Middle	-0.253	Top	-0.118		Middle	60.762	Top	-11.715
	t-stat	200.83***		222.96***		*t*-stat	-2.03**		-13.89***
Kaplan and Zingales (1997)	Bottom	0.260	Middle	0.256	Sales growth	Bottom	0.075	Middle	0.114
	Middle	0.256	Top	0.326		Middle	0.114	Top	0.069
	t-stat	-0.499		8.61***		*t*-stat	20.28***		-14.38***
Size	Bottom	7.586	Middle	5.351	R&D	Bottom	0.036	Middle	0.051
	Middle	5.351	Top	2.639		Middle	0.051	Top	0.074
	t-stat	-220.00***		-250.00***		*t*-stat	19.48***		3.77***
Age	Bottom	12.724	Middle	5.851	ROA	Bottom	0.114	Middle	0.081
	Middle	5.851	Top	5.306		Middle	0.081	Top	-0.046
	t-stat	-180.00***		-13.80***		*t*-stat	-49.14***		-5.89***
Cash flow	Bottom	0.603	Middle	0.643	Debt	Bottom	0.172	Middle	0.116
	Middle	0.643	Top	0.713		Middle	0.116	Top	0.085
	t-stat	3.89***		3.90***		*t*-stat	-51.86***		-25.86***
Cash	Bottom	0.105	Middle	0.121	Equity	Bottom	0.377	Middle	0.379
	Middle	0.121	Top	0.129		Middle	0.379	Top	0.073
	t-stat	16.01***		6.12***		*t*-stat	1.344		-3.93***
Investment	Bottom	0.206	Middle	0.262	Dividend	Bottom	0.012	Middle	0.007
	Middle	0.262	Top	0.242		Middle	0.007	Top	0.005
	t-stat	31.06***		-6.67***		*t*-stat	-35.49***		-11.34***
ΔCash	Bottom	0.012	Middle	0.075	Cash Dividend	Bottom	0.022	Middle	0.035
	Middle	0.075	Top	0.108		Middle	0.035	Top	0.038
	t-stat	74.93***		20.97***		*t*-stat	41.23***		8.24***
ΔDebt	Bottom	0.236	Middle	0.264	ΔEquity	Bottom	0.077	Middle	0.170
	Middle	0.264	Top	0.252		Middle	0.170	Top	0.191
	t-stat	19.38***		-6.738***		*t*-stat	60.57***		9.92***

This table reports mean values of the four financing constraints indexes and other firm characteristics commonly used as metrics of financing constraints. Means are calculated for the first, the second and the third regime of the Hadlock and Pierce (2010) index, respectively. We test the null hypotheses that the means of the first regime are significantly different from those of the second regime, and that the latter are significantly different from those of the third regime. ***, **, and * denote statistical significance at the 1, 5, and 10% levels, respectively.

Table 9

The Shape of ICFS: Polynomial Parametric Approach and Threshold Approach with Constant Parameters

$$\left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + \sum_{r=0}^{R} \beta_r \lambda_{mit}^{r} \times \left(\frac{Cash\ flow}{K}\right)_{it} + \mu_i + \tau_t + \varepsilon_{it}$$

Grade of Polynomial	α	β_0	β_1	β_2	β_3	Adjusted R^2	F-Statistic (p-value)
Hadlock and Pierce (2010)							
0	0.0224	0.0682				38.58%	
	(0.0019)	(0.0020)					
1	0.0227	0.0487	-0.0076			38.65%	10.06
	(0.0018)	(0.0066)	(0.0024)				(0.0015)
2	0.0231	0.0048	-0.0536	-0.0103		38.84%	22.85
	(0.0018)	(0.0096)	(0.0081)	(0.0017)			(0.0000)
3	0.0230	0.0107	-0.0371	-0.0007	0.00152	38.85%	15.33
	(0.0018)	(0.0104)	(0.0155)	(0.0081)	(0.0013)		(0.0000)
Kaplan and Zingales (1997)							
2	0.0225	0.0709	0.0028	-0.0015		38.74%	16.49
	(0.0018)	(0.0022)	(0.0019)	(0.0006)			(0.0000)
Whited and Wu (2006)							
2	0.0224	0.0317	-0.3106	-0.5419		38.78%	13.60
	(0.0018)	(0.0072)	(0.0604)	(0.1161)			(0.0000)
Almeida and Campello (2007)							
2	0.0218	0.0331	0.1525	-0.1407		38.93%	7.82
	(0.0019)	(0.0094)	(0.0386)	(0.0362)			(0.0004)

$$\left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + [\beta_1 \mathbb{I}_{(\lambda_{mit} \leq s_{1m})} + \beta_2 \mathbb{I}_{(s_{1m} < \lambda_{mit} \leq s_{2m})} + \beta_3 \mathbb{I}_{(\lambda_{mit} > s_{2m})}] \left(\frac{Cash\ flow}{K}\right)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$$

	(a)	(b)	(c)	(d)	(e)	(f)
	α	β_1	β_2	β_3	Adjusted R^2	F-Statistic (p-value)
Hadlock and Pierce (2010)		$s_1 = -3.2416, s_2 = -2.3484$				
	0.0223	0.0644	0.0745	0.0616	38.67%	377.18
	(0.0019)	(0.0034)	(0.0028)	(0.0032)		(0.000)
Kaplan and Zingales (1997)		$s_1 = 0.5007, s_2 = 1.0178$				
	0.0224	0.0665	0.0811	0.0632	38.67%	388.54
	(0.0018)	(0.0022)	(0.0038)	(0.0047)		(0.000)
Whited and Wu (2006)		$s_1 = -0.2420, s_2 = -0.2220$				
	0.0222	0.0749	0.0677	0.0638	38.64%	372.10
	(0.0019)	(0.0031)	(0.0048)	(0.0025)		(0.000)
Almeida and Campello (2007)		$s_1 = 0.6001, s_2 = 0.6466$				
	0.0222	0.0690	0.0741	0.0666	38.85%	385.83
	(0.0019)	(0.0023)	(0.0047)	(0.0031)		(0.000)

The upper panel reports estimation results for the parametric investment model using the HP, KZ, WW, and AC indexes of financing constraints (M), with M = 1, 2, 3. The lower panel reports estimation results from the estimated threshold regression model, where the number and locations of the thresholds are exogenously given. For each model, all coefficient estimates are the within fixed firm and year estimates for the full unbalanced sample of 93,107 firm-year observations with cash flow ≥ – The sample period is 1990–2013. The dependent variable is investment, normalized by the beginning-of-period net capital stock. Market-to-book ratio is measured at the end of the observation year, and cash flow is measured contemporaneously with the investment decision. In the lower panel, Column (a) reports the coefficient on market-to-book ratio; Columns (b), (c), and (d) report the coefficients on ICFS for the three regimes of financing constraints, respectively. Heteroskedasticity-consistent standard errors are in parentheses.

Appendices

Appendix 1. Data

We use a large heterogeneous sample of US corporations from 1989 to 2013, starting with all US Compustat firms. From this dataset, we eliminate financial firms (SIC codes 6020–6799) and regulated utilities (SIC codes 4011–4991) because firms in these industries often have financial metrics that are not comparable to firms in other industries. The resulting sample is well diversified by sector, as measured by primary SIC code. It comprises firms in agriculture, mining, forestry, fishing, and construction (SIC codes 100–1731); manufacturing (SIC codes 2000–3990); retail and wholesale trade (SIC codes 5000–5990); and services (SIC codes 7000–8900). Observations from 1989 were used only to construct variables with lagged terms and were not used in the regressions. Firm-year observations with negative values for total assets or sales are deleted. We focus on the period 1989-2013 for comparability with the findings from earlier relevant studies.

Like Cleary, Povel, and Raith (2007) and Lyandres (2007), we use an unbalanced panel of firm-year observations. Using firm-year observations allows firms' financial status to be reclassified every year and class composition to vary over time, so as not to "neglect … the information that the financial constraints may be binding for the same firm in some years but not in others. It would be more advisable in these cases to allow firms to transit between different financial states" (Schiantarelli 1996: 78).

Our analysis includes three key firm variables: investment (*item 128*); cash flow, defined as earnings before extraordinary items and depreciation (*item 14* + *item 18*); and market-to-book ratio, calculated as book value of assets minus book value of common equity minus deferred taxes plus market value of equity, all divided by total assets [(*item 6* − *item 60* − *item 74* + (*item 199* × *item 25*)) / *item 6*]. To control for endogeneity, we use operating cash flow instead of free cash flow, as operating cash flow is not affected by financing or investment decisions. To control for heteroskedasticity due to differences in firm size, we scale both investment and cash flow by beginning-of-period net fixed assets (*item 8*). Both net fixed assets and total assets are adjusted to 2013 prices. Age is the number of years preceding the observation year that the firm has a non-missing stock price in Compustat. Size is the log of total assets. Firm sales growth is the change compared to the previous year in the firm's inflation-adjusted annual sales, and industry sales growth is the change compared to the previous year in three-digit industry inflation-adjusted annual sales. Cash is defined as cash plus short-term investments divided by total assets (*item 1 / item 6*). Dividends are total annual dividend payments over total assets [(*item 19* + *item 21*) / *item 6*]. Debt is short-term plus long-term debt divided by total assets [(*item 9* + *item 34*) / *item 6*]. Coverage ratio is beginning-of-period operating income after depreciation over beginning-of-period interest and related expenses (*lagged item 178 / lagged item 15*). R&D is defined as research and development spending over beginning-of-period total assets (*item 46 / item 6*). Return on assets is operating income before depreciation divided by total assets (*item 13 / item 6*). Total common equity is common/ordinary equity divided by total assets (*item 60 / item 6*). Free cash flow is defined as cash flow minus investment. To mitigate the effect of outliers and potential erroneous data input, we winsorize observations at the 1st and 99th percentiles for cash flow, investment, market-to-book ratio, size, and age.

Scholars have intensely debated what metric to use to capture the degree of financing constraints (Hadlock and Pierce 2010, Hoberg and Maksimovic 2015, Farre-Mensa and Ljungquist 2016, Buehlmaier and Whited 2018). Hadlock and Pierce (2010: 1912) contend that their index of financing constraints has “many advantages over other approaches, including its intuitive appeal, its independence from various theoretical assumptions, and the presence of corroborating evidence from an alternative approach.” Moreover, their index is robustly correlated with qualitative indicators of financing constraints, corroborating the evidence of Hennessy and Whited (2007). Hoberg and Maksimovic (2015) also support Hadlock and Pierce’s index (2010) by evidencing that smaller and younger firms are more likely to be equity-constrained. We adopt the three most popular metrics of financing constraints—the Hadlock and Pierce (2010), Kaplan and Zingales (1997), and Whited and Wu (2006) indexes (KZ, WW, and HP, respectively). By construction, the three indexes increase as firm financing constraints tighten.

(A.1) $Kaplan\ and\ Zingales\ (1997) = 3.13919 total\ long\ term\ debt - 1.001909 cash\ flow - 1.314759 cash - 39.36780 dividend + 0.2826389\ market\ to\ book$

(A.2) $Whited\ and\ Wu\ (2006) = 0.021 total\ long\ term\ debt - 0.091 cash\ flow -$

$0.044 size - 0.062 dividend\ positive - 0.035 growth\ sales +$

$0.102 Industry\ growth\ sales$

(A.3) $Hadlock\ and\ Pierce\ (2010) = -0.737 size + 0.043\ size^2 - 0.040\ age$

Furthermore, we construct *Tangibility* as [(*0.715 receivables* + *0.547 inventory* + *0.535 capital stock* + *cash*) / *total assets*] following Almeida and Campello (2007) and perform our analysis using this measure of tangibility (AC) as an inverse proxy for financing constraints, in line with the view that tangibility improves the firm's ability to increase external financing (Almeida and Campello 2007).

Our main analysis uses the unbalanced panel of firm-year observations with cash flow ≥ -1, as in Hadlock and Pierce (2010), but we also use other samples for robustness exercises. Table A1 reports the mean values of the main variables in our analysis. Column (a) displays mean values for the unbalanced sample of firm-year observations with cash flow ≥ -1; Column (b) gives values for the unbalanced sample with cash flow > 0; and Column (c) gives values for the balanced sample with cash flow > 0 and dividends > 0, following Fazzari, Hubbard, and Petersen (1988) and Cleary, Povel, and Raith (2007). Unsurprisingly, results in Column (c) show that the firms with positive cash flow and positive dividends form the financially healthiest sample, with the highest net fixed assets, total assets, sales, capital expenditure, market-to-book ratio, and dividends. Their low cash flow, cash stock, and debt suggest that these firms have borrowing capacity and do not need to accumulate cash. These firms' HP, KZ, and WW indexes show them to be less financially constrained than the two unbalanced samples in Columns (a) and (b). Similarly, the net fixed assets, total assets, sales, and capital expenditure of firms with cash flow > 0 are all greater than those of firms with cash flow ≥ -1, and their indexes of financing constraints are lower.

Insert Table A1

Table A2 reports the correlations across the three indexes, HP, KZ, and WW. We find that the three indexes are all significantly positively correlated with one another, which differs from the finding of Farre-Mensa and Ljungqvist (2016) that the HP index is positively correlated with the WW index and negatively correlated with the KZ index. Our correlation coefficient between HP and KZ is 0.05, which is the same as that in Hadlock and Pierce (2010), and the correlation between HP and WW is 0.82, very similar to the value of 0.80 reported by Hadlock and Pierce (2010).

Insert Table A2

Appendix 2. Sensitivity checks

Appendix 2.1. Sensitivity checks for rejection of the joint condition

Using the HP index to sort firm-year observations, we perform several tests of the robustness of the non-monotonicity conclusion. Following most prior studies, we use the beginning-of-year market-to-book ratio to proxy for Tobin's Q. Estimation results are reported in Table A3, Panel 1. Results show that if the sample separation point is at 10% of the sample, the test rejects the equality of parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$. For a sample separation point at 35% of the sample, the test does not reject the equality of parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$. Finally, when the sample separation point is sufficiently toward the right tail of the HP index distribution, the average ICFS of the lower class, $\bar{\beta}_s$, is higher than that of the upper class, $\bar{\beta}_{n-s}$. This evidence clearly continues to reject monotonicity.

However, Tobin's Q is likely to contain substantial measurement error because of the known conceptual gap between true investment opportunities and their observable measures (Erickson and Whited 2012). Poterba (1988) points out that, because measurement error in Tobin's Q can lead to a spurious correlation between investment and cash flow, one might find non-significant ICFS parameters after accounting for this measurement error. Indeed, Erickson and Whited (2000, 2002) use a generalized method of moments (GMM) estimator based on the higher-order moments of the regression variables and show that cash flow does not affect investment when the measurement error in Tobin's Q is addressed. Cummins, Hassett, and Oliner (2006) support this finding by using a GMM estimator and an analyst-forecasts-based measure of Q as a superior proxy for Tobin's Q. Ağca and Mozumdar (2017) challenge these studies: by using the methodologies of Cummins, Hassett, and Oliner (2006) and Erickson and Withed (2000, 2002), they find a significant ICFS parameter. Additionally, they find that ICFS is higher for financially constrained firms irrespective of the metric of financing constraints used.

We therefore check whether our finding of non-monotonicity is robust to measurement errors in Tobin's Q by using the Arellano and Bond (1991) two-step difference GMM estimator as in Cummins, Hassett, and Oliner (2006) and Ağca and Mozumdar (2017). We use the standard stock-market-based measure of Q because evidence shows that an analyst-forecasts-based measure of Q is not superior (Ağca and Mozumdar 2017). As reported in Table A3, Panel 2, our results show that a GMM estimator with finite lags of Q and cash flow as instruments yields a non-monotonic ICFS parameter.[3]

ICFS studies using the stock-market-based measure of Q (i.e., the market-to-book ratio as a proxy for Tobin's Q) scale the regression variables by different measures of capital stock: investment and cash flow are scaled by net capital stock, whereas market value is scaled by book value of total assets (Kaplan and Zingales 1997). Although we follow this practice, we note that this parametrization of the model is somewhat inconsistent with Q theory and may drive unnecessary heteroskedasticity or mechanical correlations (Hayashi and Inoue 1991, Erickson and Whited 2012). Moreover, net capital stock (PPENT) is a potentially problematic variable because there are numerous built-in depreciation issues to consider. Erickson and Whited (2012) suggest scaling investment and cash flow by gross capital stock (PPEGT). Furthermore, some researchers follow Fazzari, Hubbard, and Petersen (1988) by proxying Tobin's Q with an average Q value based on the replacement cost of capital. The two proxies for Tobin's Q are quite different: the market-to-book ratio isolates variations in investment opportunities relative to total assets, whereas the average Q isolates variations in investment opportunities for property, plant, and equipment (Erickson and Whited 2012). Therefore, as robustness check, we estimate our investment model using average Q, scaled consistently with Q theory. Like Erickson and Whited (2000), we measure Tobin's Q as short-term plus long-term debt plus market

[3]We are aware of the different GMM approaches proposed for addressing the issue of appropriate instruments in ICFS estimation (Erickson and Withed 2002, Cummins, Hassett, and Oliner 2006, Ağca and Mozumdar 2017). Our major difficulty lies in comparing estimated parameters from 19 models. Given the sensitivity of parameters obtained from the GMM approach to the set of instruments, it is hard to envisage a clean, robust strategy for the set of instruments that satisfies the assumptions in all models. We therefore follow the empirical approach proposed by Ağca and Mozumdar (2017), currently regarded as the most suitable for estimating ICFS. They suggest using a two-step difference GMM estimator with long lags of market-to-book ratio and cash flow as instruments. In our exercises, all assumptions about the set of instruments are satisfied. However, we recommend cautious interpretation of these results.

value of equity minus book value of current assets, and we normalize Q, investment, and cash flow by gross capital stock. As reported in Table A3, Panel 3, the results again confirm the rejection of monotonicity.

Insert Table A3

The finding that ICFS is non-monotonic is conditional on several assumptions imposed at the outset. We therefore check whether this finding depends on the quality of the sample, the sample period, and the specification of the estimating model. We start with the sample under analysis. To systematically exclude financially weaker firms, many empirical studies of ICFS take only observations with positive cash flow or use the balanced sample (Fazzari, Hubbard, and Petersen 1988, Gilchrist and Himmelberg 1995, Kaplan and Zingales 1997, Cleary 1999, Cleary, Povel, and Raith 2007). In line with this approach, to investigate whether our finding of non-monotonicity changes with the average financial health of firms under analysis, we estimate our main model for both the sample of firm-year observations with positive cash flow and for the balanced sample. As displayed in the upper two panels of Table A4, the results confirm that the rejection of monotonicity is robust to the sample under analysis.

In addition, some studies report evidence of a change in the sensitivity of investment to cash flow during the global financial crisis of 2007–2009, although there is no consensus on the direction of this change. McLean and Zhao (2014) find that ICFS increased during the crisis, which exacerbated financing constraints, whereas Chen and Chen (2012) find that ICFS almost disappeared during the crisis, regardless of the firm's financial strength. More relevant to our analysis of monotonicity is Allayannis and Mozumdar's (2004) hypothesis that if the impact of financing constraints on firm investment declines over time, then ICFS may be almost the same across different classes of financing constraints. Because our sample period includes the financial crisis, we perform separate analyses for the pre-crisis (1990–2007) and post-crisis (2008–2013) periods. The results, reported in the lower two panels of Table A4, continue to reject monotonicity for both periods.

Insert Table A4

Finally, in re-examining the estimated model, it is worth noting that the model in Equation (12) includes the parameter γ_s , which allows for differences between the average investment value of class

s and that of its complement, n-s. If the average investment of class s is equal to that of n-s, the investment model can be specified as:

$$(A4)\quad \left(\frac{I}{K}\right)_{i,t} = \alpha Q_{i,t} + \bar{\beta}_s D_s \left(\frac{Cash\ flow}{K}\right)_{i,t} + \bar{\beta}_{n-s} D_{n-s} \left(\frac{Cash\ flow}{K}\right)_{i,t} + \mu_i + \tau_t + \varepsilon_{i,t}\ .$$

In addition, if the impact of Q is different across classes s and n-s, the investment model in Equation (12) is:

$$(A5)\quad \left(\frac{I}{K}\right)_{i,t} = \gamma_s D_s + \bar{\alpha}_s D_s Q_{i,t} + \bar{\alpha}_{n-s} D_{n-s} Q_{i,t} + \bar{\beta}_s D_s \left(\frac{Cash\ flow}{K}\right)_{i,t}$$
$$+ \bar{\beta}_{n-s} D_{n-s} \left(\frac{Cash\ flow}{K}\right)_{i,t} + \mu_i + \tau_t + \varepsilon_{i,t}\ .$$

The estimation results for the models in Equation (A4) and Equation (A5) are reported in Table A5. They again confirm the violation of monotonicity because the direction of inequality between ICFS parameters $\bar{\beta}_s$ and $\bar{\beta}_{n-s}$ changes with the sample separation point.

Insert Table A5

Appendix 2.2. Sensitivity checks for rejection of monotonicity of the sorting scheme

Insert Table A6

Insert Table A7

Appendix 2.3. Sensitivity checks of the shape to the location of regime parameters

In the threshold regression analysis with predetermined thresholds, the statistical uncertainty regarding the location of the two sample separation points, s_1 and s_2, is not adequately considered. Therefore, we check the robustness of our conclusion on the shape of ICFS in two ways. First, we estimate the threshold regression model by changing the location of each predetermined threshold parameter by ±2.5% of the observations. For brevity, we perform this exercise using the HP metric of financing constraints only. The results, reported in the upper panel of Table A8, show that when we change the threshold parameters one at a time by ±2.5% of the observations, the findings $\beta_2 > 0,\ \beta_4 = 0,$ and $\beta_6 <$

0 do not change, and ICFS is still inverse basin shaped. Second, we add the metric of financing constraints to the set of regressors and estimate this augmented threshold regression model for each metric of financing constraints. The results, displayed in the lower panel of Table A8, again confirm that ICFS is inverse basin shaped regardless of the financing constraints metric adopted.

Insert Table A8

Appendix 3. Shapes under alternative assumptions of the estimating model

Insert Figure A1

Insert Figure A2

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Table A.1
Descriptive Statistics

Variables	(a) *Cash flow* ≥ -1	(b) *Cash flow* > 0	(c) *Cash flow* > 0, *Balanced, and Dividend* > 0
Net fixed assets	1,085.93	1,261.33	4,219.43
Total assets	3,132.77	3,633.39	11,621.92
Sales	2,915.55	3,387.75	11,850.20
Capital expenditure	166.64	194.47	555.39
Market-to-book	1.41	1.41	1.48
Cash flow	0.64	0.84	0.45
Cash	0.12	0.12	0.07
Dividends	0.01	0.01	0.02
Debt	0.20	0.17	0.16
R&D	0.05	0.04	0.02
Hadlock and Pierce (2010)	-2.89	-2.97	-3.46
Kaplan and Zingales (1997)	0.27	0.20	-0.25
Whited and Wu (2006)	-0.27	-0.28	-0.41
Almeida and Campello (2007)	0.52	0.52	0.46
# Obs.	93,107.00	77,086.00	3,720.00

This table displays the mean values of the main variables used in our empirical analysis. Columns (a), (b), and (c) report the means for the samples with *cash flow* ≥ -1, with *cash flow* > 0, and with *cash flow* > 0 *dividends* > 0 after balancing, respectively. Variables are constructed using Compustat data items as follows: Net fixed assets is net property, plant, and equipment (item 8); Total assets is the book value of total assets (item 6); Net fixed assets and total assets figures are in million dollars, inflation-adjusted to 2013 values; Sales is inflation-adjusted sales (item 12); Capital expenditure is a firm's annual capital expenditure (item 128); Market-to-book ratio is calculated as book value of assets minus book value of common equity minus deferred taxes plus market value of equity, all divided by book value of assets (item 6 – item 60 − item 74 + (item 24 x item 25)) / item 6); Cash flow is earnings before extraordinary items and depreciation (item 18 + item 14) divided by the beginning-of-year net capital stock (lagged item 8); Cash is defined as cash plus short-term investment (item 1), divided by book value of assets (item 6); Dividends are the total annual dividend payments (item 19 + item 21) divided by book value of assets (item 6); Debt is defined as short-term plus long-term debt (item 9 + item 34) divided by book value of assets (item 6); R&D is the research and development expense (item 46) divided by the beginning-of-year book assets (lagged item 6); Tangibility is defined as in Almeida and Campello (2007) as 0.715 receivables + 0.547 inventory + 0.535 capital stock + cash, divided by book value of assets (item 6). See Section 1 for definitions of the Kaplan and Zingales (1997), Whited and Wu (2006), and Hadlock and Pierce (2010) metrics of financing constraints.

Table A.2
Correlation Analysis

	(a) Hadlock and Pierce (2010)	(b) Kaplan and Zingales (1997)
Number of observations = 93,107		
Kaplan and Zingales (1997)	0.0536*	
Whited and Wu (2006)	0.8218*	0.1590*

This table reports the correlations between the metrics of financing constraints attributable to Kaplan and Zingales (1997), Whited and Wu (2006), and Hadlock and Pierce (2010), for the full unbalanced sample of firm-year observations with *cash flow* ≥ -1. * indicates statistical significance at the 1% level.

Table A.3
Monotonicity Condition and Measurement of Market-to-Book Ratio

(a)	(b)	(c)	(d)	(e)	(f)	(g)
Position of s (%)	γ_s	α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic
Panel 1. Market-to-book $_{t-1}$						
10	0.0148	0.0380	0.0469	0.0639	38.44%	18.00***
	(0.0043)	(0.0021)	(0.0039)	(0.0023)		
35	-0.0035	0.0380	0.0606	0.0634	38.40%	0.49
	(0.0037)	(0.0021)	(0.0035)	(0.0025)		
85	0.0564	0.0381	0.0643	0.0569	38.60%	2.88*
	(0.0066)	(0.0021)	(0.0025)	(0.0039)		
Panel 2. Measurement errors in market-to-book ratio					J-Test	F-Statistic
10	0.2864	0.2459	0.0276	0.1112	0.146	3.73**
	(0.0836)	(0.0815)	(0.0177)	(0.0377)		
20	0.2430	0.1806	0.0733	0.1461	0.347	1.42
	(0.0829)	(0.1023)	(0.0497)	(0.0435)		
85	-1.17821	0.1469	0.1907	-0.09185	0.903	8.39***
	(0.3165)	(0.0940)	(0.0499)	(0.1002)		
Panel 3. Consistent scaling and Erickson and Whited (2000) proxy for Tobin's Q						
15	-0.0006	-0.0009	0.0998	0.1284	23.09***	34.74%
	(0.0023)	(0.0001)	(0.0058)	(0.0027)		
45	0.0111	-0.0009	0.1226	0.1257	0.49	34.67%
	(0.0020)	(0.0001)	(0.0039)	(0.0031)		
90	0.0485	-0.0009	0.1266	0.0778	57.80***	35.27%
	(0.0034)	(0.0001)	(0.0028)	(0.0059)		

This table presents results from estimating the investment model in Eq. (14) for the full unbalanced sample of 93,107 firm-year observations with cash flow ≥ -1. In Panel 1, the coefficient estimates are the within fixed firm and year estimates, and market-to-book ratio is measured at the beginning of the observation year. In Panel 2, we control for measurement errors in the market-to-book ratio using the Arellano and Bond (1991) two-step difference GMM estimator, including time dummies and lags (6–9) of market-to-book ratio and cash flow as instruments. In Panel 3, the coefficient estimates are again the within fixed firm and year estimates; however, both investment and cash flow are normalized by the beginning-of-period gross capital stock (*ppegt*), and we use the Erickson and Whited (2000) proxy for Tobin's Q, calculated as short-term plus long-term debt plus market value of equity minus book value of current assets, all normalized by annual gross capital stock (i.e., $[dltt + dlc - act + (prcc\_f * csho)] / ppegt$). In all panels, observations are sorted by the Hadlock and Pierce (2010) index of financing constraints. Column (a) reports the point of sample separation. Column (b) reports the coefficient on the class of observations $i \leq s$. Column (c) reports the coefficient on market-to-book ratio. Columns (d) and (e) report the coefficients on ICFS for the classes of observations $i \leq s$ and $i > s$, respectively. Column (f) reports the adjusted R^2 (or the p-value for the Hansen J-Test of overidentifying restrictions), and Column (g) reports the F-statistic testing the null hypothesis of equality of parameters in Columns (d) and (e). Heteroskedasticity-consistent standard errors are reported in parentheses.

Table A.4

Monotonicity Condition and Quality of the Sample

(a)	(b)	(c)	(d)	(e)	(f)	(g)
Position of s (%)	γ_s	α	$\bar{\beta}_s$	$\bar{\beta}_{n-s}$	Adjusted R^2	F-Statistic
Unbalanced sample. Cash flow > 0. # of obs: 77,086						
25	0.0142	0.0265	0.0627	0.0720	42.67%	4.61**
	(0.0042)	(0.0024)	(0.0040)	(0.0026)		
60	-0.0048	0.0266	0.0703	0.0706	42.64%	0.01
	(0.0045)	(0.0024)	(0.0030)	(0.0030)		
95	0.0423	0.0275	0.0730	0.0524	42.82%	11.44***
	(0.0112)	(0.0023)	(0.0025)	(0.0058)		
Balanced sample. # of obs: 9,672						
25	0.0117	0.0248	0.0618	0.0886	33.73%	3.84**
	(0.0101)	(0.0054)	(0.0102)	(0.0167)		
60	0.0078	0.0264	0.0687	0.0929	33.68%	1.03
	(0.0108)	(0.0051)	(0.0109)	(0.0250)		
95	0.0308	0.0257	0.0754	0.1348	33.79%	9.55***
	(0.0158)	(0.0050)	(0.0115)	(0.0196)		
Year ≤ 2007. # of obs: 73,475						
25	0.0041	0.0221	0.0640	0.0764	39.32%	6.60***
	(0.0040)	(0.0021)	(0.0046)	(0.0027)		
50	0.0077	0.0223	0.0764	0.0743	39.30%	0.19
	(0.0047)	(0.0021)	(0.0039)	(0.0030)		
95	0.0858	0.0237	0.0765	0.0579	39.54%	7.27***
	(0.0110)	(0.0021)	(0.0027)	(0.0066)		
Year > 2007. # of obs: 19,632						
20	0.0241	0.0132	0.0457	0.0609	48.01%	3.23*
	(0.0084)	(0.0053)	(0.0085)	(0.0042)		
50	0.0341	0.0131	0.0648	0.0584	48.06%	0.80
	(0.0117)	(0.0052)	(0.0064)	(0.0048)		
95	0.0725	0.0136	0.0625	0.0418	48.15%	3.43*
	(0.0305)	(0.0052)	(0.0044)	(0.0107)		

This table presents results from estimating the investment model in Eq. (14). Coefficient estimates are the within fixed firm and year estimates for different samples: the unbalanced sample of firm-year observations with cash flow > 0; the balanced sample of firm-year observations; and two different sample periods, 1990–2007 and 2008–2013. The dependent variable is investment, normalized by beginning-of-period net capital stock. Market-to-book ratio is measured at the end of the observation year, and cash flow is measured contemporaneously with the investment decision. Observations are sorted by the Hadlock and Pierce (2010) index of financing constraints. Columns (a), (b), and (c) report the point of sample separation, the coefficient on the class of observations $i \leq s$, and the coefficient on market-to-book ratio, respectively. Columns (d) and (e) report coefficients on ICFS for the classes of observations $i \leq s$ and $i > s$, respectively. Column (f) reports the adjusted R^2. Column (g) reports the F-statistic testing the null hypothesis of equality of parameters in Columns (d) and (e). Heteroskedasticity-consistent standard errors are in parentheses.

Table A.5
Monotonicity Condition and Estimating Model

Estimating Model

(a) Position of s (%)	(b) α	(c) $\bar{\beta}_s$	(d) $\bar{\beta}_{n-s}$	(e) Adjusted R^2	(f) F-Statistic
15	0.0224 (0.0019)	0.0560 (0.0039)	0.0695 (0.0021)	38.63%	11.31***
60	0.0224 (0.0019)	0.0695 (0.0026)	0.0674 (0.0025)	38.59%	0.44
90	0.0225 (0.0018)	0.0720 (0.0022)	0.0523 (0.0039)	38.70%	21.18***

(a) Position of s (%)	(b) γ_s	(c) $\overline{\alpha}_s$	(d) $\overline{\alpha}_{n-s}$	(e) $\bar{\beta}_s$	(f) $\bar{\beta}_{n-s}$	(g) Adjusted R^2	(h) F-Statistic
15	0.0433 (0.0053)	0.0131 (0.0027)	0.0230 (0.0020)	0.0529 (0.0042)	0.0700 (0.0021)	38.68%	14.43***
60	-0.0133 (0.0056)	0.0315 (0.0023)	0.0173 (0.0023)	0.0675 (0.0027)	0.0684 (0.0026)	38.64%	0.06
90	0.0187 (0.0086)	0.0348 (0.0020)	0.0022 (0.0027)	0.0690 (0.0022)	0.0584 (0.0039)	39.12%	6.05**

This table presents results from estimating the investment models in Equations (A4) and (A5). Coefficient estimates are the within fixed firm and year estimates for the full unbalanced sample of 93,107 firm-year observations. The sample period is 1990–2013. The dependent variable is investment, normalized by beginning-of-period net capital stock. Market-to-book ratio is measured at the end of the observation year, and cash flow is measured contemporaneously with the investment decision. Observations are sorted by the Hadlock and Pierce (2010) index of financing constraints. For each model, we report the adjusted R^2 and the F-statistic testing the null hypothesis of equality of the two cash flow parameters. Heteroskedasticity-consistent standard errors are in parentheses.

Table A.6

Are Sorting Metrics Monotonic? Selected Percentiles

		Percentiles											
Subsample		5	10	15	20	30	40	50	60	70	80	90	95
Panel 1: Hadlock and Pierce (2010)													
Hadlock and Pierce (2010)	f	-3.92	-3.82	-3.74	-3.68	-3.57	-3.48	-3.40	-3.32	-3.24	-3.15	-3.05	-2.98
	c	-2.83	-2.79	-2.74	-2.69	-2.60	-2.49	-2.37	-2.24	-2.07	-1.84	-1.47	-1.12
	diff	1.08***	1.03***	1.00***	0.98***	0 .97***	0.99***	1.02***	1.08***	1.17***	1.31***	1.58***	1.86***
Whited and Wu (2006)	f	-0.40	-0.39	-0.39	-0.38	-0.37	-0.36	-0.35	-0.33	-0.32	-0.30	-0.29	-0.28
	c	-0.26	-0.25	-0.24	-0.24	-0.22	-0.20	-0.18	-0.17	-0.15	-0.12	-0.09	-0.06
	diff	0.15***	0.14***	0.14***	0.14***	0.15***	0.16***	0.16***	0.17***	0.17***	0.18***	0.20***	0.22***
Kaplan and Zingales (1997)	f	0.25	0.24	0.25	0.25	0.26	0.27	0.28	0.27	0.26	0.26	0.26	0.26
	c	0.27	0.27	0.27	0.27	0.28	0.27	0.26	0.27	0.29	0.32	0.41	0.51
	diff	0.02*	0.03***	0.03***	0.02***	0.01***	0.00	-0.01*	-0.001	0.02***	0.06***	0.15***	0.25***
Almeida and Campello (2007)	f	0.45	0.46	0.46	0.47	0.48	0.48	0.49	0.50	0.51	0.51	0.52	0.52
	c	0.53	0.53	0.54	0.54	0.55	0.55	0.56	0.56	0.57	0.57	0.57	0.57
	diff	0.08***	0.08***	0.07***	0.07***	0.07***	0.07***	0.07***	0.07***	0.06***	0.05***	0.05***	0.05***
Panel 2: Kaplan and Zingales (1997)													
Kaplan and Zingales (1997)	f	-2.454	-1.616	-1.212	-0.963	-0.657	-0.464	-0.322	-0.205	-0.098	0.006	0.116	0.180
	c	0.414	0.480	0.532	0.579	0.668	0.760	0.863	0.983	1.131	1.330	1.664	1.992
	diff	2.87***	2.10***	1.74***	1.54***	1.32***	1.22***	1.18***	1.19***	1.23***	1.32***	1.55***	1.81***
Hadlock and Pierce (2010)	f	-2.984	-3.014	-2.994	-2.974	-2.933	-2.902	-2.886	-2.878	-2.877	-2.883	-2.893	-2.897
	c	-2.884	-2.875	-2.870	-2.867	-2.870	-2.880	-2.891	-2.905	-2.915	-2.912	-2.853	-2.738
	diff	0.10***	0.14***	0.12***	0.11***	0.06***	0.02***	-0.00	-0.03***	-0.04***	-0.03***	0.04***	0.16***
Whited and Wu (2006)	f	-0.316	-0.321	-0.313	-0.305	-0.291	-0.281	-0.276	-0.272	-0.270	-0.269	-0.269	-0.268
	c	-0.263	-0.259	-0.257	-0.256	-0.255	-0.255	-0.256	-0.256	-0.254	-0.251	-0.238	-0.220
	diff	0.05***	0.06***	0.06***	0.05***	0.04***	0.03***	0.02***	0.02***	0.02***	0.02***	0.03***	0.05***
Almeida and Campello (2007)	f	0.560	0.563	0.575	0.579	0.576	0.566	0.557	0.550	0.543	0.537	0.530	0.526
	c	0.523	0.520	0.516	0.511	0.502	0.496	0.492	0.487	0.482	0.476	0.478	0.487
	diff	-0.04***	-0.04***	-0.06***	-0.07***	-0.07***	-0.07***	-0.07***	-0.06***	-0.06***	-0.06***	-0.05***	-0.04***

Table A.6-continued

Panel 3: Whited and Wu (2006)													
Whited and Wu (2006)	f	-0.511	-0.477	-0.453	-0.434	-0.403	-0.379	-0.358	-0.339	-0.321	-0.304	-0.287	-0.277
	c	-0.253	-0.242	-0.233	-0.224	-0.207	-0.190	-0.173	-0.156	-0.136	-0.111	-0.076	-0.045
	diff	0.26***	0.23***	0.22***	0.21***	0.20***	0.19***	0.18***	0.18***	0.19***	0.19***	0.21***	0.23***
Hadlock and Pierce (2010)	f	-3.447	-3.510	-3.522	-3.512	-3.467	-3.413	-3.351	-3.284	-3.213	-3.132	-3.037	-2.976
	c	-2.859	-2.820	-2.777	-2.733	-2.641	-2.539	-2.427	-2.295	-2.133	-1.917	-1.558	-1.231
	diff	0.59***	0.69***	0.75***	0.78***	0.83***	0.87***	0.92***	0.99***	1.08***	1.22***	1.48***	1.74***
Kaplan and Zingales (1997)	f	-0.111	-0.024	0.021	0.069	0.115	0.155	0.187	0.206	0.216	0.226	0.240	0.248
	c	0.291	0.303	0.315	0.321	0.337	0.348	0.354	0.368	0.397	0.448	0.548	0.692
	diff	0.40***	0.33***	0.29***	0.25***	0.22***	0.19***	0.17***	0.16***	0.18***	0.22***	0.31***	0.44***
Almeida and Campello (2007)	f	0.445	0.450	0.456	0.462	0.473	0.483	0.491	0.500	0.508	0.514	0.520	0.522
	c	0.529	0.533	0.536	0.540	0.546	0.552	0.558	0.562	0.564	0.565	0.568	0.566
	diff	0.08***	0.08***	0.08***	0.08***	0.07***	0.07***	0.07***	0.06***	0.06***	0.05***	0.05***	0.04***
Panel 4: Almeida and Campello (2007)													
Almeida and Campello (2007)	f	0.208	0.257	0.292	0.319	0.360	0.392	0.417	0.438	0.456	0.474	0.495	0.507
	c	0.541	0.554	0.565	0.576	0.595	0.613	0.632	0.654	0.684	0.725	0.793	0.847
	diff	0.33***	0.30***	0.27***	0.26***	0.23***	0.22***	0.21***	0.22***	0.23***	0.25***	0.30***	0.34***
Hadlock and Pierce (2010)	f	-3.087	-3.123	-3.120	-3.115	-3.096	-3.072	-3.051	-3.020	-2.986	-2.952	-2.920	-2.903
	c	-2.878	-2.863	-2.848	-2.832	-2.800	-2.766	-2.727	-2.693	-2.663	-2.637	-2.609	-2.626
	diff	0.21***	0.26***	0.27***	0.28***	0.30***	0.31***	0.32***	0.33***	0.32***	0.31***	0.31***	0.28***
Kaplan and Zingales (1997)	f	0.564	0.524	0.497	0.480	0.446	0.422	0.411	0.401	0.382	0.352	0.311	0.289
	c	0.255	0.242	0.230	0.218	0.195	0.169	0.130	0.075	0.011	-0.057	-0.097	-0.074
	diff	-0.31***	-0.28***	-0.27***	-0.26***	-0.25***	-0.25***	-0.28***	-0.33***	-0.37***	-0.41***	-0.41***	-0.36***
Whited and Wu (2006)	f	-0.304	-0.307	-0.307	-0.306	-0.304	-0.300	-0.295	-0.290	-0.283	-0.277	-0.271	-0.268
	c	-0.264	-0.261	-0.258	-0.256	-0.249	-0.243	-0.236	-0.230	-0.224	-0.219	-0.214	-0.218
	diff	0.04***	0.05***	0.05***	0.05***	0.05***	0.06***	0.06***	0.06***	0.06***	0.06***	0.06***	0.05***

This table reports estimated subsample mean values for the four metrics of financing constraints used in our analysis. Means are estimated by sorting observations according to the KZ (Panel 1), WW (Panel 2), or AC index (Panel 3), then regressing the metric of financing constraints on two dummy variables corresponding to the first (f) 5% (10, 15, 20, etc). and the remaining (c) 95% (90, 85, 80, etc.) of the sorting metric. ***, **, and * denote significance of the difference (diff) between means at the 1, 5, and 10% levels, respectively.

Table A.7

Monotonicity of Sorting Metrics with Respect to the Hadlock and Pierce (2010) Index of Financing Constraints: Selected Percentiles

	Subsample	Percentiles											
		5	10	15	20	30	40	50	60	70	80	90	95
Hadlock and Pierce (2010)	f	-3.92	-3.82	-3.74	-3.68	-3.57	-3.48	-3.40	-3.32	-3.24	-3.15	-3.05	-2.98
	c	-2.83	-2.79	-2.74	-2.69	-2.60	-2.49	-2.37	-2.24	-2.07	-1.84	-1.47	-1.12
	diff	1.08***	1.03***	1.00***	0.98***	0 .97***	0.99***	1.02***	1.08***	1.17***	1.31***	1.58***	1.86***
Size	f	8.24	8.08	7.95	7.84	7.64	7.47	7.20	6.91	6.61	6.30	5.98	5.81
	c	5.45	5.31	5.17	5.02	4.71	4.33	3.97	3.61	3.21	2.72	2.03	1.42
	diff	-2.80***	-2.77***	-2.78***	-2.81***	-2.94***	-3.14***	-3.23***	-3.30***	-3.40***	-3.58***	-3.96***	-4.39***
Age	f	20.40	18.36	16.78	15.44	13.28	11.58	10.50	9.74	9.15	8.70	8.35	8.19
	c	7.37	6.88	6.48	6.17	5.77	5.65	5.55	5.45	5.40	5.32	5.13	4.93
	diff	-13.03***	-11.49***	-10.30***	-9.27***	-7.51***	-5.92***	-4.95***	-4.29***	-3.75***	-3.38***	-3.21***	-3.261***
Cash	f	0.11	0.11	0.11	0.11	0.11	0.10	0.10	0.11	0.11	0.11	0.12	0.12
	c	0.12	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.13	0.13	0.13
	diff	0.01***	0.00***	0.00***	0.01***	0.01***	0.02***	0.02***	0.02***	0.02***	0.01***	0.01***	0.01***
Coverage ratio	f	93.10	90.99	94.30	133.89	110.20	99.57	95.08	92.14	88.48	82.65	75.43	71.27
	c	65.51	63.92	61.24	46.15	43.35	39.24	31.57	19.98	5.55	-8.55	-23.72	-31.31
	diff	-27.59	-27.07	-33.06**	-87.74**	-66.86***	-60.33***	-63.51***	-72.15***	-82.93***	-91.20***	-99.14***	-102.57***
Debt	f	0.21	0.19	0.18	0.18	0.17	0.17	0.16	0.16	0.15	0.14	0.13	0.13
	c	0.12	0.12	0.12	0.12	0.11	0.10	0.09	0.09	0.08	0.08	0.09	0.09
	diff	-0.01 ***	-0.07***	-0.06***	-0.06***	-0.06***	-0.07***	-0.07***	-0.07***	-0.06***	-0.05***	-0.04***	-0.04***
Equity	f	0.41	0.41	0.40	0.39	0.38	0.37	0.37	0.37	0.38	0.38	0.38	0.37
	c	0.32	0.31	0.31	0.30	0.30	0.29	0.28	0.25	0.20	0.10	-0.18	-0.67
	diff	-0.09***	-0.09***	-0.09***	-0.08***	-0.08***	-0.08***	-0.09***	-0.12***	-0.18***	-0.28***	-0.55***	-1.05***
Dividends	f	0.015	0.014	0.014	0.013	0.012	0.012	0.011	0.010	0.010	0.009	0.009	0.008
	c	0.008	0.008	0.007	0.007	0.007	0.006	0.006	0.005	0.005	0.005	0.005	0.005
	diff	-0.007***	-0.007***	-0.006***	-0.006***	-0.006***	-0.006***	-0.005***	-0.005***	-0.005***	-0.004***	-0.004***	-0.004***

This table reports subsample estimated mean values of the most commonly used metrics of financing constraints. Means are estimated by sorting observations according to the HP index and then regressing the metric of financing constraints on two dummy variables corresponding to the first (f) 5% (10, 15, 20, etc.) and the remaining (c) 95% (90, 85, 80, etc.) of the Hadlock and Pierce (2010) index. ***, **, and * denote statistical significance of the difference (diff) between the two parameters at the 1, 5, and 10% levels, respectively.

Table A.8

The Shape of ICFS: Robustness to the Location of Threshold Parameters and Financing Constraints Metric

(a)	(b)	(c)	(d)	(f)	(g)	(h)	(i)	(l)	(m)
α	β_1	β_2	β_3	β_4	β_5	β_6	Adjusted R^2	F-Statistic (p-value)	θ
Hadlock and Pierce (2010)									
$\left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + [(\beta_1+\beta_4 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit}\leq s_{1m})} + \beta_2\mathbb{I}_{(s_{1m}<T\lambda_{mit}\leq s_{2m})} + (\beta_3+\beta_6 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit}>s_{2m})}](Cash\ flow/k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$									
Sample separation points: s_1 + 2.5% of obs $= -3.2109, s_2 = -2.3484$									
0.0229	0.0491	0.1443	0.0752		0.1203	-0.1242	38.84%	228.35	
(0.0018)	(0.0062)	(0.0481)	(0.0028)		(0.0122)	(0.0246)		(0.000)	
Sample separation points: s_1 − 2.5% of obs $= -3.2745, s_2 = -2.3484$									
0.0229	0.0534	0.0882	0.0757		0.1204	-0.1243	38.85%	228.70	
(0.0018)	(0.0063)	(0.0547)	(0.0028)		(0.0122)	(0.0246)		(0.000)	
Sample separation points: $s_1 = -3.2416, s_2$ + 2.5% of obs $= -2.2615$									
0.0229	0.0510	0.1206	0.0758		0.1184	-0.1212	38.85%	227.92	
(0.0018)	(0.0063)	(0.0506)	(0.0027)		(0.0131)	(0.0261)		(0.000)	
Sample separation points: $s_1 = -3.2416, s_2$ − 2.5% of obs $= -2.4228$									
0.0229	0.0509	0.1199	0.0753		0.1205	-0.1244	38.85%	228.94	
(0.0018)	(0.0063)	(0.0506)	(0.0029)		(0.0106)	(0.0225)		(0.000)	
$\left(\frac{I}{K}\right)_{it} = \alpha Q_{it} + [(\beta_1+\beta_4 T\lambda_{mit})\mathbb{I}_{(\lambda_{mit}\leq s_{1m})} + \beta_2\mathbb{I}_{(s_{1m}<T\lambda_{mit}\leq s_{2m})} + (\beta_3+\beta_6 T\lambda_{mit})\mathbb{I}_{(T\lambda_{mit}>s_{2m})}](Cash\ flow/k)_{it} + \lambda_m(k)_{it} + \mu_i + \tau_t + \varepsilon_{i,t}$									
Hadlock and Pierce (2010)									
Sample separation points: $s_1 = -3.2416, s_2 = -2.3484$									
0.0274	0.0477	0.1153	0.0739		0.1139	-0.1053	39.28%	223.47	-0.1024
(0.0018)	(0.0061)	(0.0495)	(0.0028)		(0.0119)	(0.0240)		(0.000)	(0.0068)
Kaplan and Zingales (1997)									
Sample separation points: $s_1 = 0.5007, s_2 = 1.0178$									
0.0223	0.0404	0.0496	0.0823		0.1522	-0.1023	38.89%	246.98	0.0071
(0.0019)	(0.0058)	(0.0098)	(0.0038)		(0.0445)	(0.0517)		(0.000)	(0.0016)
Whited and Wu (2006)									
Sample separation points: $s_1 = -0.24203, s_2 = -0.22199$									
0.0241	0.0218	0.1142	0.0679		0.1481	-0.1321	39.25%	205.94	-0.5533
(0.0018)	(0.0109)	(0.0265)	(0.0049)		(0.0197)	(0.0305)		(0.000)	(0.0342)
Almeida and Campello (2007)									
Sample separation points: $s_1 = 0.60008, s_2 = 0.64659$									
0.0215	0.0584	0.0296	0.0756		0.1265	-0.0698	38.94%	234.59	0.0127
(0.0019)	(0.0044)	(0.0105)	(0.0047)		(0.0221)	(0.0256)		(0.000)	(0.0145)

The upper panel displays estimation results for the threshold regression analysis when we shift the two predetermined thresholds, one at a time, to the left or right by 2.5% of observations in the sample and take the corresponding value of the location parameter as the predetermined threshold. In the lower panel, we instead control the ICFS estimates for the financing constraint metric (M), whose coefficients are reported in Column (m). Coefficients are the within-group estimates for the full unbalanced sample of firm-year observations with cash flow≥-1. The sample period is 1990–2013. The dependent variable is investment, normalized by the beginning-of-period net capital stock. Market-to-book ratio is measured at the end of the observation year, and cash flow is measured contemporaneously with the investment decision. Column (a) reports the coefficient on market-to-book ratio; Columns (b), (d), and (g) report coefficients on ICFS for the three regimes, respectively. Columns (c), (f), and (h) report coefficients on the interactions between ICFS and TM, a monotonic transformation of the financing constraints metric (M). Heteroskedasticity-consistent standard errors are in parentheses.

Figure A.1

The Inverse U-Shaped ICFS: Results from the Polynomial Parametric Approach

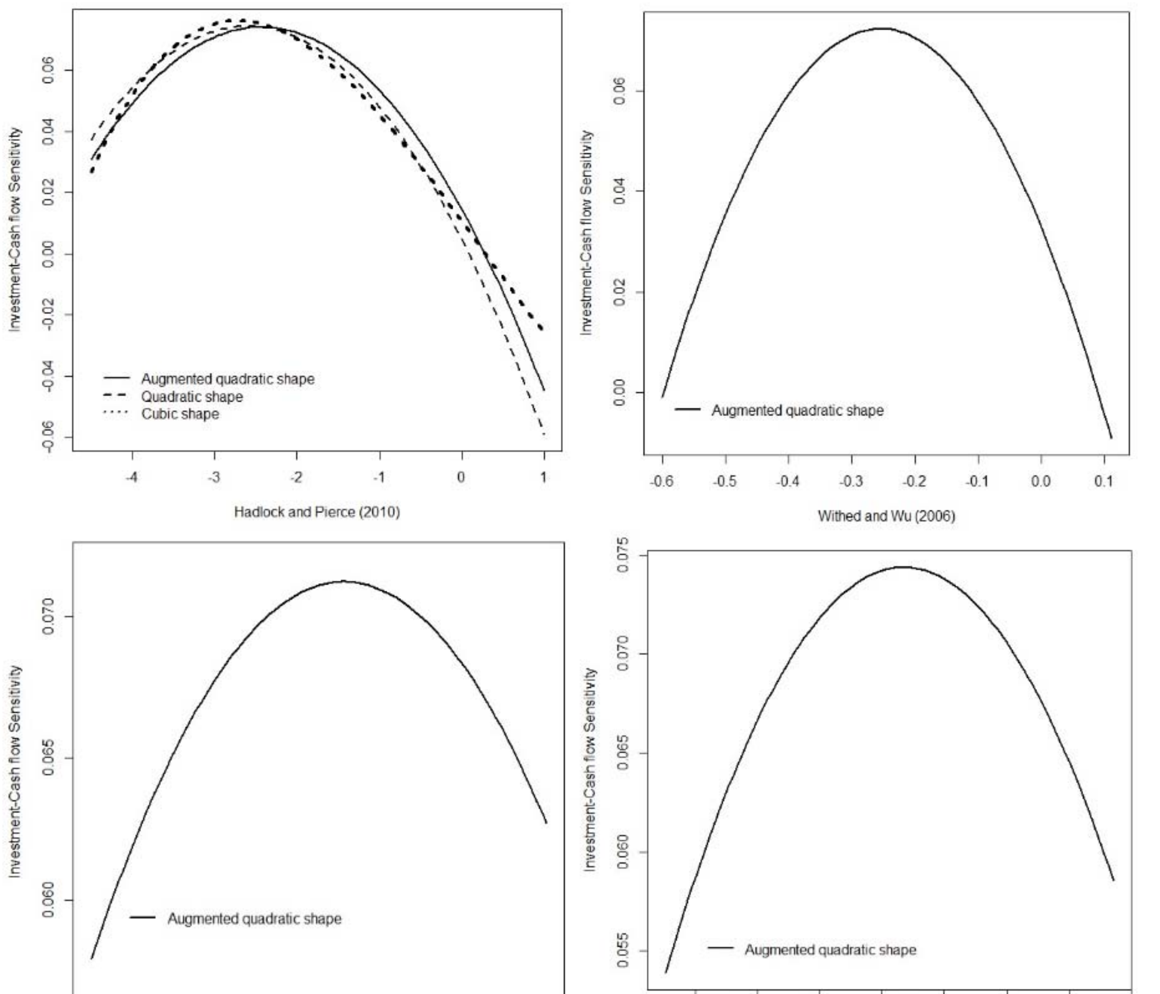
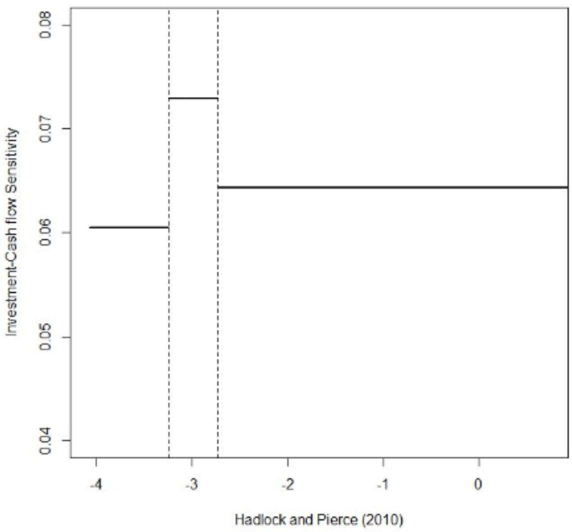


Figure A.2

The Shape of ICFS: Results from the Threshold Approach



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